"Rollover Risk and Market Freezes,"

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Journal of Finance (2011) **66**, pp. 1175-1207

Introduction

- Many financial institutions rely on short-term rollover debt to finance holdings of long-term assets.
- In August 2007, one such market – the market for asset-backed commercial paper (ABCP) – experienced a sudden "freeze."
- In mid-March 2008, the overnight secured (repo) markets froze for Bear Stearns.
- Many institutions that relied on the ABCP and repo markets have since collapsed.
- Our paper is an attempt to provide a model of such market freezes.

August 2007

- BNB Paribas' issued a press release on August 7, 2007.

  "The complete evaporation of liquidity in certain market segments of the U.S. securitization market has made it impossible to value certain assets fairly regardless of their quality or credit rating." [...] "The situation is such that it is no longer possible to value fairly the underlying US ABS assets in the three above-mentioned funds" and "therefore unable to calculate a reliable net asset value, NAV, for the funds."

- The bank suspended three funds valued at around €1.5 billion ($2.07 billion).

Collapse of the CP market

- The result was an immediate freeze in the asset backed commercial paper (ABCP) market.

- Banks had provided liquidity guarantees to funds that relied on the ABCP market. The fear that these assets would end up on the banks' balance sheets caused Libor to shoot upwards.

- Banks found it difficult to obtain funds on the overnight interbank market.

- On August 9, 2007 the ECB was forced to inject €95 billion into the overnight lending market.

Bear Sterns

- Monday, March 10, 2008: rumors spread about liquidity problems at Bear Stearns and continue through the week.
- A crisis of confidence: counterparties unwilling to make even secured funding available on customary terms.
- The holding company liquidity pool declines from $18.1 billion to $11.5 billion on Tuesday, fell sharply on Thursday and continued to fall on Friday.
- Bear Stearns' capital was adequate throughout the period March 10-17 and, at the time of its sale to JP Morgan Chase, the capital ratio was in excess of 10%, the Fed's "well capitalized" standard.

Bernanke on Bear Sterns

"[U]ntil recently, short-term repos had always been regarded as virtually risk-free instruments and thus largely immune to the type of rollover or withdrawal risks associated with short-term unsecured obligations. In March, rapidly unfolding events demonstrated that even repo markets could be severely disrupted when investors believe they might need to sell the underlying collateral in illiquid markets. Such forced asset sales can set up a particularly adverse dynamic, in which further substantial price declines fan investor concerns about counterparty credit risk, which then feed back in the form of intensifying funding pressures."

Summary

Our objective is to explain the drying up of liquidity in the absence of obvious problems of asymmetric information or fears about the value of collateral.

We study the *debt capacity* of an asset under the following three conditions:

- (i) the debt is short-term in nature and, hence, needs to be rolled over at short intervals;
- (ii) in the event of default, the assets are sold to buyers who also use short-term finance;
- (iii) liquidation costs absorb a (small) fraction of the sale price.

Results

In "efficient" markets, the debt capacity of an asset is equal to its NPV or "fundamental" value. We show in the sequel that this result almost never holds.

- We characterize the debt capacity as the solution to a simple dynamic programing problem.
- The debt capacity of an asset is *always* smaller than its fundamental value.
- When the rollover frequency is sufficiently high, the debt capacity in a given state is always less than the terminal value of the asset in that state.
- In the worst case, the debt capacity equals the *minimum possible value of the asset*.

The bank's objective

- Although there are several possible interpretations of the model, we focus on the example of an investment bank using repo finance

- A collection of assets purchased at time $t = 0$ are used as collateral for the money borrowed to pay for purchase.

- The bank's objective is to minimize the amount of capital it needs to invest in these assets, which is equivalent to maximizing the amount of finance that can be raised using the assets as collateral.

