# A Bayesian Approach to Real Options:

## The Case of Distinguishing between Temporary and Permanent Shocks

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**Implications:**

- Fixed upper trigger
- Record-setting news principle
- Maturity structure of project cash flows is irrelevant, for any given PV

# This Paper: A Bayesian Approach

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- Consider a specific application:
  - Cash flow shocks can be fundamental (**permanent**) and non-fundamental (**temporary**)
  - Firm is unable to distinguish the nature of past shocks
  - As time passes, the firm updates its prior about past shocks: the longer a shock persists, the more likely it was permanent.

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- **Implications for investment dynamics:**
  - investment trigger depends on timing of past shocks
  - investment not only in booms, but also at times of stable or decreasing cash flows
  - sluggish response to cash flow shocks
  - maturity structure of cash flows matters even for projects with the same PVs

# Key Assumptions: Motivation

1. **Both permanent and temporary shocks are important:**
   - Cash flows are more volatile than asset values
   - Correlation between cash flows and asset values is far from being perfect
   - Many shocks revert to mean

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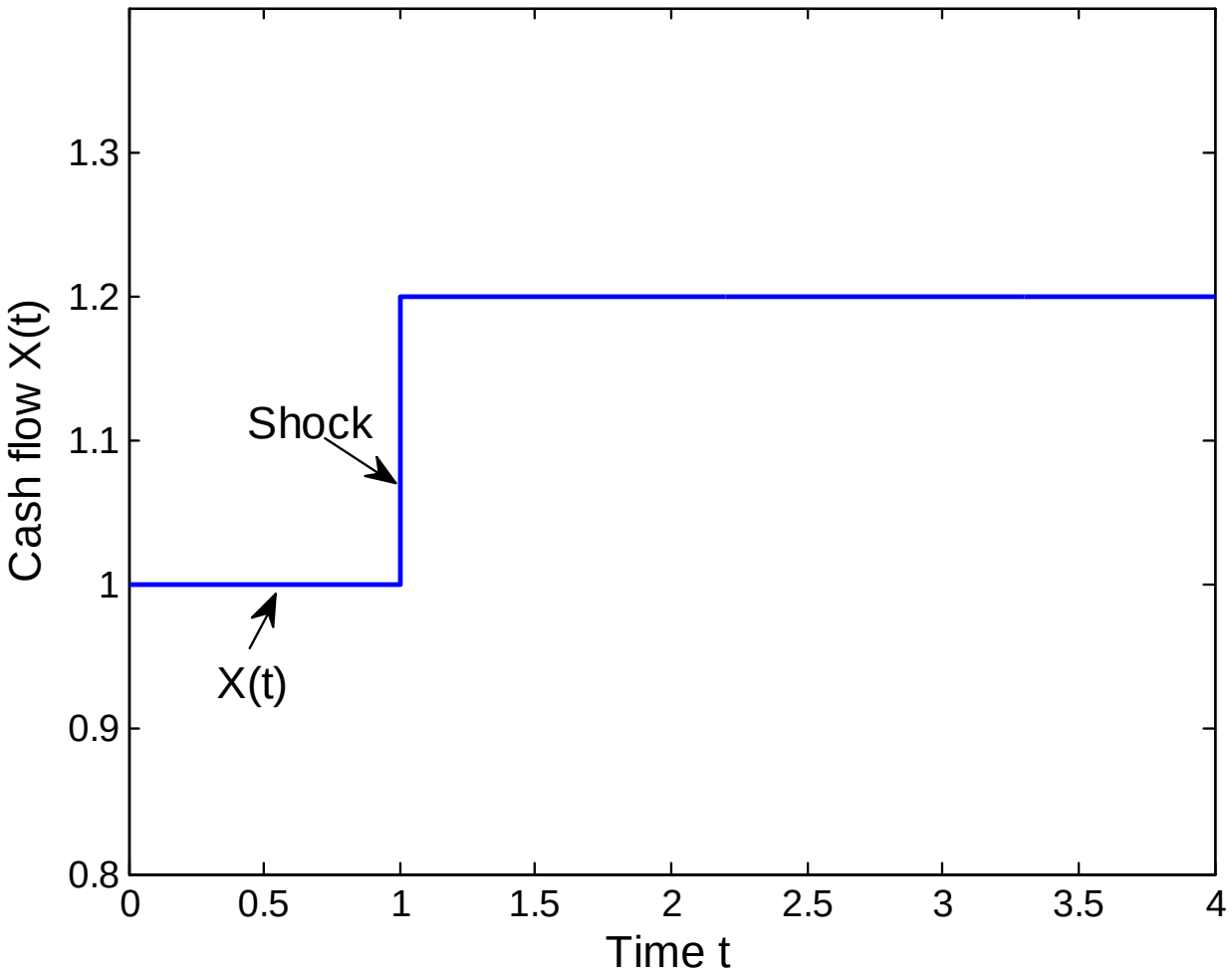
2. **Distinguishing between permanent and temporary shocks is often difficult:**

