Dynamic Trading with Predictable Returns and Transaction Costs

Dynamic Portfolio Choice with Frictions

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Motivation: Dynamic Trading

- Active investors – e.g., hedge funds, mutual funds, proprietary traders, individuals, other asset managers – try to
  - predict returns
  - minimize transactions costs
  - minimize risk
- Dynamic problem: investor trades now and in the future
- Key research questions:
  - What is the optimal trading strategy?
  - Does it work empirically?

Motivating Example

- An investor makes the following predictions:
  - Based on strong fundamentals (low M/B, P/E, low accruals, high and stable earnings, etc.) the annualized expected excess return (alpha) on Centurytel Inc. is 10%.
    - this alpha is expected to last for 2 years
  - Based on recent catalysts, improving fundamentals and pricing, the annualized alpha of Treehouse Foods Inc. is also 10%
    - this alpha is expected to last for half a year
  - Based on recent demand pressure from funds with outflow, the annualized alpha of HJ Heinz Co. is -12%
    - this alpha is expected to last for 2 weeks
  - These and other signals are collected for numerous securities
- All these stocks are positively correlated
- The investor has estimated the trading cost (incl. market impact) for these stocks based on past experience
- The investor makes a similar analysis every day

Results: Aim in Front of the Target

- Closed-form optimal dynamic trading strategy

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- ▷ Closed-form optimal dynamic trading strategy
- ▷ Two portfolio principles:
  1. Aim in front of the target
  2. Trade partially towards the current aim

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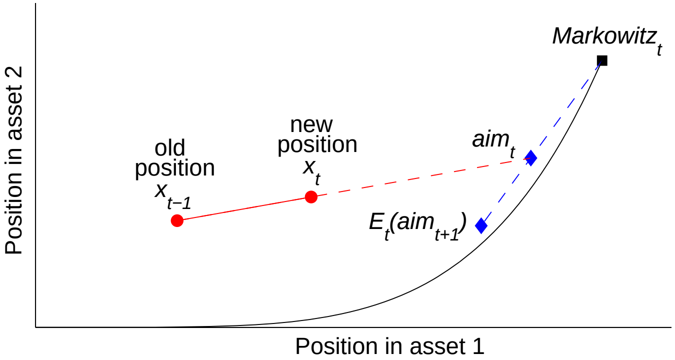
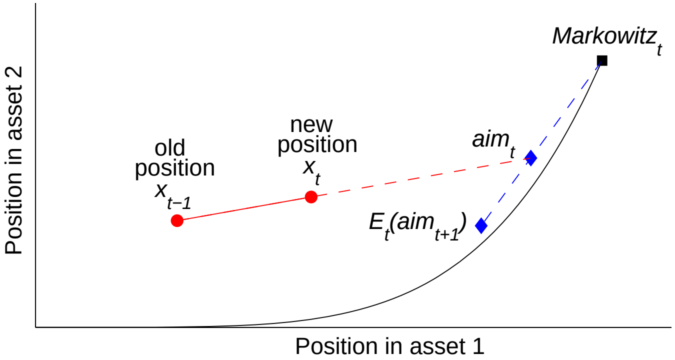


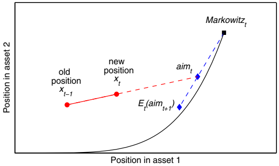
Results: Aim in Front of the Target

- ▷ Closed-form optimal dynamic trading strategy
- ▷ Two portfolio principles:
  1. Aim in front of the target
  2. Trade partially towards the current aim
- ▷ "Aim portfolio":
  - Weighted average of current and future expected Markowitz portfolios
  - Predictors with slower mean reversion: more weight
- ▷ Application to commodity futures: superior net returns

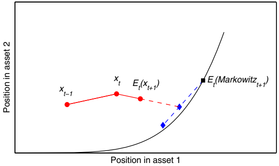


Aim in Front of the Target: Finance and Beyond

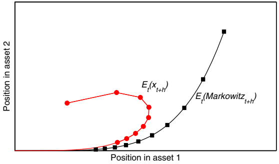
Panel A. Constructing the current optimal portfolio



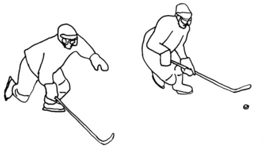
Panel B. Expected optimal portfolio next period



Panel C. Expected future path of optimal portfolio

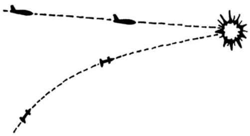


Panel D. "Skate to where the puck is going to be"