- The assets have a finite life (e.g., a pool of mortgages) and we normalize units of time so that the assets mature at date $t = 1$.
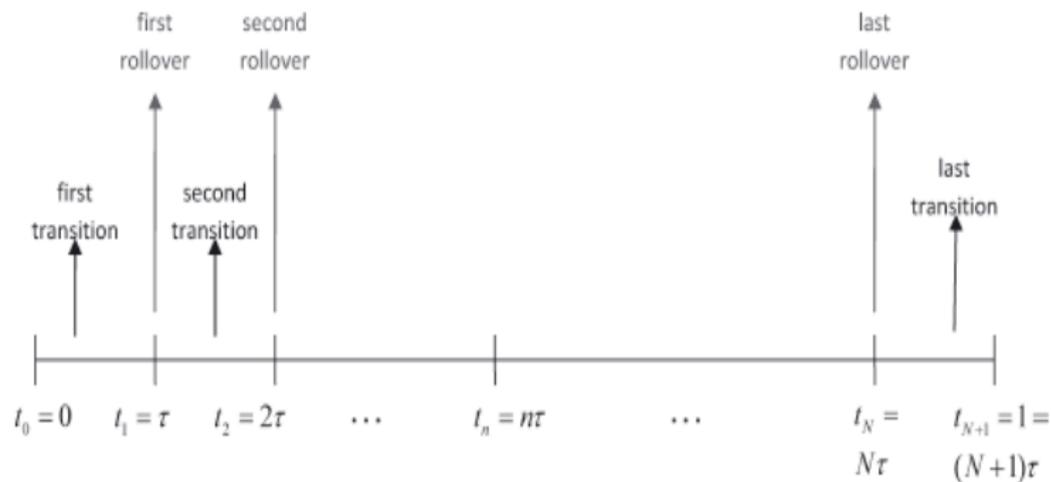
Time

- Time is represented by the unit interval, $[0, 1]$.
- The asset is purchased at the initial date $t = 0$ and has a terminal value at $t = 1$, but generates no income at the intermediate dates $0 \leq t < 1$.
- The asset is financed by debt with fixed maturity $\tau > 0$.
- The rollover dates are denoted by

$$t_n = n\tau, \text{ for } n = 1, ..., N,$$

where N is the number of rollovers and

$$\tau = \frac{1}{N+1}.$$

Timeline of N rollover dates

Uncertainty

- We begin by considering a simple example, in which there are two states of nature, a high state H and a low state L.

- Transitions are governed by a stationary probability transition matrix

$$\mathbf{P}(t) = \left[\begin{array}{cc} p_{LL}(t) & p_{LH}(t) \\ p_{HL}(t) & p_{HH}(t) \end{array} \right]$$

where

$$\begin{aligned} \mathbf{P}(t) &= e^{\mathbf{A}t} \\ &= \sum_{t=0}^{\infty} \frac{(\mathbf{A}t)^k}{k!} \end{aligned}$$

and $\mathbf{P}(t) \to \mathbf{I}$ as $t \to 0$.

Other parameters

- The terminal value of the asset is a function of the state of the economy at the terminal date $t = 1$. In the high state, the value of the asset is v^H and in the low state it is v^L, where $0 < v^L < v^H$.

- If the bank is forced to default and liquidate the assets, we assume that the assets can be sold for a fraction $\lambda \in [0, 1]$ of the *maximum amount of finance that could be raised using the asset as collateral*.

- Finally, we assume universal risk neutrality and normalize the risk-free interest rate to be equal to 0.

- Note that we are assuming that everyone in the market is using debt finance; hence, price is always bounded by debt capacity

Beginning the induction

- Let D be the face value of the debt. In equilibrium, D is chosen to maximize the market value of the debt at each date and in each state.
- We can solve the problem by backward induction, beginning at the last rollover date.
- At date t_N, the value of the assets will be either v^H or v^L.
- Note that it can never be optimal to choose D so that one of the following cases occurs:
$$D > v^H; \ v^L < D < v^H; \ D < v^L.$$
- Thus, we only have to consider $D = v^H$ and $D = v^L$.

The low state

- If we choose $D = v^L$, the debt pays v^L in both states at date t_{N+1}, so the market value of the debt at t_N is v^L.

- On the other hand, if we choose $D = v^H$, the debt pays v^H in the high state, but pays λv^L in the low state, so the market value will be $p_{LH}(\tau) v^H + p_{LL}(\tau) \lambda v^L$.

- Since $\lambda < 1$, for $\tau > 0$ sufficiently small we have

$$p_{LH}(\tau) v^H + p_{LL}(\tau) \lambda v^L < v^L.$$

- So the optimal face value is $D_N = v^L$ and the debt capacity is $B_N^L = v^L$.

The high state

- Again, if we choose $D = v^L$, the debt will be repaid in both states and the market value of the debt is v^L.