*"Most important, distinguishing between temporary and permanent shocks to commodity prices can be extraordinary difficult. The swings in commodity prices can be too large and uncertain to ascertain their causes and nature. The degree of uncertainty about duration of a price shock varies. For example, market participants could see that the sharp jump in coffee prices caused by the Brazilian frost of 1994 was likely to be reversed, assuming a return to more normal weather. By contrast, most analysts assumed that the high oil prices during the mid-1970s and early 1980s would last indefinitely."*

(World Bank's Global Economic Prospects annual report)

# Research Idea: Illustration



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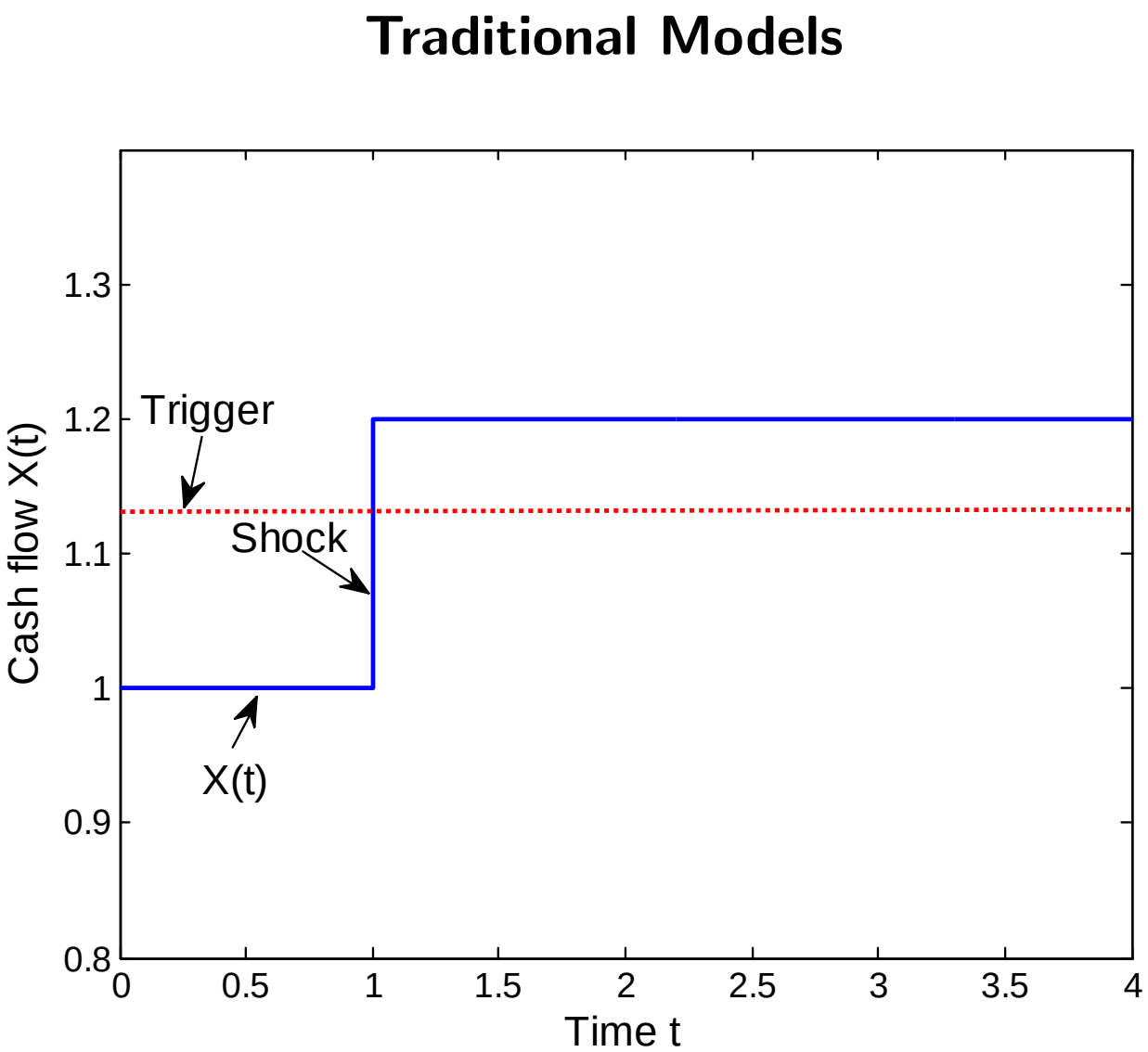

