Pricing Model Performance and the Two-Pass Cross-Sectional Regression Methodology

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Related Literature

Fama and MacBeth (1973) statistical methodology

- Tests of expected return linearity – include other variables
- Various multivariate tests against general alternative

Many use CSR R^2 for average returns as measure of goodness-of-fit

- Models are at best useful approximations
- R^2 descriptive statistic – point estimate, no formal inference
- Lewellen, Nagle, Shanken (2010) explore inference about R^2 through simulations

We derive asymptotic distribution of the CSR R^2

- Obtain distribution of difference of R^2s for competing models
- Do some multiple model comparison as well

Asset-Pricing Model Comparison

(CAPM): $\mu_2 = \gamma_0 + \beta_{vw}\gamma_{vw}$

(C-LAB): conditional CAPM of Jagannathan and Wang (1996) with labor income

$$\mu_2 = \gamma_0 + \beta_{vw}\gamma_{vw} + \beta_{lab}\gamma_{lab} + \beta_{prem}\gamma_{prem}$$

(FF3): Fama-French (1993) three-factor model

$$\mu_2 = \gamma_0 + \beta_{vw}\gamma_{vw} + \beta_{smb}\gamma_{smb} + \beta_{hml}\gamma_{hml}$$

(ICAPM): Petkova (2006) specification of Merton's (1973) intertemporal model

$$\mu_2 = \gamma_0 + \beta_{vw}\gamma_{vw} + \beta_{term}\gamma_{term} + \beta_{def}\gamma_{def} + \beta_{div}\gamma_{div} + \beta_{rf}\gamma_{rf}$$

Asset-Pricing Model Comparison 2

(CCAPM): unconditional consumption model

$$\mu_2 = \gamma_0 + \beta_{cg}\gamma_{cg}$$

(CC-CAY): conditional version of CCAPM due to Lettau and Ludvigson (2001)

$$\mu_2 = \gamma_0 + \beta_{cay}\gamma_{cay} + \beta_{cg}\gamma_{cg} + \beta_{cg \cdot cay}\gamma_{cg \cdot cay}$$

(U-CCAPM): ultimate consumption model of Parker and Julliard (2005)

$$\mu_2 = \gamma_0 + \beta_{cg36}\gamma_{cg36}$$

(D-CCAPM): durable consumption model of Yogo (2006)

$$\mu_2 = \gamma_0 + \beta_{vw}\gamma_{vw} + \beta_{cg}\gamma_{cg} + \beta_{cgdur}\gamma_{cgdur}$$

Notation

$$Y = [f',\ R']'$$

$$\begin{aligned} \mu &= E[Y] \equiv \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \\ V &= \mathrm{Var}[Y] \equiv \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \end{aligned}$$

$$\beta = V_{21} V_{11}^{-1} \qquad X = [1_N,\ \beta]$$

$$\boxed{\mu_2 = X\gamma}$$

$$\gamma_W \equiv \begin{bmatrix} \gamma_{W,0} \\ \gamma_{W,1} \end{bmatrix} = \mathrm{argmin}_\gamma (\mu_2 - X\gamma)' W (\mu_2 - X\gamma) = (X'WX)^{-1} X' W \mu_2$$

Notation 2

$$e_W = \mu_2 - X\gamma_W$$

$$Q = e_W' W e_W \qquad Q_0 = e_0' W e_0$$

$$\boxed{\rho_W^2 = 1 - \frac{Q}{Q_0}}$$

$$C = [1_N, \ V_{21}]$$

$$\lambda_W \equiv \begin{bmatrix} \lambda_{W,0} \\ \lambda_{W,1} \end{bmatrix} = \text{argmin}_\lambda (\mu_2 - C\lambda)' W (\mu_2 - C\lambda) = (C'WC)^{-1} C' W \mu_2$$

Distribution of the CSR Estimator and R^2

Fama-MacBeth – gives variance with no beta estimation error under exact linearity of expected returns in betas

EIV adjustment term - Shanken (1992), Jagannathan and Wang (1998)

Misspecification-adjustment term – Shanken and Zhou (2007)

All generalized here under very weak assumptions

CSR R^2 asymptotically normal when $0 < \rho^2 < 1$

Complicated asymptotic distributions at the extremes, $\rho^2 = 0$ or $\rho^2 = 1$

Comparing Competing Models

Let f_1, f_2, and f_3 be three sets of distinct factors

Model A uses f_1 and f_2 , Model B uses f_1 and f_3 as factors

When $K_2 > 0$ and $K_3 > 0$, the two models are non-nested

When $K_3 = 0$, model A nests model B

With **nested models**

$\rho^2_{\mathrm{A}} = \rho^2_{\mathrm{B}}$ if and only if $\lambda_{\mathrm{A},2} = 0_{\mathrm{K}2}$