- On the other hand, if we choose $D = v^H$, the debt pays v^H in the high state and λv^L in the low state, so the market value will be $p_{HH}(\tau) v^H + p_{HL}(\tau) \lambda v^L$.

- Since $p_{LH}(\tau) \approx 0$ for $\tau > 0$ sufficiently small, we have

$$p_{HH}(\tau) v^H + p_{HL}(\tau) \lambda v^L > v^L.$$

So the optimal face value is $D_N = v^H$ and the debt capacity is

$$B_N^L = p_{HH}(\tau) v^H + p_{HL}(\tau) \lambda v^L.$$

Continuing the induction

- Now consider the second last rollover date t_{N-1}. By the usual argument, it is optimal to choose the face value $D \in \left\{ B_N^H, B_N^L \right\}$.

- In the low state, we see that, for $\tau > 0$ sufficiently small,

$$p_{LH}(\tau) B_N^H + (p_{LL}(\tau)) \lambda B_N^L < v^L$$

since $B_N^H \leq v^H$ and $B_N^L = v^L$. So the debt capacity is $B_{N-1}^L = v^L$.

- In the high state, putting $D = B_N^H$ gives a market value of

$$p_{HH}(\tau) B_N^H + p_{HL}(\tau) \lambda B_N^L > B_N^L,$$

for $\tau > 0$ sufficiently small, since $B_N^H > B_N^L = v^L$.

Completing the induction

- The earlier argument can be used to show that

$$B_n^L = v^L \text{ for every } n = 1, ..., N$$

  because it is clear that $B_{n+1}^H \leq v^H$ for every $n = 1, ..., N$.

- In the high state, we have to ensure that B_{n+1}^H is bounded away from v^L in order to show that $D_n = B_{n+1}^H$ is optimal for every $n = 1, ..., N$.

- One way to do this is to assume that λ is close to 1, which implies that

$$p_{HH}(\tau)^N v^H + \left(1 - p_{HH}(\tau)^N\right) \lambda v^L > v^L.$$

Debt capacity

Theorem

Define $\left\{ \left(B_n^H, D_n^H, B_n^L, D_n^L \right) \right\}_{n=0}^N$ by setting

$$D_n^H = B_{n+1}^H, \tag{1}$$

$$B_n^H = p_{HH}(\tau) B_{n+1}^H + p(\tau) \lambda v^L, \tag{2}$$

and

$$D_n^L = B_n^L = v^L, \tag{3}$$

for $n = 1, ..., N$. The values defined by (1-3) constitute a solution to the problem of maximizing debt capacity if and only if

$$p_{LH}(\tau) v^H + p_{LL}(\tau) \lambda v^L < v^L.$$

and

$$p_{HH}(\tau)^N v^H + \left(1 - p_{HH}(\tau)^N \right) \lambda v^L > v^L.$$

Satisfying the conditions for a market freeze

- There are two necessary and sufficient conditions for the market freeze described in the preceding theorem. One ensures that it is optimal to set the face of debt in the high state equal to the high state's debt capacity at the next date:
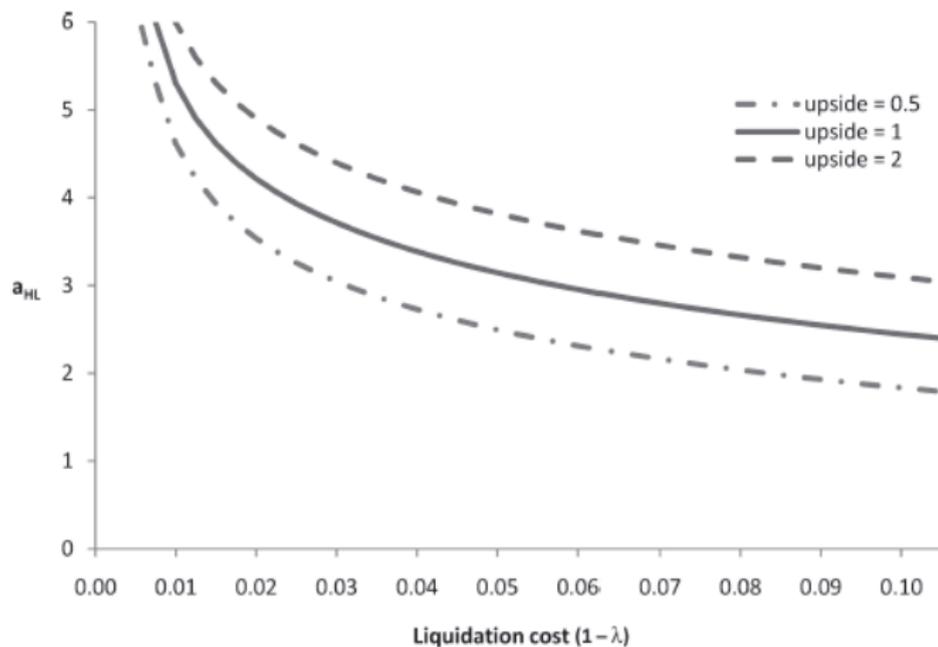
$$v^H - \lambda v^L \geq \frac{(1-\lambda) v^L}{p_{HH}(\tau)^N}.$$