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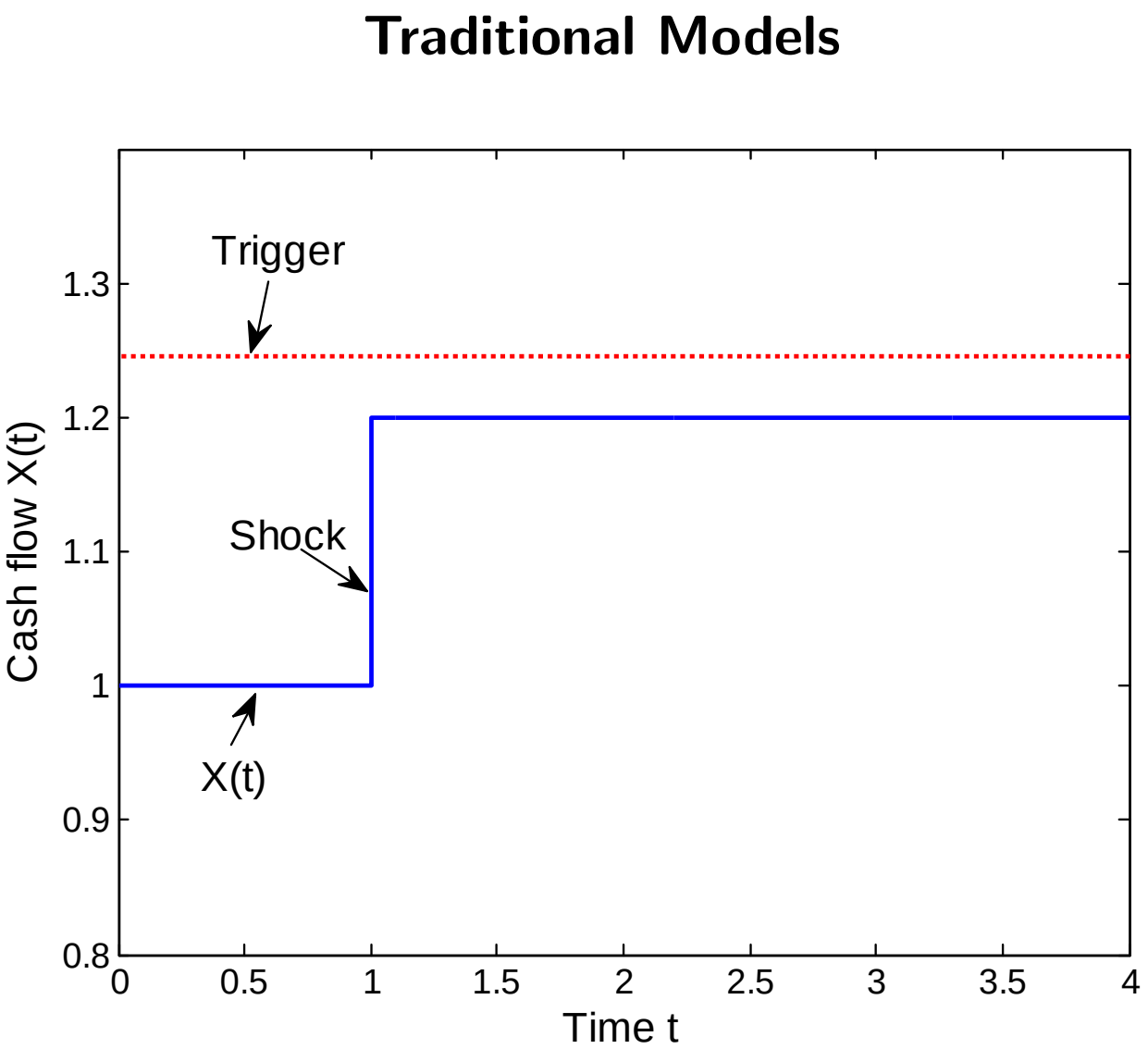

