The British Mathematical Impact: Newton, Cayley, and Beyond

One of Britain’s historical achievements is its advancement of mathematics. Another is its exploration of the earth, which, of course, required mathematical and scientific knowledge. James Cook entered the Pacific Northwest area in 1778, and George Vancouver, another British sailor, surveyed Puget Sound between 1792 and 1794. In 1810, the Canadian-British North West Company established a fur trading post named Spokane House, which was located at the confluence of the Spokane and Little Spokane Rivers and was the first permanent British settlement in the present state of Washington. Because of the influence of these explorers and settlers on this area, we thought it would be interesting to examine some of Britain’s mathematical accomplishments.

Let us begin a millennium earlier with Charlemagne (742-814) in the eighth century. Although illiterate himself, he recognized the need for education, and he launched the Carolingian Renaissance about 780 by inviting scholars, including the Englishman, Alcuin of York (ca. 735-804), to come to his court. The most accomplished intellectual of his time, Alcuin introduced a curriculum based on the seven liberal arts of the quadrivium (arithmetic, geometry, astronomy, and music) and the trivium (logic, grammar, and rhetoric). He wrote elementary texts in each of these areas, searched for additional manuscripts to be read by his students, and is thought to have composed a book entitled *Propositions for Sharpening Youthful Minds*, which consisted of fifty-three logical and arithmetic puzzles.

Another early English proponent of mathematics was Roger Bacon (1214-1292), who taught at both Oxford University and the University of Paris, where he was known as the “wonderful teacher.” Although he was more interested in science than mathematics, he understood the value of quantification in the pursuit of scientific knowledge. His writings contain such statements as:

> Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy;

> If in the other sciences we should arrive at certainty without doubt and truth without error, it behooves us to place the foundations of knowledge in mathematics;

> Mathematics is the gate and the key of science.

Among his achievements, Bacon was the first person to advocate reform of the Julian calendar. In one of his most revolutionary insights, he sometimes hinted at the use of experimentation in science, an idea that was several centuries ahead of its
time. He also grasped the possibility of applying scientific discoveries to invention. There are suggestions in his writings of submarines, automobiles, and airplanes.

Bacon died on the threshold of the fourteenth century. There was little intellectual activity in Britain for the next two hundred years, in part because of the spread of the Black Death in Europe and the Hundred Years’ War (1337-1453) between England and France. During the sixteenth century, Robert Recorde (1510-1558) founded the British school of mathematics by publishing the first English book on algebra. Entitled The Whetstone of Witte, it encouraged the study of mathematics in England during the Renaissance. He also produced an abridgement of Euclid’s Elements, which was the first English work on geometry, and one on astronomy, called Castle of Knowledge, which supported the Copernican theory of the solar system. Another of his contributions was the modern symbol for equality. He devised it, he said, because he could think of nothing more equal than two parallel lines.

Castle of Knowledge appeared in 1551. Copernicus, who died in 1543, did not publish his book on the heliocentric system until late in his life because he feared persecution for challenging the dominant geocentric view. The fact that Recorde’s work was issued only eight years after Copernicus’ death illustrates the intellectual freedom in Britain at the time and boded well for future scholarly activity there.

At the same time, advances in astronomy emphasized the need for improved methods of computation. John Napier (1550-1617) of Scotland responded to this challenge by developing the first system of logarithms. His method was geometric rather than algebraic, and his approach led to logarithmic properties that were somewhat different from the modern ones. He also had no concept of a logarithmic base. Nevertheless, his work created a system essentially equivalent to one with the base of 1/e. The English mathematician, Henry Briggs (1561-1639), modified Napier’s work to define the common logarithms we use today.

Napier coined the term “logarithm” to mean “reckoning number,” which emphasized the computational value of the idea. The French mathematician and mathematical physicist, Pierre-Simon de Laplace, commented that “by shortening labors” logarithms had “doubled the lives of astronomers” by cutting their work in half.