where $\frac{v^H - v^L}{v^L}$ is the *upside* in the low state; $1 - \lambda$ is the *liquidation* cost; and a_{HL} is the *arrival rate* of a switch from the high to the low state.

- A sufficient condition for this inequality to hold is

$$\frac{v^H - v^L}{v^L} \geq (1-\lambda) a_{HL} \tag{4}$$

Parameter values satisfying (4)

The second condition

- A second condition ensures that it is optimal to set the face value of the debt equal to v^L in the low state; this can be rewritten as:

$$\frac{p_{LH}(\tau)}{1 - p_{LH}(\tau)} \frac{v^H - v^L}{v^L} \leq 1 - \lambda, \tag{5}$$

where $\frac{p_{LH}(\tau)}{1 - p_{LH}(\tau)}$ is the *odds ratio*; $\frac{v^H - v^L}{v^L}$ is the *upside*; and $1 - \lambda$ is the *liquidation cost*.

- In addition, however, we want the change in debt capacity to be large relative to the change in fundamentals, say, the difference in fundamentals should be at most 10% of $v^H - v^L$.

- A sufficient condition for these inequalities to hold is that the number of roll overs be at least N^*, where

$$N^* = 2.3026 \left(1 + \frac{v^H - v^L}{v^L} \frac{1}{1 - \lambda}\right). \tag{6}$$

Parameter values satisfying (6)
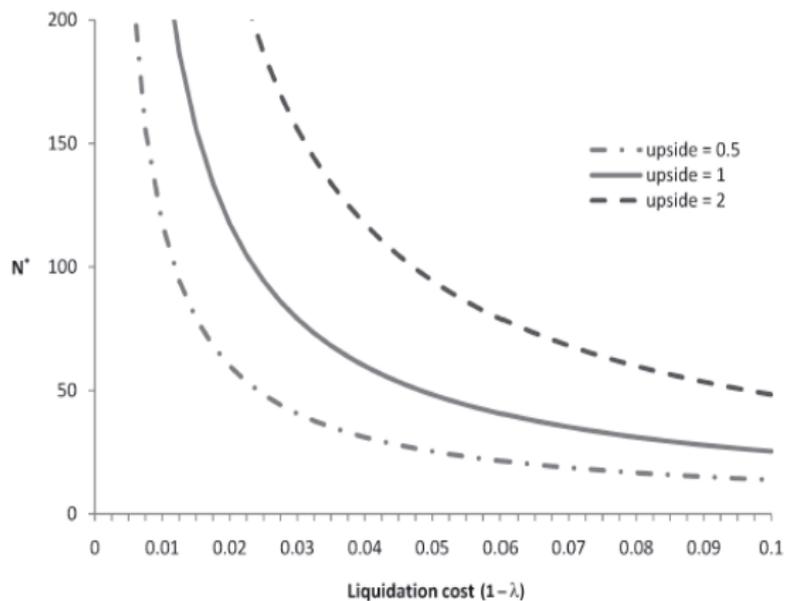
Illustrative examples

Example

The asset has a maturity of six months and is funded by overnight repos. So the debt must be rolled over approximately 162 times. In order for the market to freeze in the low state (debt capacity equal to v^L), the value of the liquidation cost must be at least $1 - \lambda = 0.0144$. or around 1.5% of the upside.