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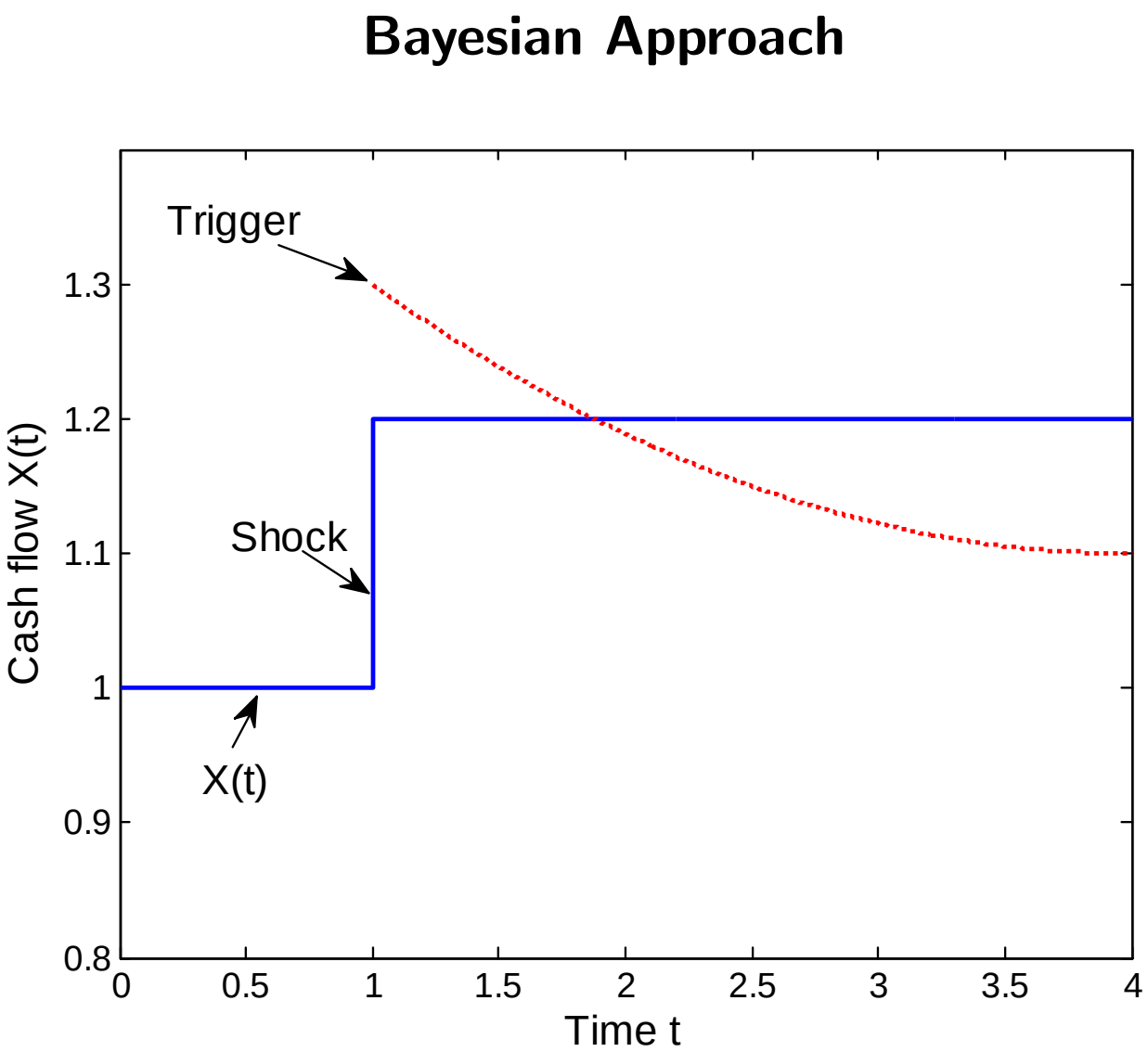

