Internet Appendix for "A Habit-based Explanation of the Exchange Rate Risk Premium"*

In this appendix, I derive the optimal foreign and domestic consumption allocations in a two-country model where agents are characterized by habit preferences as in the main text (Verdelhan 2010). Agents can trade, but incur proportional and quadratic trade costs. This model replicates the empirical forward premium and equity premium puzzles as well as interest rate and exchange rate volatility. Finally, I investigate the impact of nontradable goods on the correlation between exchange rates and consumption growth rates. In the Annex at the end of this document, I provide details on the simulation method used in this appendix and in Verdelhan (2010).

*Citation format: Verdelhan, Adrien, 2010, Internet Appendix to "A Habit-based Explanation of the Exchange Rate Risk Premium," Journal of Finance 65, 123-145, http://www.afajof.org/IA/2010.asp. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

In Verdelhan (2010), I start from consumption allocations and show that a two-country model with external habit preferences replicates the uncovered interest rate parity (UIP) or forward premium puzzle when risk-free rates are procyclical. In order to derive closed-form expressions for interest rates, exchange rates, and the UIP slope coefficient, I abstract from international trade. In this appendix, I start from endowment allocations and consider international trade. I obtain a one-good, two-country model that replicates the empirical forward premium and equity premium puzzles as well as interest rate and exchange rate volatility.

The model has two key components: habit preferences and international trade costs. As in the main text, agents have habit preferences that imply time-varying stochastic discount factors. As consumption declines toward habit levels in a business cycle trough, the curvature of the utility function rises. As a result, risky asset prices fall and expected returns rise. Contrary to the main text, agents trade across countries. But international trade is costly. I assume that shipping costs have two components. The first is the typical iceberg-like trade cost: when a unit of the good is shipped, only a fraction arrives on foreign shores. The second component is a quadratic cost, which captures the capacity constraints of international trade and ensures that the total trade cost increases with the volume of trade. I use the planner's problem to derive the optimal consumption allocations and trade flows. As in Dumas (1992), the model implies a cone of no-trade: when marginal utilities of consumption do not differ enough to justify the expense of shipping costs, countries do not trade. When countries do not trade, real exchange rates move freely with the ratio of the countries' marginal utilities of consumption. This is the case considered in the main text. When the foreign country exports, the marginal utility of consumption in the foreign country is equal to the marginal utility of consumption in the domestic country multiplied by the marginal shipping cost. The ratio of the foreign to domestic marginal utilities of consumption is thus equal to the marginal shipping cost. The reverse is also true: when the domestic country exports, this ratio is equal to the inverse of the previous marginal shipping cost. As a result, when countries trade, real exchange rates are determined by marginal trade costs. When trade costs are proportional, real exchange rates bounce back and forth between the two constant values implied by international trade. With proportional and quadratic costs, real exchange rates are no longer constant even when countries trade. The opportunity to trade when marginal utilities differ considerably limits the volatility of changes in real exchange rates.

A lower exchange rate volatility is a welcome feature of the model because many models imply exchange rates that are too volatile. Brandt, Cochrane, and Santa-Clara (2006) make this point forcefully. In complete markets, the change in real exchange rates is theoretically equal to the ratio of foreign and domestic stochastic discount factors. Since Mehra and Prescott (1985) and Hansen and Jagannathan (1991), we know that stochastic discount factors must have a large variance in order to price stock excess returns. Thus, volatile stochastic discount factors imply very volatile exchange rates unless these discount factors are highly correlated. But, assuming power utility, actual consumption data suggest that the correlation between discount factors is low. A high risk aversion delivers high Sharpe ratios but also volatile exchange rates. Using habit preferences and trade costs, this appendix offers a novel resolution of the tension between the large implied volatility of stochastic discount factors and the comparatively low volatility of changes in exchange rates.

I simulate the model for a range of trading costs found in the international economics literature. Endowment shocks are Gaussian, independent, and identically distributed. Their standard deviation and cross-country correlation are in line with the data. The model reproduces the first two moments of consumption growth and interest rates, as well as volatile stochastic discount factors that match the equity and currency risk premia. The model implies a mean excess return on the stock market that is lower than what is found in the U.S. in the post-WWII sample. But it leads to the same Sharpe ratio. The simulation recovers the usual result in tests of the UIP condition: a negative slope coefficient in a regression of changes in exchange rates on interest rate differentials. The model also reproduces the variance of changes in the real exchange rate.

The model, however, has one main weakness: it does not account for the disconnect between consumption and exchange rates, known as the Backus and Smith (1993) puzzle. When markets are complete and agents have constant relative risk aversion, the correlation between differences in consumption growth and changes in real exchange rates is equal to one. In this model with habit preferences, the correlation is no longer one, but it remains higher than what is found in the data because one source of shocks drives all variables. A complete solution to this puzzle might require introducing market incompleteness and several types of goods, and is therefore beyond the scope of this paper. The model nonetheless offers a glimpse at a potential solution. I introduce nontradable goods in the utility function and interpret them as a source of measurement error on the real exchange rate. As a result, real exchange rates are more volatile and the correlation with relative consumption growth rates decreases. This brings the model closer to the data and echoes Burstein, Eichenbaum, and Rebelo's (2005) estimate, which suggests that at least half of the variation in real exchange rates stems from changes in the relative prices of nontradable goods across countries.

