Validating Solutions to Equations

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Why Validate?

Self-monitoring is one of the “Ten principles of Learning” (1998)

The more responsibility students take for their learning, the greater the results (Bransford, 1999)

The ability to self-validate one’s knowledge is central for lifelong learning and personal growth” (Armstrong, 2007)
Part of the Process

Step 4: Verification (Rubenstein, 1975)
Step 4: Look Back (check and interpret) (Polya, 1957)
Step 5: Look Back (Woods, 2000)
Step 8: Test/validate (Myrvaagnes, 1999)
Definition

A number $a$ is a **solution** to an algebraic equation $f(x)=g(x)$ iff $f(a)=g(a)$

The **solution set** for $f(x)=g(x)$ is the set of all numbers that are solutions to $f(x)=g(x)$
The Basic Check (no error)

\[2x + 3 = 5\]

\[2x = 2\]

\[x = 1\]

\[2(1) + 3 = 5\]

\[5 = 5\]

Solution Set \[\{1\}\]
The Basic Check (error)

\[ 2x + 3 = 5 \]
\[ 2x = 8 \]
\[ x = 4 \]

\[ 2(4) + 3 = 5 \]
\[ 11 \neq 5 \]
The Step-by-Step Check

\[ 6 - 2(x + 1) = 14 \]
\[ 6 - 2x - 2 = 14 \]
\[ 6 - 4x = 14 \]
\[ -4x = 8 \]
\[ x = -2 \]

\[ 6 - 2(-2 + 1) \neq 14 \]
\[ 6 - 2(-2) - 2 \neq 14 \]
\[ 6 - 4(-2) = 14 \]
\[ -4(-2) = 8 \]
\[ -2 = -2 \]
Activity: Try the Step-by-Step Check

\[ 12 - 5(x + 3) = 2x - 5 \]

1. Solve the equation making a **single** mistake.
2. Exchange papers
3. Use the step-by-step check to find the mistake
When two mistakes make a correct solution

\[5 + 2(x - 4) = 9\]
\[5 + 2x - 4 = 9\]
\[1 + 2x = 9\]
\[2x = 8\]
\[x = 6\]
How many solutions should we get?

$$4x^2 + 15x + 9 = 0$$

$$(4x + 3)(x + 3) = 0$$

$$x = -\frac{3}{4} \quad x = 3$$

$$4\left(-\frac{3}{4}\right)^2 + 15\left(-\frac{3}{4}\right) + 9 = 0$$

$$4(3)^2 + 15(3) + 9 = 0$$

$$0 = 0 \quad \times \quad 90 \neq 0$$
### How many solutions should we get?

<table>
<thead>
<tr>
<th>Equation Type</th>
<th>No Solutions</th>
<th>One Solution</th>
<th>Two Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>contradiction</td>
<td>Typical</td>
<td>Not Possible</td>
</tr>
<tr>
<td>Absolute Value</td>
<td>$</td>
<td>f(x)</td>
<td>= a$ where $a &lt; 0$</td>
</tr>
<tr>
<td>Quadratic</td>
<td>contradiction</td>
<td>repeated factor discriminant = 0</td>
<td>Typical</td>
</tr>
</tbody>
</table>
Extraneous Solutions - Rational Equations

\[
\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}
\]

\[
(x - 6)(x - 2) \left( \frac{1}{x-6} + \frac{x}{x-2} \right) = \left( \frac{4}{x^2 - 8x + 12} \right) (x - 6)(x - 2)
\]

\[
x - 2 + x(x - 6) = 4
\]

\[
x^2 - 5x - 6 = 0
\]

\[
x \neq 6, x = -1 \quad \{ -1 \}\]
When do we get extraneous solutions?

<table>
<thead>
<tr>
<th>Equation Type</th>
<th>Why</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational</td>
<td>Multiplying by zero</td>
<td>Check for division by zero in initial equation</td>
</tr>
<tr>
<td>Radical</td>
<td>Squaring both sides</td>
<td>Check for opposite bases on the squaring step</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>Log Properties</td>
<td>Check for negative arguments on the log property step</td>
</tr>
</tbody>
</table>
Error Logs

Record all mistakes, errors, and failures
Include what, when, and correction
Based on “Issues Log” from Kegan (2016)
### Sample Error Log

<table>
<thead>
<tr>
<th>Error in original work</th>
<th>Corrected work</th>
<th>Description of error</th>
<th>When and where it occurred</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 - 2x - 2 = 14$</td>
<td>$5 - 2x - 2 = 14$</td>
<td>Combining unlike terms</td>
<td>Solving linear equation in HW 2 (10/5)</td>
</tr>
<tr>
<td>$6 - 4x = 14$</td>
<td>$3 - 2x = 14$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-2x = 8$</td>
<td>$-2x = 8$</td>
<td>Adding/subtracting when I should be</td>
<td>Solving linear equation in HW 3 (10/7)</td>
</tr>
<tr>
<td>$x = 10$</td>
<td>$x = -4$</td>
<td>dividing</td>
<td></td>
</tr>
<tr>
<td>$5 - 2(x+1) = 14$</td>
<td>$5 - 2(x+1) = 14$</td>
<td>Not distributing</td>
<td>Solving linear equation in HW 4 (10/12)</td>
</tr>
<tr>
<td>$5 - 2x + 1 = 14$</td>
<td>$5 - 2x - 2 = 14$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Blank Error Log

<table>
<thead>
<tr>
<th>Error in original work</th>
<th>Corrected work</th>
<th>Description of error</th>
<th>When and where it occurred</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>
Best Practices for Orientation

- Convince students of the importance of validation
- Establish expectations for the course
- Share success stories with these techniques
Homework

Assign problems with no answer key
Have students grade their homework
Spot check in class
Tests

Require validation as part of written work

Indicate value of check on test

Discuss errors in class
Sample Test

MAT 121 Exam

Prof. Watts

Allowed Tools: Scientific Calculator, Ruler

Allowed Resources: None

Directions:

- Show all work - Minimum number of steps required is given. Write out anything put into a calculator.
- Check all work - show validation for all problems. Find and correct all mistakes.
- No communicating with others or using phones for any reason during the exam.

Total Points: 100

1. (5 points) Solve the following equation. Show at least 4 steps (1 point each) plus a check (1 point)

\[ 3(x + 2) - 12 = 5(x + 1) \]
Best Practices for the Facilitator

Model self-validation
Keep your own error log
Explore all errors
Getting Student Buy-In

Your student asks “Is this right?”, what do you say?

Your student asks “I can’t find my mistake?”, what do you do?
Validation Activity

Find a validation technique for one of the following processes:

1. Graphing a Linear Function
2. Solving a Quadratic Inequality
3. Finding the Derivative of a Function
Thank You AMATYC!

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Works Cited