Example

The asset has a maturity of two years and is funded by short term loans that are rolled over weekly. In total the debt must be rolled over 104 times. In order for the market to freeze in the low state, the value of the liquidation cost must be at least $1 - \lambda = 0.02264$ or around 2.25% of the upside.

Example

The asset has a maturity of ten years and is funded by one month loans, so the debt must be rolled over 120 times. In order for the market to freeze in the low state, the value of the liquidation cost must be at least $1 - \lambda = 0.01956$ or around 2.0% of the upside.

Numerical example

We can illustrate the theorem with a numerical example. Let the asset values be

$$v^H = 100 \text{ and } v^L = 50;$$

let the generator be

$$\mathbf{P} = \left[\begin{array}{cc} -8.0 & 8.0 \\ 0.1 & -0.1 \end{array} \right].$$

Fundamental values

- The fundamental value of the assets — the value for which they could be sold in an efficient market — is simply the expected value of the terminal payoff.
- The transition probabilities between the first and last dates are

$$\mathbf{P}(1) = \left[\begin{array}{cc} 1.2645 \times 10^{-2} & 0.98735 \\ 1.2342 \times 10^{-2} & 0.98766 \end{array} \right].$$

- Then the fundamental values at date $t_0 = 0$ are

$$\left[\begin{array}{c} V_0^L \\ V_0^H \end{array} \right] = \left[\begin{array}{cc} 1.2645 \times 10^{-2} & 0.98735 \\ 1.2342 \times 10^{-2} & 0.98766 \end{array} \right] \left[\begin{array}{c} 50 \\ 100 \end{array} \right]$$

$$= \left[\begin{array}{c} 99.367 \\ 99.383 \end{array} \right].$$

Debt capacity at the last rollover date

- The transition probabilities between rollover dates are

$$\mathbf{P}(0.01) = \left[\begin{array}{cc} 0.92315 & 7.6846 \times 10^{-2} \\ 9.6057 \times 10^{-4} & 0.99904 \end{array} \right].$$

- It is always optimal to choose the face value of the debt equal to v^H or v^L. Suppose the economy is in the high state.
- If $D = v^H = 100$, the market (expected) value of the debt is

$$0.99904 \times 100 + 9.6057 \times 10^{-4} \times 0.9 \times 50 = 99.947.$$

- If $D = v^H = 50$, the market (expected) value of the debt is 50, so

$$B_N^H = 99.947.$$

The last date (continued)

- Now suppose the economy is in the low state. If $D = v^H = 100$, the market (expected) value of the debt is

$$7.6846 \times 10^{-2} \times 100 + 0.92315 \times 0.9 \times 50 = 49.226.$$

- If $D = v^L = 50$, the market (expected) value of the debt is 50.

- So clearly it is optimal to put $D_N^L = 50$ and the debt capacity in the low state is

$$B_N^L = 50.$$

Debt capacity at intermediate dates

- In general, we choose the face value of the debt to be either $D = B_{n+1}^H$ or $D = B_{n+1}^L$.
- If we choose $D = B_{n+1}^L$ then the market (expected) value of the debt is B_{n+1}^L in both states.
- Consider the decision in state L. It is clear that

$$B_{n+1}^H \leq v^H \text{ and } B_{n+1}^L \geq v^L,$$

  so if it is optimal to set $D_N^L = v^L$ at date t_N it must be optimal to set $D_n^L = B_{n+1}^L$ at t_n.
- By induction, the debt capacity is $B_n^L = v^L$ for any rollover date.

Intermediates dates (continued)

- We obtain a lower bound for debt capacity assuming $D_n = B_{n+1}^H$ for all n.