Panel E. Shooting: lead the duck

Panel F. Missile systems: lead homing guidance



Related Literature

- Optimal trading with transactions costs, no predictability
  - Constantinides (86), Amihud and Mendelson (86), Vayanos (98), Liu (04)
- Predictability, no transactions costs
  - Merton (73), Campbell and Viceira (02)
- Optimal trade execution with exogenous trade:
  - Perold (88), Almgren and Chriss (00)
- Numerical results with time-varying investment opportunity set
  - Jang, Koo, Liu, and Loewenstein (07), Lynch and Tan (08)
- Quadratic programming
  - Used in macroeconomics (Ljungqvist and Sargent (04)) and other fields: solve up to Ricatti equations
  - Grinold (06)

Outline of Talk

- Basic model
- Optimal portfolio strategy: Aim in front of the target
- Persistent price impact
- Application: Commodity futures

Discrete-Time Model

Returns: $$r^s_{t+1} = \underbrace{\sum_k \beta^{sk} f^k_t}_{=E_t(r^s_{t+1})} + u^s_{t+1}$$

Risk: $$\text{var}_t(u_{t+1}) = \Sigma$$

Alpha decay: $$\Delta f^k_{t+1} = -\sum_j \Phi^{kj} f^j_t + \varepsilon_{t+1}$$

Transaction costs: $$TC(\Delta x_t) = \frac{1}{2}\Delta x_t^\top \Lambda \Delta x_t$$

Assumption A: $$\Lambda = \lambda\Sigma$$

Objective: $$\max_{x_t} \mathrm{E} \sum_t (1-\rho)^{t+1} \left(x_t^\top r_{t+1} - \frac{\gamma}{2} x_t^\top \Sigma x_t \right) - \frac{(1-\rho)^t}{2} \Delta x_t^\top \Lambda \Delta x_t$$

Solution Method: Dynamic Programming

Introduce value function V that solves the Bellman equation:

$$V(x_{t-1}, f_t) = \max_{x_t} \left\{ -\frac{1}{2}\Delta x_t^\top \Lambda \Delta x_t + (1-\rho)\left(x_t^\top E_t(r_{t+1}) - \frac{\gamma}{2} x_t^\top \Sigma x_t + E_t[V(x_t, f_{t+1})] \right) \right\}$$

Proposition

The model has a unique solution and the value function is given by

$$V(x_t, f_{t+1}) = -\frac{1}{2} x_t^\top A_{xx} x_t + x_t^\top A_{xf} f_{t+1} + \frac{1}{2} f_{t+1}^\top A_{ff} f_{t+1} + A_0.$$

The coefficient matrices A_{xx}, A_{xf}, A_{ff} can be solved explicitly and A_{xx} is positive definite.

Trade Partially Towards the Aim

Proposition (Trade Partially Towards the Aim)

i)The optimal dynamic portfolio x_t is:

$$x_t = x_{t-1} + \Lambda^{-1} A_{xx} \left(aim_t - x_{t-1} \right)$$

with "trading rate" $\Lambda^{-1} A_{xx}$ and

$$aim_t = A_{xx}^{-1} A_{xf} f_t$$

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$$aim_t = A_{xx}^{-1} A_{xf} f_t$$

ii) Under Assumption A, the trading rate is the scalar

$$a/\lambda = \frac{-(\gamma + \lambda\rho) + \sqrt{(\gamma + \lambda\rho)^2 + 4\gamma\lambda(1-\rho)}}{2(1-\rho)\lambda} < 1$$

The trading rate is decreasing in transaction costs λ and increasing in risk aversion γ.

What is the Target and What is the Aim?

- What is the moving target, i.e., the optimal position in the absence of transaction costs?

$$Markowitz_t = (\gamma\Sigma)^{-1} Bf_t$$

- What is the aim portfolio?