CSR coefficient on the simple beta (not the usual multiple regression beta) indicates whether a factor adds to model explanatory power

Tests based on asymptotic distribution of $\hat{\rho}^2_{\mathrm{A}} - \hat{\rho}^2_{\mathrm{B}}$ under null $\rho^2_{\mathrm{A}} = \rho^2_{\mathrm{B}}$

Test of Equivalent Performance for Non-Nested Models

In general, 3 different ways we can have $\rho_A^2 = \rho_B^2$

Both models perfect even though factors differ: $\rho_A^2 = \rho_B^2 = 1$

The non-common factors are totally irrelevant for each model, so pricing errors are identical

Non-common factors relevant but both models are still imperfect: $0 < \rho_A^2 = \rho_B^2 < 1$

Three different asymptotic distributions possible for $\hat{\rho}_A^2 - \hat{\rho}_B^2$

General test requires sequential inference

We focus mainly on the last scenario – normal distribution

Test Portfolios

Asset Returns: February 1959 to July 2007 (T=582)

Main Analysis:

25 FF size-B/M portfolios + 5 industry portfolios

Robustness tests:

25 FF size-B/M portfolios + 5 industry portfolios + 3 FF3 factors

25 FF size-B/M portfolios (fewer rejections in model comparison)

25 size-beta portfolios

TABLE 1

SAMPLE CROSS-SECTIONAL R^2S AND SPECIFICATION TESTS OF THE MODELS

PANEL A: OLS

	CAPM	C-LAB	FF3	ICAPM	CCAPM	CC-CAY	U-CCAPM	D-CCAPM
$\hat{\rho}^2$	0.115	0.548	0.747	0.766	0.044	0.366	0.473	0.772
$p(\rho^2 = 1)$	0.000	0.051	0.002	0.327	0.000	0.001	0.116	0.237
$p(\rho^2 = 0)$	0.258	0.042	0.009	0.009	0.510	0.256	0.005	0.007
$\text{se}(\hat{\rho}^2)$	0.200	0.221	0.117	0.145	0.130	0.211	0.244	0.125
$\hat{Q}_c$	0.131	0.060	0.098	0.058	0.137	0.102	0.100	0.067
$p(Q_c = 0)$	0.000	0.170	0.001	0.135	0.000	0.001	0.002	0.077
No. of par.	2	4	4	6	2	4	2	4

PANEL B: GLS

	CAPM	C-LAB	FF3	ICAPM	CCAPM	CC-CAY	U-CCAPM	D-CCAPM
$\hat{\rho}^2$	0.107	0.109	0.298	0.242	0.011	0.015	0.034	0.239
$p(\rho^2 = 1)$	0.000	0.000	0.001	0.024	0.000	0.000	0.000	0.007
$p(\rho^2 = 0)$	0.005	0.337	0.000	0.076	0.547	0.933	0.280	0.016
$\text{se}(\hat{\rho}^2)$	0.069	0.071	0.101	0.137	0.036	0.040	0.059	0.133
$\hat{Q}_c$	0.126	0.128	0.099	0.086	0.143	0.141	0.149	0.084
$p(Q_c = 0)$	0.000	0.000	0.001	0.004	0.000	0.000	0.000	0.010
No. of par.	2	4	4	6	2	4	2	4

Risk-Premia Coefficients - Gammas

Ultimate and durable consumption factors *cg*36 and *cgdur*, value-growth factor *hml* and *prem* state variable reliably positive at 5% level

TABLE 2
ESTIMATES AND t-RATIOS OF ZERO-BETA RATE AND RISK PREMIA
PANEL B: GLS

	ICAPM						U-CCAPM	
	$\hat{\gamma}_0$	$\hat{\gamma}_{vw}$	$\hat{\gamma}_{term}$	$\hat{\gamma}_{def}$	$\hat{\gamma}_{div}$	$\hat{\gamma}_{rf}$	$\hat{\gamma}_0$	$\hat{\gamma}_{cg36}$
Estimate	1.48	−0.51	0.16	−0.04	0.01	−0.27	1.28	0.92
t-ratio$_{fm}$	4.71	−1.82	3.07	−0.98	0.45	−2.66	6.13	1.72
t-ratio$_s$	3.93	−1.62	2.58	−0.82	0.39	−2.24	5.91	1.66
t-ratio$_{jw}$	3.89	−1.61	2.59	−0.74	0.37	−2.09	5.97	1.73
t-ratio$_{pm}$	2.94	−1.31	1.61	−0.55	0.27	−1.58	5.27	1.08