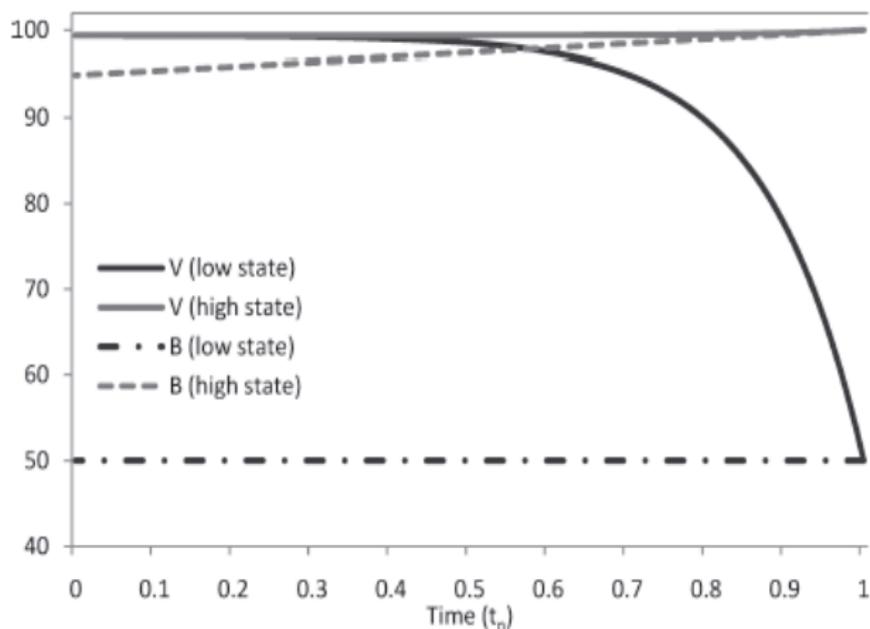- The probability of staying in the high state until date 1 is

$$(0.99904)^{100-n+1} \geq (0.99904)^{100} = 0.90842.$$

- The debt capacity at time t_n is at least as great as

$$(0.90842)\,100 + (0.9)\,(1 - 0.90842)\,(50) = 94.963.$$

- This proves that it is optimal to set $D_n^H = B_{n+1}^H$ at each rollover date t_n.

Fundamental value and debt capacity

Figure shows curves: V (low state), V (high state), B (low state), B (high state) plotted against Time (t_n) from 0 to 1, with vertical axis from 40 to 100.

Sensitivity at the first date

- We have shown that the debt capacities at date 0 are

$$B_0^H = 94.963 \text{ and } B_0^L = v^L = 50,$$

whereas the fundamental values are

$$V_0^H = 99.383 \text{ and } V_0^L = 99.367.$$

- So the arrival of bad news in the high state occasions fall in debt capacity and fundamental:

$$\Delta B = 50 - 94.963 = -44.963$$
$$\Delta V = 99.367 - 99.383 = -0.016.$$

Intuition

- As $\tau \to 0$, the probability of "no news" approaches one.
- In the low state, the increase in market value of debt from setting a high face value converges to zero.
- In the limit it is offset by the (small) liquidation costs.
- The borrower in the low state is forced to treat his asset as if it were only worth V^L, regardless of the fundamental value.
- In the high state, there is a positive probability of transition to the low state.
- Since the debt capacity is below the fundamental in the low state, it must also be below the fundamental in the high state.

Market freezes when the conditions of the theorem are not satisfied
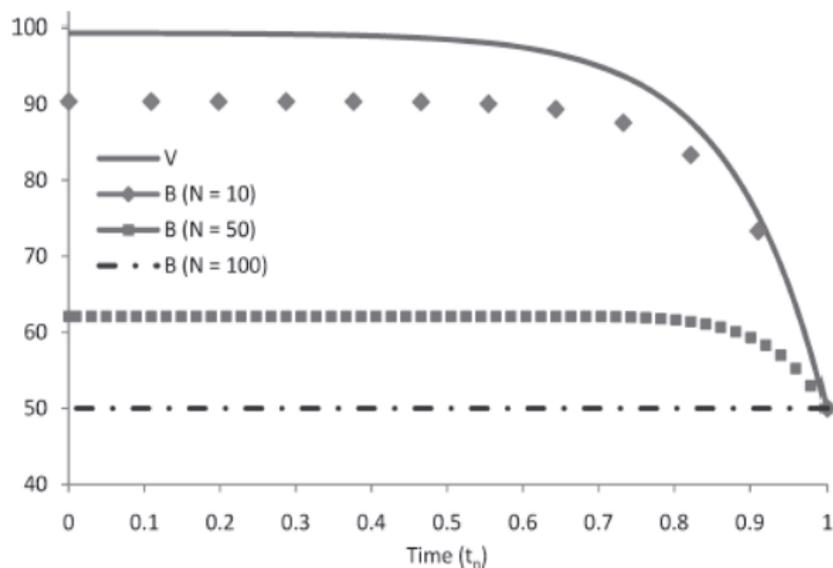
Drivers of the market freeze