Aim in Front of the Target

Proposition (Aim in Front of the Target)

(i) The aim portfolio is the weighted average of the current Markowitz portfolio and the expected future aim portfolio. Under Assumption A, letting $z = \gamma/(\gamma + a)$*:*

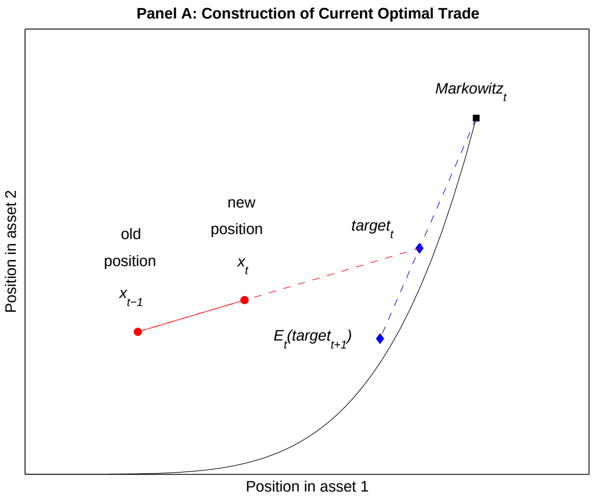
$$aim_t = z\, Markowitz_t + (1 - z)\, E_t(aim_{t+1}).$$

(ii) The aim portfolio is the weighted average of the current and future expected Markowitz portfolios. Under Assumption A,

$$aim_t = \sum_{\tau=t}^{\infty} z(1-z)^{\tau-t} E_t(Markowitz_\tau)$$

The weight of the current Markowitz portfolio z *decreases with transaction costs* λ *and increases in risk aversion* γ*.*

Aim in Front of the Target: Illustration



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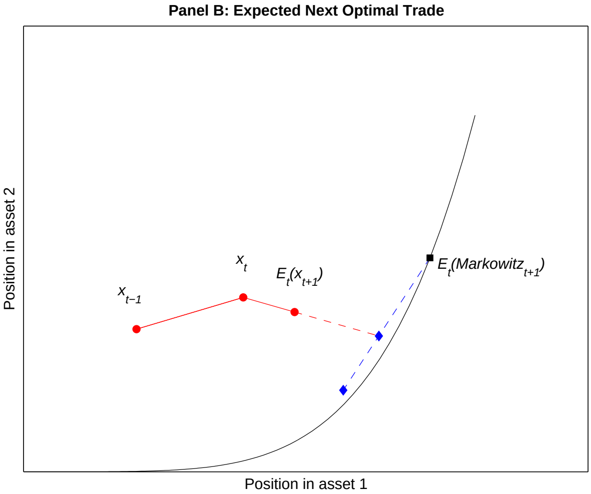

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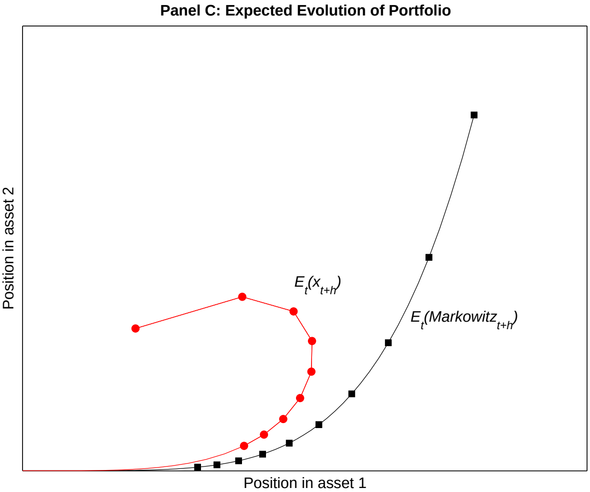


Weight Signals Based on Alpha Decay

Proposition (Weight Signals Based on Alpha Decay)

(i) Under Assumption A, the aim portfolio is:

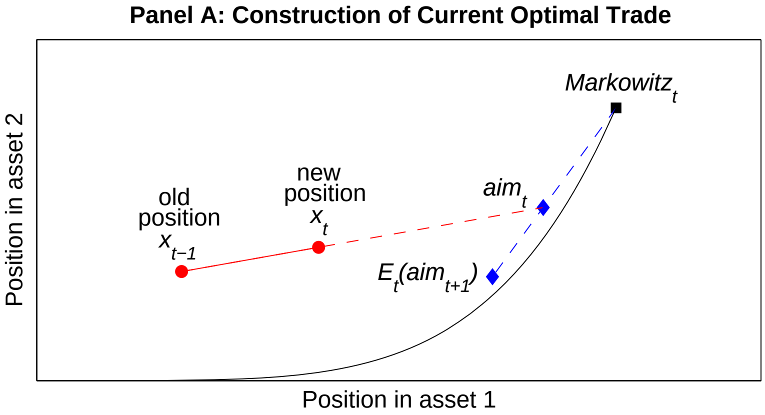
$$aim_t = (\gamma\Sigma)^{-1} B \left(I + \frac{a}{\gamma}\Phi \right)^{-1} f_t$$

(ii) If the matrix Φ is diagonal, $\Phi = \mathrm{diag}(\phi^1, ..., \phi^K)$, then the aim portfolio is:

$$aim_t = (\gamma\Sigma)^{-1} B \left(\frac{f_t^1}{1+\phi^1 a/\gamma}, \ldots, \frac{f_t^K}{1+\phi^K a/\gamma} \right)^\top$$

I.e., the aim pf. is the Markowitz pf. with factors f_t^k
scaled down based on their own alpha decay given by Φ.