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  - $dX(t)$ can also have drift ($\alpha X dt$) and diffusion ($\sigma X dB_t$) components

# Bayesian Learning Process

- $p(t)$: conditional probability of the past shock being temporary
- Using Bayes rule:

$$dp(t) = -\lambda_3 p(t)(1 - p(t))\,dt \text{ with } p(t_0) = \frac{\lambda_2}{\lambda_1 + \lambda_2}$$

- Hence,

$$p(t) = \frac{\lambda_2}{\lambda_1 e^{\lambda_3 (t - t_0)} + \lambda_2}$$

# Investment Trigger

$$\bar{X}(p) = rI + \overbrace{\frac{\sigma^2}{2}\bar{X}(p)^2 G_{XX}\left(\bar{X}(p), p\right)}^{\text{Brownian effect}}$$
$$+\underbrace{p\lambda_3\left[H\left(\frac{\bar{X}(p)}{1+\varphi}\right) - \left(\frac{\bar{X}(p)}{(1+\varphi)(r-\alpha)} - I\right)\right]}_{\text{Bayesian effect}}$$

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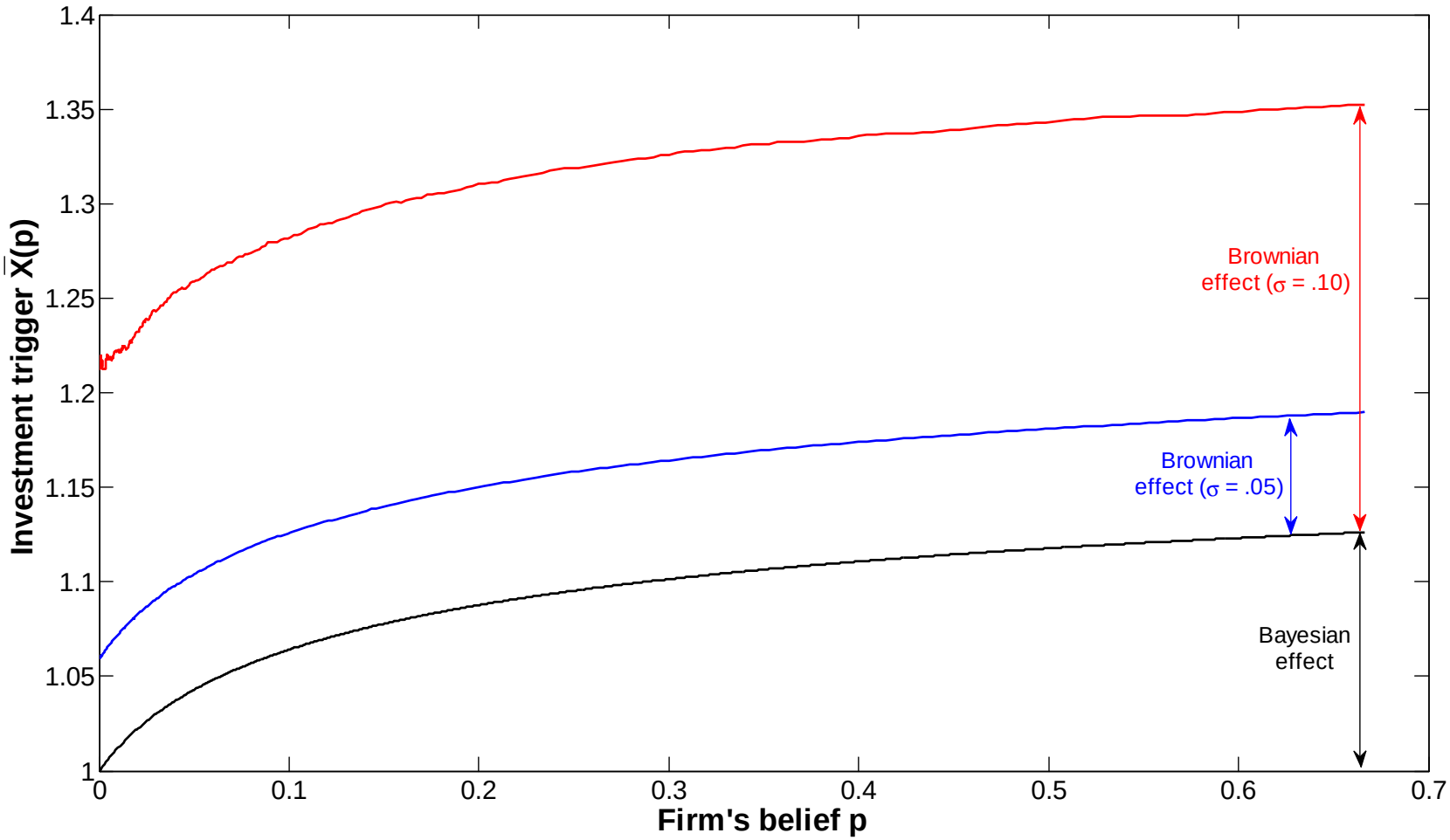