The model presented in this appendix relates to a large literature in international economics and international finance. Proportional (iceberg-like) shipping costs were first proposed by Samuelson (1954), and then used by Dumas (1992), Sercu, Uppal, and Hulle (1995), Sercu and Uppal (2003), Obstfeld and Rogoff (2000), and Fitzgerald (2008) to study real exchange rates. None of these papers tackle the equity and forward premium puzzles. Hollifield and Uppal (1997) show further that proportional trade costs are not enough to reproduce the forward premium puzzle when agents are characterized by power utility, not even at extreme levels of constant risk aversion or high trading costs. A recent literature that does not take into account international trade offers interesting solutions to the equity and currency puzzles; I review this literature in the main text. Finally, three papers make progress on the Backus and Smith (1993) puzzle. Colacito and Croce (2008) show that cross-country-correlated long-run risk with Epstein and Zin (1989) preferences successfully reconciles international prices and quantities in complete, frictionless markets. Corsetti, Dedola, and Leduc (2008) find two ways to reproduce the puzzle. A first solution assumes a low implied elasticity of substitution between traded and nontraded goods and incomplete markets, where only non-state-contingent bonds are traded. A second solution assumes a high implied elasticity of substitution between traded and nontraded goods, incomplete markets, and nearly permanent productivity shocks. Bodenstein (2008) develops a two-country model with complete asset markets and limited enforcement that reproduces the correlation between exchange rates and consumption growth rates.

The rest of the appendix is organized as follows: Section I presents the habit model and shows how to compute optimal international trade starting from endowment processes. Section II reports simulation results obtained with proportional and quadratic trade costs. Section III investigates the role of nontradable goods as sources of measurement errors and their impact on the previous results. Section IV concludes.

I. Model

In the model, there are two endowment economies with same initial wealth and one good. In this section, I describe the preferences of the representative agent in each country and the optimal trade and consumption allocation problem. I assume that agents can trade across countries but incur international shipping costs.

Preferences and trade costs: Let X_t denote the amount of the good exported from a domestic to a foreign country at time t. A superscript * refers to the same variable for the foreign country. The proportional trade cost is captured by the parameter τ. The quadratic cost is assumed to be proportional (with coefficient δ) to the ratio of exports to endowments as in Backus, Kehoe, and Kydland (1992). To find the optimal amount of exports $X_t \geq 0$ and $X_t^\star \geq 0$ and consumption allocations C_t and $C_t^\star$, I consider the following planning problem:

$$\text{Max } E\sum_{t=0}^{\infty}\beta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma} + E\sum_{t=0}^{\infty}\beta^t \frac{(C_t^\star - H_t^\star)^{1-\gamma} - 1}{1-\gamma},$$

subject to

$$C_t = Y_t - X_t + X_t^\star(1 - \tau - \frac{\delta}{2}\frac{X_t^\star}{Y_t}) \text{ and } C_t^\star = Y_t^\star - X_t^\star + X_t(1 - \tau - \frac{\delta}{2}\frac{X_t}{Y_t^\star}),$$

where γ denotes the risk-aversion coefficient in the two countries, H_t the external domestic habit level, and Y_t the domestic endowment. The external habit level can be interpreted as a subsistence level or as a social externality. In each country, the habit level is related to consumption through the following autoregressive process of the surplus consumption ratio $S_t \equiv (C_t - H_t)/C_t$:

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g_c). \tag{IA.1}$$

Lowercase letters correspond to logs, and g_c is the average consumption growth rate. The sensitivity function $\lambda(s_t)$ describes how habits are formed from past aggregate consumption. The same features apply to the foreign representative agent. I assume that in both countries endowment growth shocks are lognormally distributed:

$$\Delta y_{t+1} = g_y + u_{t+1}, \text{ where } u_{t+1} \sim i.i.d.\ N(0, \sigma_y^2),$$

with mean g_y and volatility σ_y. Domestic and foreign shocks u and $u^\star$ can be correlated. I refer to "bad times" as times of low surplus consumption ratio (when the consumption level is close to the habit level), and use "negative shocks" to refer to negative endowment growth shocks u. The dynamics of the surplus consumption ratio are then fully described by specifying the sensitivity function $\lambda(s_t)$ as

$$\lambda(s_t) = \left\{ \begin{array}{ll} \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1 & \text{when } s \leq s_{\max}, \\ 0 & \text{elsewhere,} \end{array} \right\}$$

where $\bar{S}$ and $s_{\max}$ are respectively the steady-state and upper bound of the surplus consumption ratio. The term $\bar{S}$ measures the steady-state gap, in percentage points, between consumption and habit levels.

First-order conditions: Two assumptions simplify the solution of the maximization problem. First, in each country, the habit level depends only on domestic, not foreign, consumption, and on aggregate, not individual, consumption. In this case, the local curvature of the utility function, or local risk-aversion coefficient, is simply $\gamma_t = -C_t U_{cc}(t)/U_c(t) = \gamma/S_t$. When consumption is close to the habit level, the surplus consumption ratio is low and the agent is very risk averse. Second, there is only one good in the model. As a result, if one country exports, the other does not.

Let us assume first that the domestic country exports ($X_t \geq 0$, $X_t^\star = 0$). The first order condition with respect to X_t is then

$$-[Y_t - X_t - H_t]^{-\gamma} + [1 - \tau - \delta\frac{X_t}{Y_t^\star}][Y_t^\star + X_t(1 - \tau - \frac{\delta}{2}\frac{X_t}{Y_t^\star}) - H_t^\star]^{-\gamma} = 0. \tag{IA.2}$$

The optimal amount of exports is the solution to equation (IA.2) provided that it is positive and satisfies the following conditions: exports are below endowments, consumption is above habit in both countries, and a positive fraction of the export makes it to the shore. A closed-form solution can be found for log utility ($\gamma = 1$) or when there is no quadratic cost. The Annex at the end of this document details the simulation method in the general case with quadratic costs.

The case of foreign country exports is obviously symmetric. If the foreign country exports ($X_t = 0$, $X_t^\star \geq 0$), the first order condition with respect to $X_t^\star$ is then:

$$-[Y_t^\star - X_t^\star - H_t^\star]^{-\gamma} + [1 - \tau - \delta\frac{X_t^\star}{Y_t}][Y_t + X_t^\star(1 - \tau - \frac{\delta}{2}\frac{X_t^\star}{Y_t}) - H_t]^{-\gamma} = 0. \qquad \text{(IA.3)}$$

If there are no positive solutions to both export problems, then countries consume their endowments. There is a no-trade zone in which the marginal utility gain of shipping a good is more than offset by the trade cost. Figure IA.1 summarizes the different cases.