Risk-Premia Coefficients FF3 (OLS)

	FF3			
	$\hat{\gamma}_0$	$\hat{\gamma}_{vw}$	$\hat{\gamma}_{smb}$	$\hat{\gamma}_{hml}$
Estimate	1.94	−0.95	0.16	0.41
t-ratio$_{fm}$	5.64	−3.00	1.18	3.41
t-ratio$_s$	5.45	−2.93	1.18	3.41
t-ratio$_{jw}$	5.53	−2.93	1.19	3.44
t-ratio$_{pm}$	5.17	−2.75	1.19	3.42

	FF3			
	$\hat{\lambda}_0$	$\hat{\lambda}_{vw}$	$\hat{\lambda}_{smb}$	$\hat{\lambda}_{hml}$
Estimate	1.94	−5.25	4.63	3.33
t-ratio$_{pm}$	5.17	−2.25	2.79	1.60

TABLE 4
Tests of Equality of Cross-Sectional R^2s

Panel A: OLS

	C-LAB	FF3	ICAPM	CCAPM	CC-CAY	U-CCAPM	D-CCAPM
CAPM	−0.432 (0.034)	−0.631 (0.001)	−0.651 (0.055)	0.072 (0.818)	−0.251 (0.440)	−0.358 (0.367)	−0.657 (0.009)
C-LAB		−0.199 (0.341)	−0.219 (0.369)	0.504 (0.066)	0.182 (0.484)	0.075 (0.812)	−0.224 (0.306)
FF3			−0.020 (0.865)	0.703 (0.000)	0.380 (0.031)	0.274 (0.226)	−0.025 (0.742)
ICAPM				0.723 (0.000)	0.400 (0.067)	0.293 (0.279)	0.006 (0.967)
CCAPM					−0.322 (0.171)	−0.429 (0.032)	−0.728 (0.002)
CC-CAY						−0.107 (0.701)	−0.406 (0.037)
U-CCAPM							−0.299 (0.199)

$R^2_{row} - R^2_{column}$

Panel B: GLS

	C-LAB	FF3	ICAPM	CCAPM	CC-CAY	U-CCAPM	D-CCAPM
CAPM	−0.002 (0.980)	−0.191 (0.001)	−0.135 (0.418)	0.096 (0.268)	0.092 (0.303)	0.074 (0.452)	−0.132 (0.141)
C-LAB		−0.189 (0.025)	−0.133 (0.325)	0.098 (0.263)	0.094 (0.295)	0.075 (0.433)	−0.130 (0.257)
FF3			0.055 (0.696)	0.287 (0.008)	0.283 (0.008)	0.264 (0.021)	0.059 (0.608)
ICAPM				0.231 (0.110)	0.227 (0.117)	0.209 (0.170)	0.003 (0.986)
CCAPM					−0.004 (0.950)	−0.023 (0.715)	−0.228 (0.008)
CC-CAY						−0.019 (0.764)	−0.224 (0.065)
U-CCAPM							−0.205 (0.140)

$R^2_{row} - R^2_{column}$

Additional Results

A misspecified model will perform better on some test assets than others

Explore robustness using **25 size-beta portfolios**

CC-CAY OLS R^2 now 87.4% (36.6% earlier), about same as ICAPM, and dominates FF3 at 5% level

CC-CAY highest GLS R^2 43.2% and only model not rejected

In earlier analysis,

Zero-beta rates ranged from 0.96% to 2.2% per month

Coefficient on market beta always negative and often significant

Should model be given “credit” for explanatory power in these cases?

Explore *excess-returns specification* with **excess zero-beta rate constrained to equal 0**

CSR Analysis with Constrained Zero-Beta Rate

Unconstrained OLS: D-CCAPM was top model, followed by ICAPM and FF3

Unconstrained GLS: FF3 was top model, followed by ICAPM and D-CCAPM

TABLE 5

SAMPLE CROSS-SECTIONAL R^2S AND SPECIFICATION TESTS OF THE MODELS USING EXCESS RETURNS

PANEL A: OLS

	CAPM	C-LAB	FF3	ICAPM	CCAPM	CC-CAY	U-CCAPM	D-CCAPM
$\hat{\rho}^2$	0.858	0.893	0.958	0.972	0.880	0.886	0.946	0.883
$p(\rho^2 = 1)$	0.000	0.010	0.000	0.414	0.006	0.003	0.358	0.001