Thomas Harriot (1560-1621) established the study of the theory of equations in England by solving linear, quadratic, cubic, and quartic equations, constructing an equation from its roots, and finding relationships between the solutions and the coefficients of an equation. Just as Recorde introduced the symbol for equality, Harriot created the inequality signs that have become standard. He also advanced astronomy by discovering sunspots and by identifying the moons of Jupiter independently of Galileo. He taught navigation to the sea captains of his patron, Sir Walter Raleigh, and in 1585 became the first European mathematician to have a connection with America when Raleigh sent him to survey what is now North Carolina.
William Oughtred (1574-1660), a younger contemporary of Harriot, wrote a popular textbook on arithmetic and algebra that contributed significantly to the dissemination of mathematical knowledge more widely in England. He recognized the value of logarithms and used the concept to invent the slide rule. As with Recorde and Harriot, another of his interests was the improvement of mathematical notation. He introduced the cross (\(\times\)) for multiplication, the four dots (\(\cdot \cdot \cdot \)) for a proportion, and the abbreviations we use for the sine and cosine functions.

John Wallis (1616-1703) was one of the most significant British mathematicians prior to Newton. He was tutored by Oughtred, taught at Oxford University for more than fifty years, and was an expert in cryptography. An author of several books, including one on analytic geometry that developed the conic sections algebraically rather than as sections of cones, he was best known for his *Arithmetica Infinitorum*, which helped to prepare the way for Newton’s development of calculus. In it, Wallis obtained the power rule for integration by generalizing from specific cases. He also produced a work on algebra that was the first major book in England to discuss the history of mathematics. Unlike Harriot and Oughtred, he endorsed the use of negative and imaginary numbers, represented the latter as points on an axis perpendicular to the real axis, and prepared the way for negative and fractional exponents.

Christopher Wren (1632-1723) was also a student of Oughtred. He taught mathematics and astronomy at Oxford and wrote on solid analytic geometry, optics, mathematical physics, and celestial mechanics. Indeed, if not for the Great Fire of 1666 in London, he might be better known as a mathematician than as an architect. The conflagration, however, turned his attention to architecture and the reconstruction of numerous churches and public buildings, including St. Paul’s Cathedral. He is also credited with the design of the Wren Building on the campus of the College of William and Mary in Williamsburg, Virginia. William and Mary is the second oldest college in the United States, and the Wren Building is the oldest academic building still in use in the country.

Isaac Barrow (1630-1677) became a professor of mathematics at Cambridge University in 1663, by which time he was recognized as one of the leading British mathematicians of his era. Also renowned as a classical scholar, he translated works of Euclid, Apollonius, and Archimedes. His research on finding tangents to curves and calculating areas bounded by curves led him to the threshold of calculus. His method of finding the slope of a tangent to a curve is similar to the standard explanation of differentiation found in calculus textbooks today. However, he lacked the concept of a limit to make his argument rigorous.

Barrow offered a series of lectures at Cambridge concerning his work on tangents and areas, which showed that he fully understood the inverse relationship between differentiation and integration. However, he did not grasp the possibility of generalizing his insights to create a new branch of mathematics. With his
encouragement, that task fell to Isaac Newton (1642-1727), one of the students who attended the class.

As a child, Newton demonstrated little of the precocity that characterized his later groundbreaking achievements. However, an uncle recognized his potential and arranged for him to attend Cambridge. He entered in 1661, and as mentioned above, was strongly influenced by the arrival of Isaac Barrow in 1663. In 1665, the university closed for two years because the Great Plague swept through London, and Newton sought safety by returning to his rural boyhood home.

During the next two years, Newton made three profound discoveries that had an enduring effect on mathematics and science. He developed the fundamentals of calculus, formulated the law of universal gravitation, and separated sunlight into its spectrum of colors. Any one of these would have established his scientific reputation, but he did not announce them. Rather, he returned to Cambridge in 1667 after the abatement of the plague and received his master’s degree the following year. In 1669, Barrow resigned his professorship shortly after being called to the court of King Charles II and was replaced by Newton, whom he endorsed for the position.

Newton spent the next thirty-five years at Cambridge, where he continued to work on mathematics, physics, and astronomy. Among his interests were the study of power series and their use in calculus, the development of the generalized binomial theorem, the theory of equations, the nature of light, and Kepler’s laws of planetary motion.

In 1687, with the encouragement and financial assistance of the astronomer, Edmond Halley, Newton published his greatest work, *Mathematical Principles of Natural Philosophy*, widely considered to be the most important scientific book ever written. Often referred to as Newton’s *Principia*, it provided an integrated treatment of his discoveries. While discussing calculus, he almost succeeded in defining the concept of the limit of a function although this issue was not resolved until early in the nineteenth century. Through stating his three laws of motion and using them to explain numerous natural phenomena, he revolutionized physics. By deriving Kepler’s laws from his three laws and the law of universal gravitation, he gave a unified explanation of the solar system.