[Figure 1 about here.]

Real exchange rates: I assume that there are no arbitrage opportunities and that financial markets are complete. The Euler equation for a foreign investor buying a foreign bond with return $R_{t+1}^\star$ is: $E_t(M_{t+1}^\star R_{t+1}^\star) = 1$, where $M^\star$ denotes the intertemporal marginal rate of substitution or stochastic discount factor (SDF) of the foreign investor. The Euler equation for a domestic investor buying the same foreign bond is: $E_t(M_{t+1}R_{t+1}^\star \frac{Q_{t+1}}{Q_t}) = 1$, where M is the SDF of the home investor and Q is the real exchange rate expressed in domestic goods per foreign good. Because the stochastic discount factor is unique in complete markets, the change in the real exchange rate is defined as the ratio of the two stochastic discount factors at home and abroad:

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{t+1}^\star}{M_{t+1}}. \qquad \text{(IA.4)}$$

The SDF in this model is given by

$$M_{t+1} = \beta\frac{U_c(C_{t+1}, H_{t+1})}{U_c(C_t, H_t)} = \beta(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t})^{-\gamma} = \beta e^{-\gamma\left[g + (\phi - 1)(s_t - \bar{s}) + (1 + \lambda(s_t))(\Delta c_{t+1} - g)\right]}. \qquad \text{(IA.5)}$$

I now turn to the value of the real exchange rate in the model. When there is trade, one first-order condition (IA.2) or (IA.3) of the social planner's problem is satisfied, and the countries share risk. When there is no trade, the real exchange rate is determined in the asset market as the ratio of the two marginal utilities of consumption. To summarize, the real exchange rate Q_t can take the following values:

- If the domestic country exports, $Q_t = \frac{1}{1 - \tau - \delta X_t / Y_t^\star}$;

- If the foreign country exports, $Q_t = 1 - \tau - \delta X_t^\star / Y_t$;
- If there is no trade, $Q_{t+1} = (\frac{Y_t^\star - H_t^\star}{Y_t - H_t})^{-\gamma}$.

Introducing quadratic costs has an interesting implication for real exchange rates. Without quadratic costs, real exchange rates fluctuate between two constant boundaries when there is no trade and remain on a boundary when one country exports, as shown by Dumas (1992). With quadratic costs, real exchange rates are never constant even when countries export. I now turn to the simulation of this two-country model.

II. Simulation Results

I first review the calibration exercise and then report simulation results.

A. Calibration

This model is similar to the one studied in Verdelhan (2010), and I use the same preference parameters. Here I focus on the trading costs and endowment processes, which differ from those used in the main text.

Anderson and Van Wincoop (2004) provide an extensive survey of the trade cost literature and conclude that total international trade costs, which include transportation costs and border-related trade barriers, represent an ad-valorem tax of about 74%. This total trading cost encompasses border-related trade barriers, which represent a 44% cost and are estimated through direct observation and inferred costs. Transportation costs stricto sensu represent 21%. This value is close to the 25% used in Obstfeld and Rogoff (2000). I simulate the model with a proportional trade cost τ equal to 25%, 50%, and 75%, and a quadratic trade cost of $\delta = 0.2$ as in Backus, Kehoe, and Kydland (1992). This parameter ensures that trade costs increase with trade, but reasonably so: when a country imports the equivalent of 20% of its endowment, trade cost increase by two percentage points.

I now turn to the other parameters. I estimate the mean (g_y) and standard deviation (σ_y) of real per capita net income rates, and the cross-country correlation of GDP growth rates (ρ). As in the main text, I fix γ to two, which is a common value in the real business cycle literature and the value chosen by Campbell and Cochrane (1999) and Wachter (2006) in their simulations. To determine the remaining three independent parameters of the model, I target three simple statistics: the mean ($\overline{r}$) and standard deviation (σ_r) of the real interest rates and the mean Sharpe ratio ($\overline{SR}$). Matching these moments does not pin down the pro- or countercyclical behavior of real interest rates. Building on Verdelhan (2010) and the evidence therein, I impose the additional condition that real interest rates are low when conditional Sharpe ratios are high. As a result, real interest rates are procyclical in the model.

The six target moments are measured from 1947:II to 2004:IV for the U.S. economy. Net income is defined as the sum of consumption in nondurables and services plus net exports. Per capita consumption data for nondurables and services and per capita data for exports and imports come from the Bureau of Economic Analysis. Net income is more volatile than consumption growth: its standard deviation is 0.66% per quarter versus 0.51% for consumption growth. I fix the correlation between domestic and foreign endowment shocks to 0.20. This value corresponds to the correlation of the U.S. and U.K. real GDP growth rates (0.19 in the 1957:II to 2004:IV period).[1] Using the correlation of net income growth rates, as defined above, would lead to a much lower correlation (0.06). The latter, however, would be misleading as export and import series correspond to all international trade with the rest of the world and not only bilateral U.S.-U.K. trade.

U.S. interest rates, inflation, and stock market excess returns are from the Center for Research in Security Prices. The real interest rate is the return on a 90-day Treasury bill minus expected inflation. I compute expected inflation with a one-lag two-dimensional VAR using inflation and interest rates. The Sharpe ratio is the ratio of the unconditional mean of quarterly stock excess returns on their unconditional standard deviation. Table IA.I summarizes the parameters used in this paper. They are close to the ones proposed by Campbell and Cochrane (1999) and Wachter (2006). The habit process is very persistent ($\phi = 0.995$), and consumption is on average 7% above the habit level, with a maximum gap of 12% (respectively, 6% and 9% in Campbell and Cochrane (1999)).

[Table 1 about here.]