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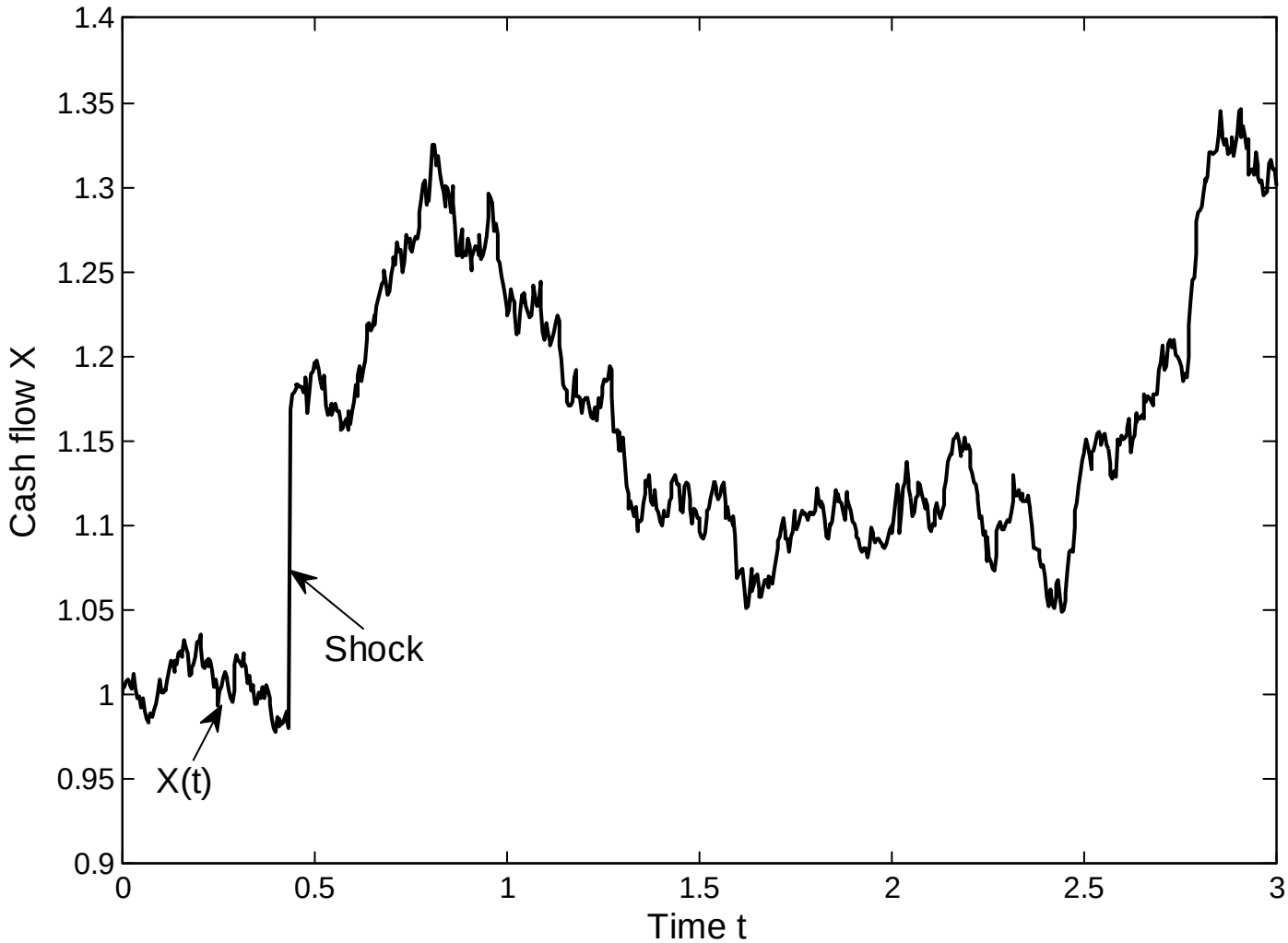