In 1696, following his retirement from Cambridge, Newton was appointed warden and in 1699, master of the British mint. The position was intended to be a sinecure, but he did not treat it as such. Counterfeiting had become a major problem in England during the seventeenth century. Newton used his knowledge of chemistry to restore the integrity of the country’s currency by minting coins that could not be debased. When asked why he was interested in doing so, he replied: “In applied mathematics you must describe your unit.” Today we would say that he was an opponent of fiat currencies.
Newton died in 1727 and was buried in Westminster Abbey. The French writer, François Voltaire, who attended the funeral, was amazed at the profound esteem in which Newton was held. He wrote: “I have seen a professor of mathematics, only because he was great in his profession, buried like a king who had done good to his subjects.”

Newton’s vast range of achievements inspired other mathematicians, including the Englishman, Brook Taylor (1685-1731), who studied the infinite series that bears his name. He was also quite interested in the concept of perspective and wrote two books on the subject. Another contributor to the topic of series was the Scottish mathematician, Colin Maclaurin (1698-1746), whose series, of course, is a special case of Taylor’s. Maclaurin also obtained results on conic sections, improved the understanding of the concept of a limit, and published the method known as Cramer’s Rule before Gabriel Cramer (1704-1752) himself.

Mary Fairfax Somerville (1780-1872) was a self-educated native of Scotland, who mastered Laplace’s monumental work, Celestial Mechanics, his completion of Newton’s analysis of the solar system. Her accomplishment was so impressive that she was asked to write an explanation of the treatise that would make it more accessible to the general public. The result, complete with the necessary mathematical details, was widely used as a textbook for mathematics and astronomy students for almost a century.

Despite these achievements, British mathematics went into decline during the eighteenth century. It is well known that the German mathematician, Gottfried Wilhelm Leibniz (1646-1716), also created calculus as a separate branch of mathematics. Although Newton’s work in this area preceded Leibniz’, the latter published his first. As a result, a bitter dispute concerning priority arose between the two. Today it is recognized that they worked independently and deserve equal credit. However, as a result of the quarrel, British mathematicians became increasingly isolated from their European colleagues and failed to keep pace with advances on the Continent.

Mathematics in Britain began to revive early in the nineteenth century because of several students at Cambridge, including Charles Babbage (1791-1871). Motivated by having read Elementary Treatise on Differential Calculus and Integral Calculus by the French mathematician, S. F. Lacroix, they formed an organization that they called the Analytical Society. Their goal was to promote Leibniz’ analytic methods and differential notation in calculus as opposed to Newton’s geometric methods and dot notation, which students found difficult to understand. By 1820, Leibniz’ approach had begun to replace Newton’s in British universities.

In the same year Babbage began work on the project for which he is best remembered, the invention of the modern computer. He called his earliest model the Difference Engine, which operated by means of repeated additions and could work with six-figure numbers. He then designed an enlarged version, one that could handle twenty digits. However, despite receiving a grant of 20,000 pounds from the
government and investing 17,000 pounds of his own money in the project, he was unable to build a working prototype because the technology of the time could not produce sufficiently exact parts.

While working on the Difference Engine, Babbage conceived of a more sophisticated version he called the Analytical Engine, one that would be able to add, subtract, multiply, and divide. He designed it to run on steam power, to use punched cards to input instructions, and to store one thousand numbers of fifty digits each. He appealed again to the government for financial support, but his request was denied because of the failed attempt to build the Difference Engine.

Mary Fairfax Somerville was a friend of Babbage, and she introduced him to Ada Lovelace (1815-1852), the mathematically talented daughter of the British poet Byron. Babbage asked her to translate a paper on the Analytical Engine, which was written by the Italian engineer, L. F. Menabrea, who later became prime minister of Italy. Lovelace added extensive notes to her translation, including explicit instructions that would allow the machine to solve specific problems. The result was so detailed that many consider her to be the first computer programmer.