B. Results

Dynamics: For a preview of the model's dynamics, let us first consider the case of proportional trade costs but no quadratic costs. Figure IA.2 reports the time series of the real exchange rate, the surplus consumption ratios, and the export/endowment ratios for both countries during the first 3,000 periods of a simulation. Countries trade when their endowments imply differences in marginal utility of consumption that are not offset by trade costs. When countries trade, the real exchange rate is constant, equal to $1/(1-\tau)$ or $1-\tau$ depending on whether the domestic or foreign country exports. When there is no trade, the real exchange rate fluctuates between these bounds. Thus, with a low trade cost, the exchange rate mostly bounces back and forth between two boundaries and spends most of its time on the boundaries. The real exchange rate is in this case often constant, which is counterfactual. Adding quadratic trade costs leads to more reasonable patterns as shown in Figure IA.3. Even when countries trade, the real exchange rate is no longer constant and it can exceed the previous two fixed boundaries. The increasing marginal trade cost works against large import volumes, even when endowments imply large differences in marginal utility of consumption. I turn now to the summary statistics of the simulation, reported in Table IA.II, starting with exchange rate volatility.

[Figure 2 about here.]

[Figure 3 about here.]

Exchange rates and asset prices: When countries trade, SDFs become positively correlated. The lower the trade cost, the smaller the no-trade zone and the lower the exchange rate variance. At the limit, when there is no trade cost, countries share risk perfectly and the real exchange rate is constant. When proportional trade costs are equal to 50%, the volatility of the real exchange rate is in line with its empirical counterpart. This appealing result obtains in a model with volatile SDFs and reasonable asset prices.

The model is consistent with equity excess returns. When countries trade, they share risk and their consumption growth is less volatile than their endowment shocks. This in turn decreases slightly the standard deviation of real interest rates, but the model reproduces approximatively the mean and standard deviation of risk-free rates. The model implies a conservative equity premium that is lower and less volatile than the empirical average value for the U.S. from 1947 to 2004, but still leads to a Sharpe ratio of 0.5 in line with the data.

As a result, this appendix offers a novel resolution of the tension between the large implied volatility of stochastic discount factors and the comparatively low volatility of changes in exchange rates.

Note, however, that the model cannot reproduce with the same set of parameters both the pre- and post-Bretton Woods exchange rate volatilities because we know since Baxter and Stockman (1989) that real consumption growth shocks have similar volatilities in both subperiods. Explaining differences in exchange rate regimes is beyond the scope of this paper.

[Table 2 about here.]

UIP puzzle: The model also reproduces the forward premium puzzle. The UIP slope coefficient is negative and in the 95% confidence interval of its empirical counterpart. The same logic as in Verdelhan (2010) applies here. When the domestic investor is more risk-averse, the foreign currency is dominated by domestic consumption growth shocks, and it is a risky investment for the domestic investor. Moreover, when countries share risk, consumption growth shocks are positively correlated. In this case, when the domestic investor is less risk-averse than the foreign investor, the foreign currency can even provide a consumption hedge. Finally, the model implies a downward-sloping real yield curve, whereas Wachter (2006) obtains an upward-sloping real yield curve because she assumes that real interest rates are countercyclical and not procyclical as in this paper and in Verdelhan (2010). The five-year yield implied by the model is in line with the estimates from Ang, Bekaert, and Wei (2008), who find that the unconditional real rate curve is fairly flat around 1.3%.

Trade: I now turn to the trade dynamics. With only one good, the model is not suited to and does not match interesting stylized facts on trade. As previously noted, the model implies that when one country exports, the other does not, and this is clearly at odds with the data. I simply check here that the volume of trade implied by the model is reasonable. The model leads to an openness ratio of 8%, computed as the average of imports and exports divided by net income, for proportional trade costs equal to 50%. This value is in line with the empirical estimate for the U.S. from 1947:I to 2004:IV (8%). Note, however, that this empirical estimate takes into account all international trade with the U.S. and not only bilateral U.S.-U.K. trade. One would expect the openness ratio to be smaller and more volatile for one particular bilateral trade than for the sum of all exports and imports.

Exchange rates and consumption growth: The major weakness of the model lies in the implied strong and positive correlation between changes in exchange rate and consumption growth that is not apparent in the data. Backus and Smith (1993) find that the actual correlation between exchange rate changes and consumption growth rates is low and often negative. Chari, Kehoe, and McGrattan (2002), Corsetti, Dedola, and Leduc (2008), and Benigno and Thoenissen (2008) confirm their findings. Lustig and Verdelhan (2007) show that the correlation between consumption growth and exchange rates depends on interest rate differentials. Because the correlation switches sign when the interest rate differential fluctuates, a simple unconditional measure might underestimate the link between exchange rates and consumption growth. Yet, even conditionally, the correlation between consumption growth and exchange rates is low in the data and high in the model.

Backus and Smith (1993) note that in complete markets and with power utility, the change in the real exchange rate is equal to the relative consumption growth in two countries times the risk-aversion coefficient ($\Delta q_{t+1} = -\gamma[\Delta c^{\star}_{t+1} - \Delta c_{t+1}]$), thus implying a perfect correlation between the consumption growth and real exchange rate variation. In the model presented here, the presence of habits leads to a lower correlation than with power utility. But the model still implies too high a correlation between real exchange rates and consumption growth rates because a single source of shocks drives all variables. This is a major quandary in international economics, and this model does not offer a new solution to this puzzle. Instead, I use the model to investigate the impact of measurement errors on real exchange rates and consumption data. These measurement errors might stem from the presence of nontradable goods that the model has so far ignored.

III. Nontradable Goods and Measurement Errors

Thus far I consider only one good and assume that it is tradable. Yet, Burstein, Eichenbaum, and Rebelo (2005, 2006) estimate that at least 50% of the variation in real exchange rates is due to changes in the relative prices of nontradable goods across countries. I now introduce nontradable goods in the model.[2] I do not consider a different model, but simply reinterpret nontradables as measurement errors and investigate their impact on the main outcome of the model.