Weight Signals Based on Alpha Decay: Illustration



Position Homing In

Proposition (Position Homing In)

Suppose that the agent has followed the optimal trading strategy from time $-\infty$ until time t. Then the current portfolio is an exponentially weighted average of past aim portfolios. Under Assumption A,

$$x_t = \sum_{\tau=-\infty}^{t} \frac{a}{\lambda}\left(1 - \frac{a}{\lambda}\right)^{t-\tau} aim_\tau \tag{1}$$

Example: Timing a Single Security

A security has risk $\Sigma = \sigma^2$ and return

$$r_{t+1} = \underbrace{\sum_k \beta^k f_t^k}_{=E_t(r_{t+1})} + u_{t+1}$$

The optimal strategy is

$$x_t = \left(1 - \frac{a}{\lambda}\right) x_{t-1} + \frac{a}{\lambda} \frac{1}{\gamma\sigma^2} \sum_{i=1}^{K} \frac{\beta^i}{1 + \phi^i a/\gamma} f_t^i.$$

Example: Relative-Value Trades w/ Security Characteristics

Each security s (e.g., IBM) has its own characteristics $f_t^{i,s}$ (e.g., its value and momentum) and characteristics predict returns for all securities, with the same coefficients:

$$E_t(r_{t+1}^s) = \sum_i \beta^i f_t^{i,s}$$

Each characteristic has the same mean-reversion speed for all securities

$$\Delta f_{t+1}^{i,s} = -\phi^i f_t^{i,s} + \varepsilon_{t+1}^{i,s}.$$

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The optimal characteristic-based strategy is

$$x_t = \left(1 - \frac{a}{\lambda}\right) x_{t-1} + \frac{a}{\lambda} (\gamma\Sigma)^{-1} \sum_{i=1}^{I} \frac{\beta^i}{1 + \phi^i a/\gamma} f_t^i.$$

Example: Static Model

When the future is completely discounted ($\rho = 1$), objective is

$$\max_{x_t} \left(x_t^\top E_t(r_{t+1}) - \frac{\gamma}{2} x_t^\top \Sigma x_t - \frac{\lambda}{2} \Delta x_t^\top \Sigma \Delta x_t \right)$$

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Solution

$$x_t = \frac{\lambda}{\gamma + \lambda} x_{t-1} + \frac{\gamma}{\gamma + \lambda} (\gamma \Sigma)^{-1} E_t(r_{t+1}).$$

No choice of γ, λ recovers the dynamic solution.

Example: Signals (Equally) Valuable for K Days

Suppose:

- All factors equally good $B = (\beta, ..., \beta)$
- Today's yesterday is tomorrow's day-before-yesterday:

$$\begin{aligned} f_{t+1}^1 &= \varepsilon_{t+1}^1 \\ f_{t+1}^k &= f_t^{k-1} \quad \text{for } k > 1 \end{aligned}$$

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Optimal strategy:

$$x_t = \left(1 - \frac{a}{\lambda}\right) x_{t-1} + \frac{a}{\lambda} \frac{\beta}{\sigma^2 (1-z)} \sum_k \left(1 - z^{K+1-k}\right) f_t^k,$$

where $z = a/(a+\gamma) < 1$.

More weight to recent signals even if they don't predict better.