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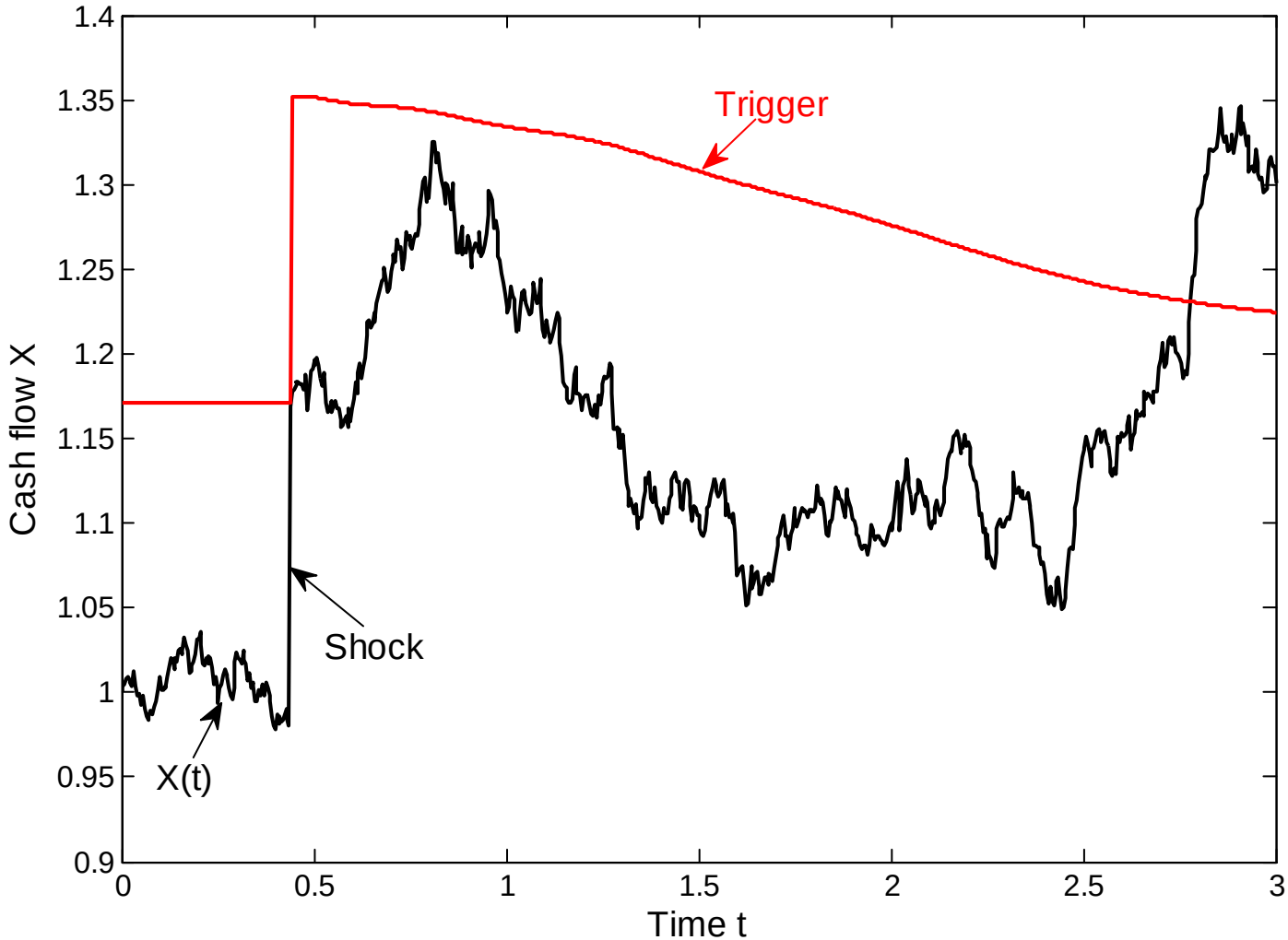