Let us assume that preferences are defined over total consumption C, which combines consumption over tradable goods C^T and nontradable goods C^N through a CES function:

$$C_t = [\psi_t^{\frac{1}{\theta}}(C_t^T)^{\frac{\theta-1}{\theta}} + (1-\psi_t)^{\frac{1}{\theta}}(C_t^N)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}},$$

where ψ_t denotes a preference shock and θ the elasticity of substitution between tradable and nontradable goods. The domestic household receives an endowment each period in tradable and nontradable goods. Assume that the domestic and foreign household can also trade a bond denominated in units of aggregate consumption. The Euler equations of the domestic and foreign investors imply that the change in the real exchange rate is still equal to the ratio of the domestic and foreign SDFs as before, but the marginal utilities of consumption are now defined with respect to tradable goods. Let us define ξ_t as

$$\xi_t = \frac{\partial C_t}{\partial C_t^T} = (\frac{\psi_t C_t}{C_t^T})^{\frac{1}{\theta}}.$$

The real exchange rate is the ratio of two marginal utilities of consumption times the ratio of domestic and foreign wedges ξ_t and $\xi_t^\star$:

$$Q_t = \frac{U^\star_{C^T,t}}{U_{C^T,t}} = \frac{U^\star_{C,t}}{U_{C,t}}\frac{\xi_t^\star}{\xi_t}. \qquad \text{(IA.6)}$$

The first term on the right-hand side, $U^\star_{C,t}/U_{C,t}$, corresponds to the ratio of marginal utilities of aggregate consumption; this is the object studied so far in this paper. The second ratio, $\xi_t^\star/\xi_t$ is new; it is time-varying if the relative endowments of tradable and nontradable goods or the relative preference parameters change. We can map the previous simulations into this framework by reinterpreting consumption growth shocks as shocks to aggregate consumption, and not simply shocks to consumption of tradable goods. In this case, the law of motion of the state variable and real interest rates are not modified, but the real exchange rate should now be computed according to equation (IA.6). I interpret the ratio of the domestic and foreign wedges ξ_t and $\xi_t^\star$ as a measurement error affecting changes in real exchange rates. I simulate the model with the same parameters as before (with $\tau = 0.25$ for trading costs).

Table IA.III shows that small measurement errors greatly reduce the correlation between real exchange rates and relative consumption growth. The simulated correlation is then within two standard deviations of the actual point estimate. Compared to the case of a single tradable good, the volatility of real exchange rates nearly doubles, and the model thus attributes 50% of the real exchange rate variation to nontradable goods.

[Table 3 about here.]

This is, however, not a complete solution to the Backus and Smith (1993) puzzle. Such a solution would require an independent calibration of the wedges ξ_t and $\xi_t^\star$, and certainly a departure from the complete market assumption. Note that market incompleteness per se does not solve the puzzle, as shown by Chari, Kehoe, andMcGrattan (2002). This is intuitive: we know from Baxter and Crucini (1995) that the equilibrium allocation in economies that only trade in uncontingent bonds is close to the first best, provided that shocks are not permanent. But Benigno and Thoenissen (2008) find that a model with incomplete markets and nontraded intermediate goods goes a long way towards its solution. This puzzle is beyond the scope of this paper, and I leave it for further research.

IV. Conclusion

This appendix presents a one-good, two-country model in which the stand-in agents have Campbell and Cochrane (1999) habit preferences. The model is parameterized to produce countercyclical risk aversion and procyclical real risk-free rates. Agents can trade, but incur proportional and quadratic trading costs. This model reproduces the first two moments of consumption growth and interest rates as well as the forward premium puzzle. It also delivers sizable stock market and currency excess returns and volatile real exchange rates. The model has one main weakness: it implies a high and positive correlation between changes in exchange rates and consumption growth because, in the model, markets are complete and only one source of shocks drives all variables. In the data this correlation is low, and even often negative. Using the model as a laboratory, I find that measurement errors in marginal utilities of consumption, possibly due to nontradable goods, might alleviate the model's weakness.