Persistent Transaction Costs Model

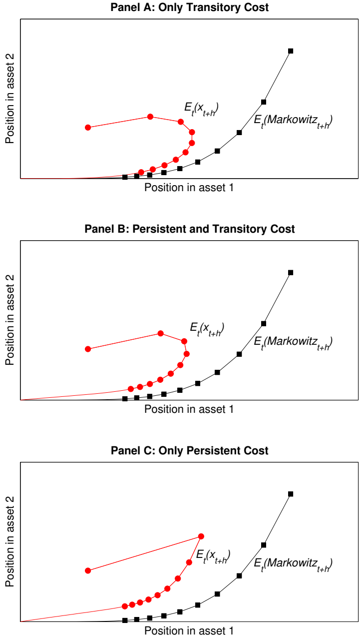
Proposition

With temporary and persistent transaction costs, the optimal portfolio x_t *is*

$$x_t \quad = \quad x_{t-1} + M^{rate}\left(aim_t - x_{t-1}\right),$$

which tracks an aim portfolio, $aim_t = M^{aim} y_t$*, that depends on the return-predicting factors and the price distortion.*

Persistent Transaction Costs Model



Application: Dynamic Trading of Commodity Futures

Data on liquid futures without tight price limits 01/01/1996 – 01/23/2009:

- Aluminum, Copper, Nickel, Zinc, Lead, Tin from London Metal Exchange (LME)
- Gas Oil from the Intercontinental Exchange (ICE)
- WTI Crude, RBOB Unleaded Gasoline, Natural Gas from New York Mercantile Exchange (NYMEX)
- Gold, Silver is from New York Commodities Exchange (COMEX)
- Coffee, Cocoa, Sugar from New York Board of Trade (NYBOT)

Predicting Returns and Other Parameter Estimates

Pooled panel regression:

$$r_{t+1}^{s} = \underset{(0.17)}{0.001} + \underset{(2.22)}{10.32}\ f_t^{5D,s} + \underset{(2.82)}{122.34}\ f_t^{1Y,s} - \underset{(-1.79)}{205.59}\ f_t^{5Y,s} + u_{t+1}^{s}$$

Alpha decay:

$$\begin{aligned}
\Delta f_{t+1}^{5D,s} &= -0.2519 f_t^{5D,s} + \varepsilon_{t+1}^{5D,s} \\
\Delta f_{t+1}^{1Y,s} &= -0.0034 f_t^{1Y,s} + \varepsilon_{t+1}^{1Y,s} \\
\Delta f_{t+1}^{5Y,s} &= -0.0010 f_t^{5Y,s} + \varepsilon_{t+1}^{5Y,s}
\end{aligned}$$

Risk: Σ estimated using daily price changes

Absolute risk aversion: $\gamma = 10^{-9}$

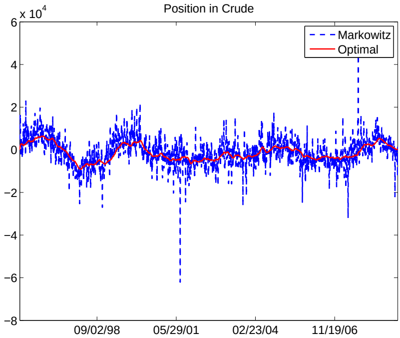
Time discount rate: $\rho = 1 - \exp(-0.02/260)$

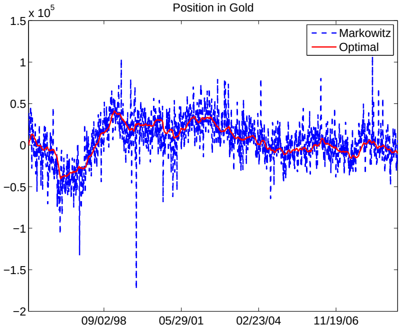
Transactions costs: $\lambda = 3 \times 10^{-7}$, as well as $\lambda^{high} = 10 \times 10^{-7}$