One of the problems that concerned nineteenth century mathematicians was the precise formulation of the concept of a complex number. The method of completing the square to solve quadratic equations had been known since antiquity, but whenever square roots of negative numbers were encountered, they were rejected as meaningless. However, after Carl Gauss proved the Fundamental Theorem of Algebra in 1799, the importance of complex numbers came to be recognized. Caspar Wessel (1743-1818) of Norway and Jean-Robert Argand (1768-1822) of France represented them as two-dimensional vectors, and Gauss considered them to be points in the complex plane. The final step was taken by the Irish mathematician, William Rowan Hamilton (1805-1865), who was the first to define them in purely arithmetic terms as ordered couples of real numbers.

Hamilton wanted to extend the idea of a complex number to three dimensions to represent forces that do not lie in the same plane. After some fifteen years of effort, he realized that he had to abandon the commutative property of multiplication in order to do so. He introduced the concept of a quaternion, an expression of the form $a + bi + cj + dk$, where $a$ is a scalar and the symbols $i$, $j$, and $k$ have a function similar to the complex number $i$. Specifically, the square of each has the value of $-1$, as does the product $ijk$. The lack of commutativity results from the fact that $ij = k$, $jk = i$, and $ki = j$, but $ji = -k$, $kj = -i$, and $ik = -j$. Hamilton’s identification of these relationships came to him in a flash of insight as he was walking across a bridge in his home city of Dublin, Ireland, and to commemorate his discovery, he carved the inscription $i^2 = j^2 = k^2 = ijk = -1$ into the bridge with his knife.
Hamilton’s hope that quaternions would become the basis of mathematical physics was not realized, but his work still led to a significant advance. By dispensing with the scalar term of a quaternion and defining $i$, $j$, and $k$ to be the unit vectors along the $x$-, $y$-, and $z$-axes respectively, the Englishman, Oliver Heaviside (1850-1925), and the American, Josiah Willard Gibbs (1839-1903), independently laid the foundation for three-dimensional vector analysis. They defined the dot and cross products of two vectors, and Gibbs introduced the standard notation for each one. As we know, the cross product is noncommutative, and unlike the case with numbers, it can be zero even though neither vector is. This latter property is also true of the dot product although it is commutative.

Hamilton’s introduction of noncommutative multiplication led to the investigation of new algebraic structures. A key figure in this regard was Arthur Cayley (1821-1895), who, like Newton, attended Cambridge and made fundamental contributions to matrix theory by formulating many of its basic concepts. In particular, he defined the addition of matrices, the zero and identity matrices, the transpose and inverse of a matrix, and symmetric and skew symmetric matrices. One of his most innovative ideas was the definition of matrix multiplication, which, of course, is noncommutative. He observed that a system of linear equations in the same number of unknowns can be expressed in matrix form and solved by using the inverse of the coefficient matrix. He also noted that if the determinant of the coefficient matrix is zero, then the matrix is not invertible.

Cayley’s interests spanned a wide variety of mathematical fields, including n-dimensional analytic geometry, the theory of curves and surfaces, properties of determinants, and especially invariant theory, an area in which he collaborated with James J. Sylvester (1814-1896) and which was used by Albert Einstein (1879-1955) in his theory of relativity. The origin of invariant theory lies in the work of Gauss and George Boole, whom we will discuss shortly. It deals with the properties of algebraic expressions, such as polynomials, that do not change when linear transformations are applied to them. Although opposite in temperament, Cayley, who was serene and systematic, and Sylvester, who was volatile and disorganized, had a long, fruitful mathematical partnership. When Johns Hopkins University opened in Baltimore in 1876, it hired Sylvester to become a professor of mathematics, and while there, he launched the American Journal of Mathematics, the first such publication in the United States. In 1882, when the university invited Cayley to deliver a lengthy series of lectures, Baltimore was hailed as the center of mathematics in America.

Another of Cayley’s contributions was the idea of an abstract group. This notion has roots in the work of Lagrange and Galois, but it was restricted originally to permutation groups. Cayley introduced groups the elements of which are not permutations, and although his work in this respect received little attention initially, it was a step toward the formal definition of the concept. With the development of modern algebra, its importance became clear.
To support himself after graduating, Cayley established a lucrative law practice. At the same time he continued his mathematical research, and when offered the opportunity to return to his alma mater as a professor, he abandoned his legal career to devote himself exclusively to mathematics. A prolific writer, he published 966 articles during his lifetime. Only Euler and Cauchy published more. Among the many outstanding mathematicians who studied and worked at Cambridge, he is ranked second only to Newton.