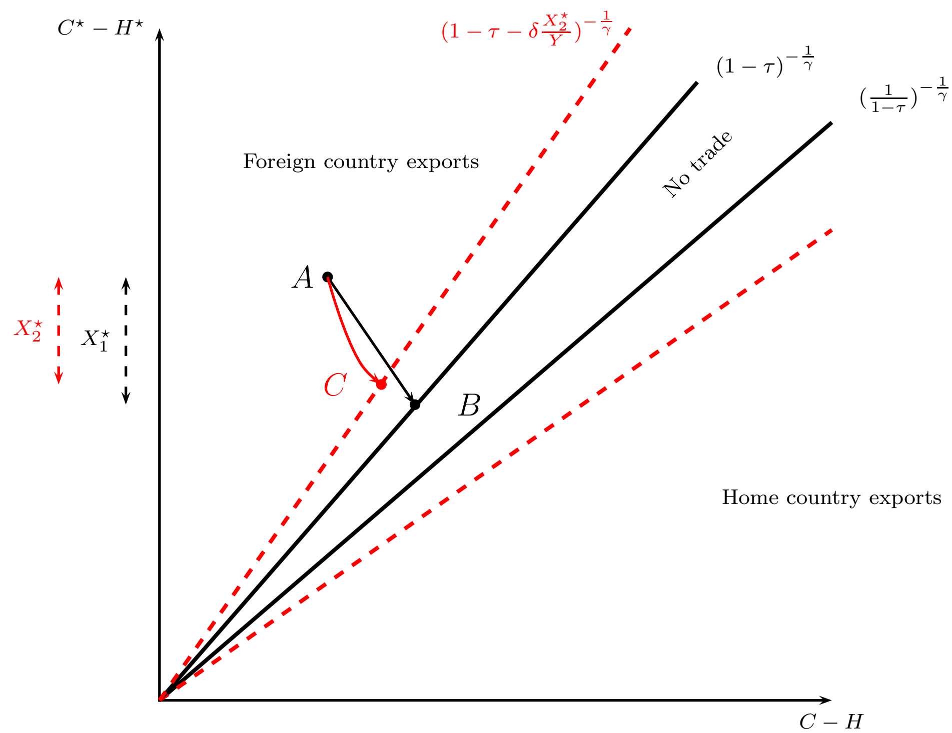

Figure IA.1. Optimal Trade The figure presents the optimal export problem with proportional and quadratic trading costs. The horizontal axis corresponds to domestic consumption net of domestic habit, $C - H$. The vertical axis corresponds to foreign consumption net of foreign habit, $C^\star - H^\star$. Assume that the two countries are characterized by the point A, where endowments (net of habit levels) are given. If there are only proportional costs, the foreign country exports $X_1^\star$ units. For each unit that the foreign country exports, the domestic country receives $(1 - \tau)$. Thus, the slope between A and B is $-1/(1 - \tau)$. At point B, the real exchange rate is equal to $(1 - \tau)$. If there are proportional and quadratic costs, the foreign country exports $X_2^\star$ units. The quadratic trading cost incurred is equal to $\delta\frac{X_2^\star}{Y}$. At point C, the real exchange rate is equal to $(1 - \tau - \delta\frac{X_2^\star}{Y})$.

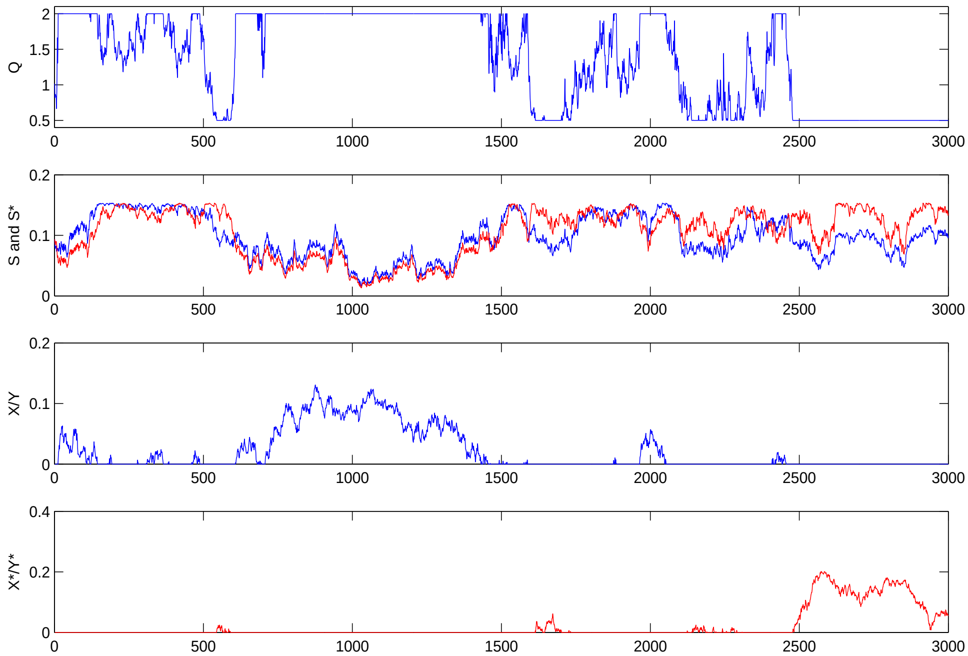

Figure IA.2. Snapshot of a simulation with proportional trading costs (first 3,000 periods). The first panel presents the real exchange rate. The second panel presents the surplus consumption ratios in the two countries (S and $S^{\star}$). The last two panels present the export/endowment ratios (X/Y and $X^{\star}/Y^{\star}$) at home and abroad. The trade cost parameters are $\tau = 0.5$ and $\delta = 0$.

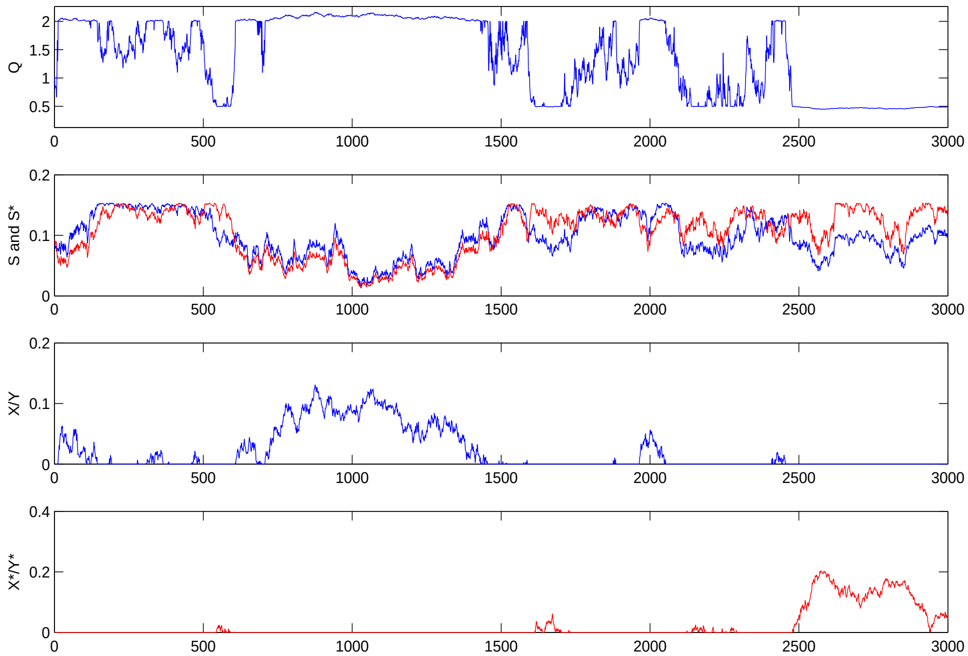

Figure IA.3. Snapshot of a simulation with proportional and quadratic trading costs (first 3,000 periods). The first panel presents the real exchange rate. The second panel presents the surplus consumption ratios in the two countries (S and $S^\star$). The last two panels present the export/endowment ratios (X/Y and $X^\star/Y^\star$) at home and abroad. The trade cost parameters are $\tau = 0.5$ and $\delta = 0.2$.