Performance of Trading Strategies Before and After TCs

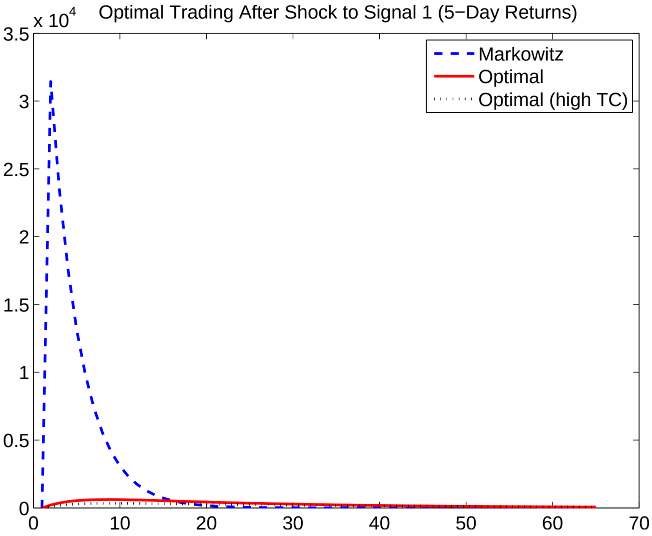
	Panel A: Benchmark Transaction Costs		**Panel B: High Transaction Costs**	
	Gross SR	**Net SR**	**Gross SR**	**Net SR**
Markowitz	0.83	-9.38	0.83	-10.11
Dynamic optimization	**0.63**	**0.60**	**0.58**	**0.53**
Static optimization				
Weight on Markowitz = 10%	0.63	0.00	0.63	-1.45
Weight on Markowitz = 9%	0.62	0.10	0.62	-1.10
Weight on Markowitz = 8%	0.62	0.20	0.62	-0.78
Weight on Markowitz = 7%	0.62	0.29	0.62	-0.49
Weight on Markowitz = 6%	0.62	0.36	0.62	-0.22
Weight on Markowitz = 5%	0.61	0.43	0.61	0.00
Weight on Markowitz = 4%	0.60	0.48	0.60	0.19
Weight on Markowitz = 3%	**0.58**	**0.51**	0.58	0.33
Weight on Markowitz = 2%	0.52	0.49	**0.52**	**0.39**
Weight on Markowitz = 1%	0.36	0.34	0.36	0.31

Positions in Crude and Gold Futures

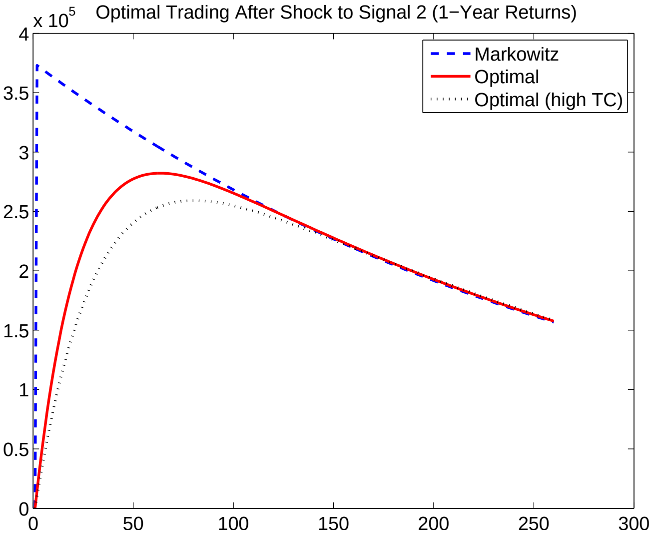




Optimal Trading in Response to Shock to 5-Day Return-Predicting Signal



Optimal Trading in Response to Shock to 1-Year Return-Predicting Signal



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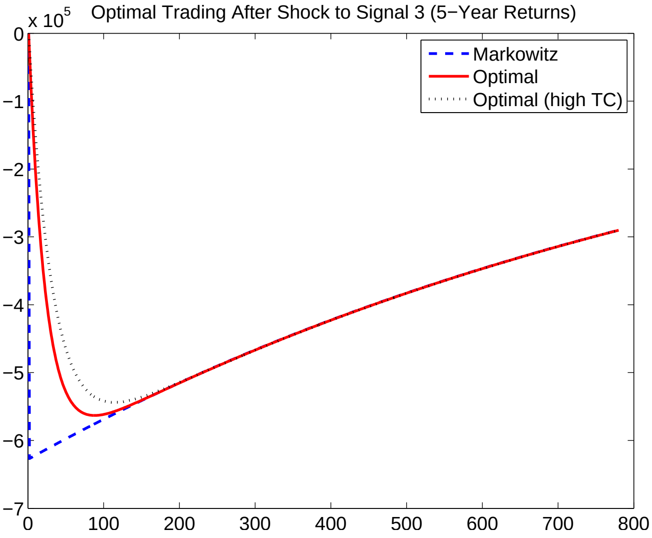

New paper: Dynamic Portfolio Choice with Frictions

What's different in this paper:

- Continuous time
- Micro foundation for transaction costs
- Connection between discrete and continuous time
  - What happens when trading becomes more frequent?
- Generalized factor dynamics and return dynamics, including stochastic volatility
- Equilibrium implications

Conclusion: Aim in Front of the Target

- Derive the closed-form optimal dynamic portfolio strategy
  1. Aim in front of the target
  2. Trade partially towards the current aim at constant rate
  3. Give more weight to persistent factors
- Superior net returns in application