George Boole (1815-1864) expanded the use of algebra in a manner that differed from the work of Hamilton and Cayley. Because his economic circumstances did not allow him to pursue higher education, he learned mathematics on his own by studying the works of Newton, Lagrange, and Laplace. The first mathematics book to influence him was Lacroix’ calculus text, which, as we mentioned, inspired Babbage.

Boole’s major goal was to develop a method of expressing the relationships of Aristotelian logic in symbolic terms. To do so, he created an algebra of sets, known today as Boolean algebra. Decades later, his work had a profound impact on an American electrical engineering student named Claude Shannon (1916-2001), who applied it to the design of electronic switching circuits as part of his master’s thesis. Because of its impact on the burgeoning field of computer science, the thesis has been called one of the most important of the twentieth century.

During World War II, Shannon worked in the area of cryptanalysis, as did the English mathematician, Alan Turing (1912-1954), who played a key role in deciphering the Nazis’ Enigma code at Bletchley Park in England. The two discovered that they had complementary ideas concerning the design of computers. During and after the war, Turing made fundamental contributions to computer science, and Shannon founded the field of information theory.

Boole also contributed to invariant theory, which led to a friendship with Cayley, and to differential equations, in which he introduced the concept of a differential operator. In the latter area he began with linear equations with constant coefficients and then extended his method to ones with variable coefficients. His approach constructed a polynomial equation in which the differential operator is the variable. Finding its roots allowed him to solve the original equation.

The effort to reconnect British mathematics with work on the European continent continued into the late nineteenth and early twentieth centuries. One of the leaders in this regard was Godfrey Harold Hardy (1877-1947). His book, *A Course in Pure Mathematics*, published in 1908, had the same effect on British mathematics education as Lacroix’ calculus book had on Babbage and the other members of the Analytical Society.
Just as Cayley and Sylvester benefited from their collaboration, so did Hardy and John Edensor Littlewood (1885-1977). They made numerous contributions to number theory, their primary area of interest, including work on the Riemann Hypothesis, which remains one of the major unsolved problems in mathematics to this day. Their partnership spanned a period of thirty-five years and yielded almost one hundred papers.

Hardy also sponsored the career of the Indian prodigy, Srinivasa Ramanujan (1887-1920). With the encouragement of friends, Ramanujan sent some of his results to Hardy, who recognized their significance immediately. He invited Ramanujan to Cambridge, where he was working at the time. Between 1914 and 1917, Ramanujan wrote thirty-two papers under the direction of Hardy and Littlewood, seven of them in collaboration with Hardy. In 1917, Ramanujan’s health began to fail, apparently from tuberculosis. He returned to India in 1919 following the end of World War I and died the following year at the age of thirty-two. Despite his short life, his notebooks are still being deciphered, including one discovered in 1976.

In the years since Hardy and Littlewood were active, other British mathematicians have continued to contribute to number theory. The most famous example is Andrew Wiles’ (b. 1953) proof of Fermat’s Last Theorem in 1994.

We mentioned Voltaire’s comment about Newton after attending his funeral. Let us close by examining Newton’s cultural impact, which extended far beyond the shores of Britain. His work launched the Age of Reason by demonstrating the power of the human mind to grasp the laws of nature and prepared the way for the Industrial Revolution, which began about 1760 in England, less than a hundred years after publication of the *Principia*. The philosopher, John Locke (1632-1704), was a friend of Newton and understood in a general way what he had accomplished mathematically and scientifically. Locke advocated extending Newton’s approach by applying reason to all questions, both scientific and humanistic. His influence led to the designation of the eighteenth century portion of the Age of Reason as the Enlightenment.

Locke wrote that “in all sorts of reasoning, every single argument should be managed as a mathematical demonstration.” His influence led to the use of the axiomatic method and quantitative reasoning in fields of knowledge previously approached qualitatively. It is not an exaggeration to say that each of us has benefited from the cultural influence of Newton, both mathematically and otherwise.

Bibliography


Videos

The Bit Player (Shannon)

The Genius of George (Boole)

The Imitation Game (Turing)

The Man Who Knew Infinity (Ramanujan)

All are available at Prime Video.