Table IA.I
Calibration Parameters

The table presents the parameters of the model and their corresponding values in Campbell and Cochrane (1999)and Wachter (2006). Data are quarterly. The reference period here is 1947:II to 2004:IV (1947 to 1995 in Campbell and Cochrane (1999), 1952:II to 2004:III in Wachter (2006)). Net income is defined as the sum of consumption in nondurables and services plus net exports. Per capita consumption, exports, and imports are from the Bureau of Economic Analysis web site. Interest rates and inflation data are from the Center for Research in Security Prices. The real interest rate is the return on a 90-day Treasury bill minus expected inflation, which is computed using a one-lag two-dimensional VAR using inflation and interest rates. The UIP coefficient is computed using U.S.-U.K. exchange rates and interest rate differentials. U.K. consumption (1957:II to 2004:IV), population, interest rates, inflation rates, and exchange rates come from Global Financial Data.

	My parameters	Campbell and Cochrane (1999)	Wachter (2006)
		Endowments	
$g_y(\%)$	0.47	0.47	0.55
$\sigma_y(\%)$	0.66	0.75	0.43
ρ	0.20	–	–
		Preferences	
$\overline{r}(\%)$	0.34	0.23	0.66
γ	2.00	2.00	2.00
ϕ	0.99	0.97	0.97
$\overline{S}$	0.07	0.06	0.04
		Trade costs	
τ	0/0.25/0.5/0.75	–	–
δ	0.20	–	–

Table IA.II
Simulation: Main Results

The table presents the standard deviation (σ_y and σ_c) of real per capita net income and consumption growth, the mean ($\overline{r}$) and standard deviation (σ_r) of the real interest rate, the mean ($\overline{Rm}$) and standard deviation (σ_{Rm}) of the market return, and the standard deviation ($\sigma_{\Delta q}$) of the real exchange rate. All moments are annualized. $\rho_{\Delta q_t, \Delta c_t^\star - \Delta c_t}$ denotes the correlation between the consumption growth differential and changes in the real exchange rate. $\overline{T}$ denotes the mean openness ratio. α_{UIP} denotes the UIP slope coefficient. The parameter τ determines the size of the proportional trading cost. The model parameters are reported in Table IA.I. The last column corresponds to actual data for the U.S., and the U.S.-U.K. exchange rate over the 1947:II to 2004:IV period (1952:II to 2004:IV for $\rho_{\Delta q_t, \Delta c_t^\star - \Delta c_t}$ because of U.K. consumption series). Data are quarterly.

	Simulation Results			Data
	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	
σ_y (%)	1.32	1.32	1.32	1.32
σ_c (%)	1.02	1.05	1.16	1.02
$\overline{r}$ (%)	1.80	1.70	1.71	1.40
σ_r (%)	1.43	1.44	1.43	1.99
$\overline{Rm}$ (%)	5.43	5.35	5.35	8.63
σ_{Rm} (%)	6.64	7.02	7.71	16.70
$\sigma_{\Delta q}$ (%)	4.37	9.17	16.60	10.29
$\rho_{\Delta q_t, \Delta c_t - \Delta c_t^\star}$	0.73	0.71	0.66	−0.04
$\overline{T}$ (%)	0.12	0.10	0.05	0.08
α_{UIP}	−1.60	−1.39	−0.88	−1.29

Table IA.III
Simulation: Impact of Measurement Errors

The table first reports the standard deviation of consumption growth Δc and changes in the real exchange rate Δq. The moments are annualized. The table then reports the correlation between the consumption growth differential and changes in real exchange rate $\rho_{\Delta q_t, \Delta c_t^\star - \Delta c_t}$ and the UIP slope coefficient α_{UIP}. The first column corresponds to the benchmark model. The second column corresponds to series simulated with measurement errors. In both cases, the proportional trading cost is equal to 0.5. The last column corresponds to the data.

	No noise	Noise	Data
Δc	1.05	1.21	1.35
Δq	9.17	13.77	12.67
$\rho_{\Delta q_t, \Delta c_t^\star - \Delta c_t}$	0.71	0.31	−0.04
α_{UIP}	−1.39	−1.30	−1.29

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Annex: Simulation Method

I first draw $20,000$ independent and identically distributed endowment shocks and delete the first $10,000$. From the $10,000$ endowment shocks and the parameters of the model, I build the endowment process. To compute the price-dividend ratio and bond prices as a function of the surplus consumption ratio, I use the numerical algorithm developed by Wachter (2005). I choose a grid of 100 points in which S ranges from $5e^{-4}$ to S_{max}. I refer the reader to Wachter (2005) for details, and I focus here on the specific difficulties of this two-country economy. Solving the social planner program presents two challenges that I briefly describe below.

A. Habit and Consumption

Trade at date $t+1$ in equations (IA.2) and (IA.3) depends on the habit level at date $t+1$. The habit level cannot be computed using its exact law of motion because it requires the value of consumption at date $t+1$, which in turn depends on trade at date $t+1$ (see equation IA.1). But Campbell and Cochrane (1999) choose the sensitivity function $\lambda(s_t)$ so that the habit level at date $t+1$ does not actually depend on the consumption level at the same date. This can be shown using a first-order Taylor approximation of the law of motion of the habit level h_{t+1} when s_t is close to its steady-state value $\bar{s}$ and the consumption growth Δc_{t+1} is close to its average g_c (see footnote 1, page 6 of Campbell and Cochrane (1995)):

$$h_{t+1} = \phi h_t + [(1-\phi)h + g_c] + (1-\phi)c_t. \tag{IA.7}$$

Equation (IA.7) gives a first guess for the habit level at date $t+1$, thus allowing for the computation of trade and consumption at date $t+1$. This new estimate of consumption is used to compute the habit level using the exact law of motion, and the process is iterated until convergence.