PANEL B: GLS

	CAPM	C-LAB	FF3	ICAPM	CCAPM	CC-CAY	U-CCAPM	D-CCAPM
$\hat{\rho}^2$	0.058	0.091	0.274	0.339	0.044	0.105	0.110	0.083
$p(\rho^2 = 1)$	0.000	0.000	0.000	0.075	0.000	0.000	0.000	0.000

Now D-CCAPM in 6th place and *cgdur* no longer significantly priced

Constrained Model Comparison Results

FF3 dominates CAPM (OLS and GLS), CCAPM and D-CCAPM (both GLS) at the 1% level; almost beats C-LAB (GLS) at 5% level

ICAPM almost dominates CAPM (OLS and GLS) at 5% level

FF3 dominates more models statistically, yet ICAPM has higher R^2s

- ICAPM has highest standard error of GLS R^2
- About twice as large as FF3 standard error

Conclusions

ICAPM and FF3 stand out as the best performing models

ICAPM never dominated but doesn’t dominate other models very often

FF3 frequently dominates other models, but shows vulnerability with size-beta portfolios

Important to take precision into account in comparing R^2 statistics

Simulations

25 FF size-B/M portfolios + 5 industry portfolios, T = 600

Report rejection rates for nominal 5% level tests

Specification tests (FF3 true null)

R^2-based test: (5.0% OLS, 7.8% GLS)

F-test: (5.5% OLS, 5.6% GLS)

Tests of $\rho^2 = 0$ (FF3 true null) and nested model comparison test

All correct size

Normal test for equality of true R^2s (FF3 and C-LAB equal under null)

(5.8% OLS, 2.7% GLS)

Multiple model comparison inequality test (all models same under null)

(3.3% to 5% OLS, 2.7% to 6% GLS)

Multiple Model Comparison

Searching for significant results overstates statistical significance

True p-value larger than nominal level

Ex. FF3 dominates C-LAB (p-value 2.5%) with GLS estimation

Can we reject C-LAB from multiple model comparison perspective?

Develop test of joint hypothesis that a given model is at least as good as a set of p alternative models

Let ρ_i^2 = population CSR R^2 of model i and let $\delta = (\delta_2, \ldots, \delta_p)$, where $\delta_i^2 = \rho_1^2 - \rho_i^2$

H_0: $\delta \geq 0_r$ with $r = p-1$ Model 1 is not dominated by any other model

Multiple Model Comparison 2

Non-nested models:

Adapt likelihood ratio test of Wolak (1987, 1989):

Consider

$$\min_{\delta}(\hat{\delta} - \delta)'\hat{\Sigma}_{\hat{\delta}}^{-1}(\hat{\delta} - \delta) \quad \text{s.t.} \quad \delta \geq 0_r$$

LR = T*min has “chi-bar” distribution under null

Nested models: e.g., CCAPM in CC-CAY and D-CCAPM

Create single expanded alternative model: *cg, cay, cay*cg, vw, cgdur* and use earlier pairwise model comparison test

TABLE 6
Multiple Model Comparison Tests

Panel A: OLS

Benchmark	$\hat{\rho}^2$	r	LR	p-value	s	$\hat{\rho}_M^2 - \hat{\rho}^2$	p-value
CAPM	0.115	2	0.844	0.259	4	0.734	0.057
C-LAB	0.548	5	1.056	0.330			
FF3	0.747	5	0.129	0.901			
ICAPM	0.766	5	0.002	0.825			
CCAPM	0.044	4	21.12	0.000	2	0.733	0.009
CC-CAY	0.366	5	4.728	0.059			
U-CCAPM	0.473	5	1.646	0.222			
D-CCAPM	0.772	5	0.000	0.921			

Panel B: GLS

Benchmark	$\hat{\rho}^2$	r	LR	p-value	s	$\hat{\rho}_M^2 - \hat{\rho}^2$	p-value
CAPM	0.107	2	0.000	0.607	4	0.337	0.458
C-LAB	0.109	5	5.399	0.073			
FF3	0.298	5	0.000	0.866			
ICAPM	0.242	5	0.153	0.567			
CCAPM	0.011	4	7.349	0.021	2	0.248	0.092
CC-CAY	0.015	5	7.695	0.025			
U-CCAPM	0.034	5	5.456	0.053			
D-CCAPM	0.239	5	0.264	0.563			

Multiple comparison tests also confirm the decline of D-CCAPM with excess returns, and the decline of FF3 with size-beta portfolios (rejections at 5% level)