- *Credit risk*: If $V^H = V^L$ the debt capacity equals the fundamental, so some credit risk is needed, but $V^H - V^L > 0$ can be arbitrarily small.
- *Liquidation costs*: If $\lambda = 1$ the debt capacity equals the fundamental, so some liquidation cost is needed, but $1 - \lambda > 0$ can be arbitrarily small.
- *Short-term debt*: We take as an empirical fact, but theoretical motivations are available (Calomiris-Kahn; Diamond-Rajan; Brunnermeier-Oehmke).
- *Rollover risk*: Clearly, this is the key assumption. A high rollover frequency implies that information arrives relatively slowly. Then the Poisson structure implies "no news" is very likely.

The general case

- We allow for an arbitrary finite set of states

$$\mathcal{S} = \{s_1, ..., s_I\}$$

and an arbitrary set of values

$$0 < v_1 < \ldots < v_I.$$

- The transition probability over time t is defined by

$$\mathbf{P}(t) = e^{\mathbf{A}t}$$

where the $I \times I$ matrix $\mathbf{A}$ is the generator and $t \in [0, 1]$

Fundamental values

- Let V_n^i denote the fundamental value of the asset at date t_n in state i.
- The values $\left\{ V_n^i \right\}$ are defined by putting

$$V_{N+1}^i = v_i \text{ for } i = 1, ..., I$$

and

$$V_n^i = \sum_{j=1}^{I} p_{ij} \left(1 - t_n \right) v_j, \text{ for } n = 0, ..., N \text{ and } i = 1, ..., I,$$

where $p_{ij} \left(1 - t_n \right)$ is the (i, j) component of the matrix $\mathbf{P} \left(1 - t_n \right)$.

Debt capacity

Let B_n^i denote the equilibrium debt capacity of the assets in state s_i at date t_n. By convention, we set $B_{N+1}^i = v_i$ for all i.

Theorem

The equilibrium values of $\left\{ B_n^i \right\}$ must satisfy

$$B_n^i = \max_{k=1,...,I} \left\{ \sum_{j=1}^{k-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=k}^{I} p_{ij}(\tau) B_{n+1}^k \right\}$$

for $i = 1, ..., I$ and $n = 0, ..., N$.

Proof.

Immediate once one realizes that $D_n^i \in \left\{ B_{n+1}^1, ..., B_{n+1}^I \right\}$. $\qquad\qquad\square$

Limit theorem

To prove the next theorem we need to assume that higher information states are "better" in the sense that

$$V_{in} < V_{i+1,n}, \text{ for all } i = 1, ..., I-1 \text{ and } n = 0, ..., N+1. \qquad (7)$$

A sufficient condition for (7) is that $\{p_{i+1,j}(\tau)\}$ strictly dominates $\{p_{i,j}(\tau)\}$ in the sense of first-order stochastic dominance.

Theorem

Suppose that (7) is satisfied. Then there exists $\tau^* > 0$ such that for all $0 < \tau < \tau^*$, for any $n = 0, ..., N$ and any $i = 1, ..., I$,

$$B_n^i = \sum_{j=1}^{i-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=i}^{I} p_{ij}(\tau) B_{n+1}^i.$$

Corollaries

- In the worst state s_1, the debt capacity B_n^1 equals v_1, for all n.

- Debt capacity in state s_i is less than or equal to v_i:

$$B_n^i \leq v_i, \text{ for all } i \text{ and } n.$$

- Debt capacity is less than or equal to the fundamental value:

$$B_n^i \leq V_n^i, \text{ for all } n \text{ and } i.$$

- Debt capacity is monotonically non-decreasing with respect to n:

$$B_n^i \leq B_{n+1}^i \text{ for all } i \text{ and } n.$$

Conclusion

- We have presented a model of a "market freeze" caused by:
  - ▸ credit risk
  - ▸ liquidation costs
  - ▸ and rollover risk
- Policy implications:
  - ▸ Reliance on wholesale markets for short term debt backed by illiquid collateral
  - ▸ Capital requirements, regulatory arbitrage
- Future work:
  - ▸ Strategic disclosure of information
  - ▸ Micro-foundations for short-term debt backed by illiquid assets
  - ▸ Applications, related issues: freeze in interbank markets due to expectations about future liquidity needs.