B. Optimal Trade

No quadratic cost: When there is no quadratic cost, the domestic country exports when $(Y_t^\star - H_t^\star)(1-\tau)^{-\frac{1}{\gamma}} < (Y_t - H_t)$. If this condition is verified, the optimal amount of exports is derived from the first-order condition (IA.2):

$$X_t = \frac{Y_t - H_t - (1-\tau)^{-\frac{1}{\gamma}}(Y_t^\star - H_t^\star)}{1 + (1-\tau)^{1-\frac{1}{\gamma}}}.$$

Similarly, the foreign country exports when $(Y_t^\star - H_t^\star) > (1-\tau)^{-\frac{1}{\gamma}}(Y_t - H_t)$. If this condition is verified, the optimal amount of exports is equal to

$$X_t^* = \frac{Y_t^\star - H_t^\star - (1-\tau)^{-\frac{1}{\gamma}}(Y_t - H_t)}{1 + (1-\tau)^{1-\frac{1}{\gamma}}}.$$

As a result, there is no trade when $(1-\tau)^{\frac{1}{\gamma}} \leq (Y_t^\star - H_t^\star)/(Y_t - H_t) \leq (1-\tau)^{-\frac{1}{\gamma}}$.

Quadratic costs: In the presence of quadratic costs, there is no closed-form solution for the optimal amount of exports (except for log utility).

To find the optimal amount of exports, let us define and minimize the following function f derived from the first-order condition (IA.2):

$$f(X_t) = -[Y_t - X_t - H_t]^{-\gamma} + [1 - \tau - \delta \frac{X_t}{Y_t^\star}][Y_t^\star + X_t(1 - \tau - \frac{\delta}{2}\frac{X_t}{Y_t^\star}) - H_t^\star]^{-\gamma}.$$

The solution X_t to $f(X_t) = 0$ has to satisfy three conditions. First, a country cannot export more than its endowment; thus X_t is in the interval $0 \leq X_t \leq Y_t$. Second, habit preferences prevent consumption from falling below the habit level in both countries; thus $X_t \leq Y_t - H_t$ and $Y_t^\star + X_t(1-\tau-\frac{\delta}{2}\frac{X_t}{Y_t^\star}) - H_t^\star \geq 0$. The latter condition imposes $X_t \in [x_{1,t}, x_{2,t}]$, where $x_{1,t} = Y_t^\star(1-\tau-\sqrt{\Delta_t})/\delta$ and $x_{2,t} = Y_t^\star(1-\tau+\sqrt{\Delta_t})/\delta$ when $\Delta_t = (1-\tau)^2 + 2\delta(Y_t^\star - H_t^\star)/Y_t^\star > 0$. Third, the foreign country imports X_t only if a positive fraction of the good makes it to its shore, and thus $0 \leq X_t \leq 2Y_t^\star(1-\tau)/\delta$. To satisfy the three conditions X_t has to be in the interval $[0, min(Y_t - H_t, 2Y_t^\star(1-\tau)/\delta)] \bigcap [x_{1,t}, x_{2,t}]$.

Note that when the endowment level is above the habit ($Y_t^\star - H_t^\star > 0$), then $\Delta_t > 0$, $x_{1,t} < 0$, and $x_{2,t} > 2Y_t^\star(1-\tau)/\delta$. Thus, the solution of the maximization problem is in the interval $[0, min(Y_t - H_t, 2Y_t^\star(1-\tau)/\delta)]$. In this case, over this simple interval, a solution exists if and only if

$$\frac{Y_t^\star - X_t^\star}{Y_t - X_t} < (1-\tau)^{\frac{1}{\gamma}}. \tag{IA.8}$$

Note that f is decreasing:

$$\begin{aligned} f'(X_t) &= -\gamma[Y_t - X_t - H_t]^{-\gamma-1} - \frac{\delta}{Y_t^\star}[Y_t^\star + X_t(1-\tau-\frac{\delta}{2}\frac{X_t}{Y_t^\star}) - H_t^\star]^{-\gamma} \\ &- \gamma[1-\tau-\delta\frac{X_t}{Y_t^\star}]^2[Y_t^\star + X_t(1-\tau-\frac{\delta}{2}\frac{X_t}{Y_t^\star}) - H_t^\star]^{-\gamma-1}. \end{aligned}$$

Thus, there exists an optimal amount of exports if $f(0) > 0$ and $f(min[Y_t - H_t, 2Y_t^\star(1-\tau)/\delta]) < 0$. The first boundary condition $f(0) > 0$ is equivalent to condition (IA.8). This boundary condition also defines cases when the domestic country exports under no quadratic costs.

Let us check that the second boundary condition $f(min[Y_t - H_t, 2Y_t^\star(1-\tau)/\delta]) < 0$ is always satisfied. When $Y_t - H_t \geq 2Y_t^\star(1-\tau)/\delta$, the boundary condition $f(2Y_t^\star(1-\tau)/\delta) < 0$ is always satisfied:

$$f(2Y_t^\star(1-\tau)/\delta) = -[Y_t - 2Y_t^\star(1-\tau)/\delta) - H_t]^{-\gamma} - [1-\tau][Y_t^\star - H_t^\star]^{-\gamma} < 0.$$

When $Y_t - H_t \leq 2Y_t^\star(1-\tau)/\delta$, there also exists a solution to $f(X_t) = 0$ because $f_{X_t \to Y_t - H_t}(X_t) \to -\infty$.

Notes

[1]The correlation coefficient is lower than in Backus, Kehoe, and Kydland (1992). I use growth rates over the 1952:II to 2004:IV period, whereas Backus, Kehoe, and Kydland (1992) use HP-filtered series over the 1960:I to 1990:I period.

[2]Crucini, Telmer, and Zachariadis (2005) show that most of the price dispersion across countries is attributable to the tradeability of inputs, not the tradeability of the final good. The model in this paper does not have a production sector. As a result, "non tradable goods" can also be interpreted here as referring to the nontradeability of inputs.