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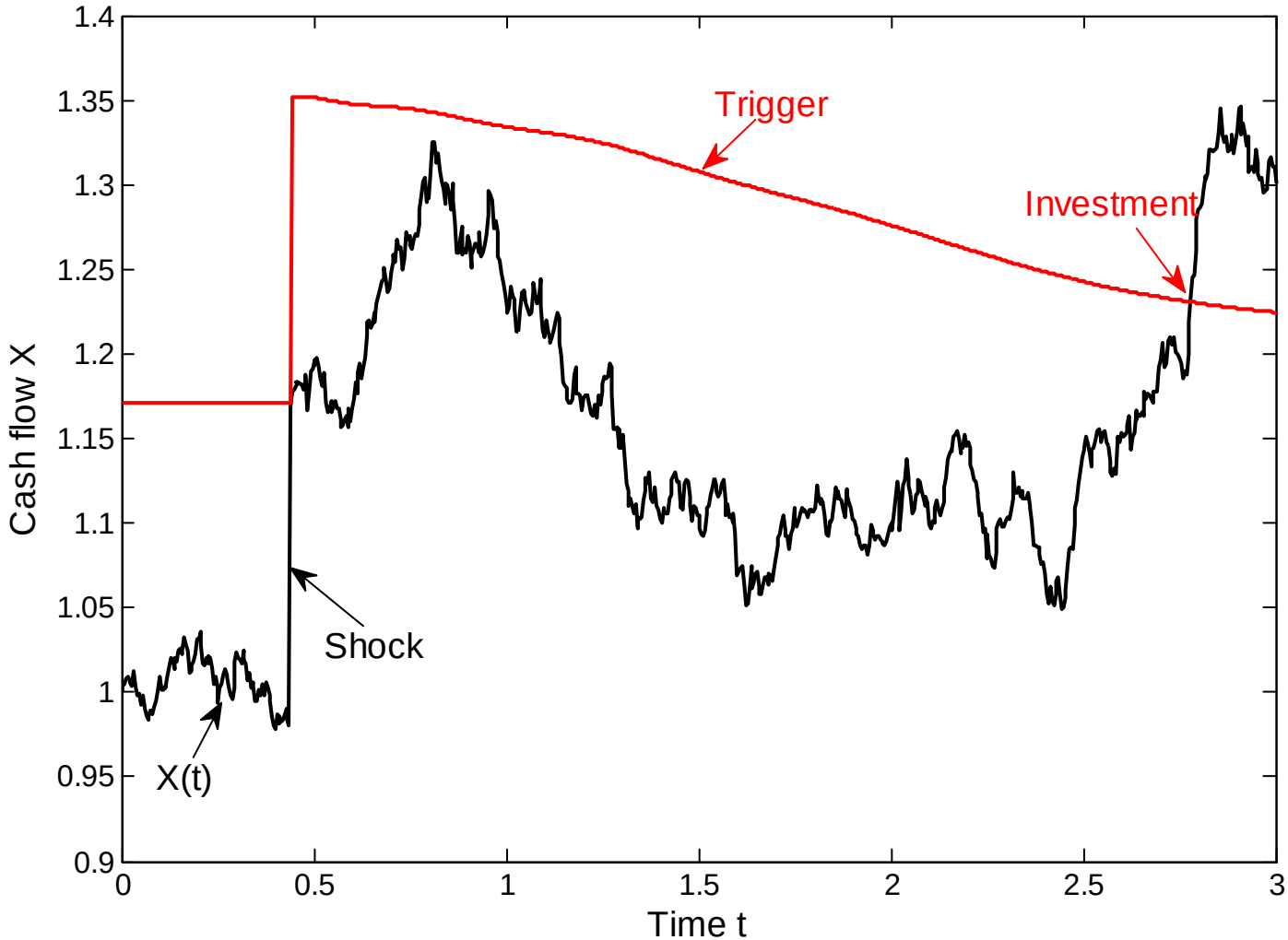

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- **Different levels of lumpyness of project cash flows**
  - Investment in a more "front-loaded" project occurs earlier *even when the projects are identical in other dimensions*
  - Example: immediate sale of the asset vs development of an oil well

# Conclusion

- Introduce a novel kind of real options problem
- Bayesian uncertainty about past shocks leads to
  - investment not only in booms, but also at times of stable or decreasing cash flows
  - sluggish response to shocks
  - importance of maturity structure of project cash flows
- Uncertainty about past shocks may be as important as uncertainty about future shocks