All Math Students Are Mathematicians: Active Learning and Conceptual Understanding in CUNY Start

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AMATYC 2021 Conference
October 29th, 2021
Phoenix, AZ
Agenda

1. Introduction to CUNY and CUNY Start
2. Developmental math students’ views of math
3. Learning goals in CUNY Start Math
4. Sample lesson from CUNY Start curriculum
5. Effective Questioning
6. Questions
The City University of New York

The City University of New York (CUNY) is the largest urban university system in the US, with over 270,000 students enrolled.

CUNY includes 25 campuses located across all five boroughs of New York City, including 7 community colleges, 4 comprehensive colleges, and 7 senior colleges.

In the 2021-22 academic year, CUNY Start/Math Start programs are running at 8 CUNY colleges.
2007 • College Transition Program (part of CUNY Adult Literacy/HSE program)

2011 • CUNY Start expanded to seven CUNY colleges

2014 • Math Start added as a shorter intervention

2019 • CUNY Start & Math Start served over 4,000 students at 10 colleges
      • Quantitative Reasoning Curriculum and Co-requisite-Like Attached Credit Courses Piloted

2021 • CUNY Start and Math Start continue to serve students at these colleges in online, hybrid, and in-person formats
➢ Intensity/Hours:
  ➢ **CUNY Start FT program (Math and R/W):**
    25 hours per week (11 hours of math)
  ➢ **CUNY Start PT program (Math or R/W):**
    12 hours per week (10.5 hours of math)
  ➢ **Math Start:**
    Up to 20 hours per week (18.5 hours of math)

➢ Advising & College Success Seminar
➢ Instructor/Advisor collaboration
➢ Built-in tutoring by trained alumni
➢ Same CUNY-wide proficiency standards
➢ Phase 1 and Phase 2
➢ $75 student fee, no financial aid used
CUNY Start Outcomes

http://www1.cuny.edu/sites/cunystart/resources/how-are-we-doing/

Summer 2018

- 77% of 841 students initially requiring remediation in math, 77% achieved proficiency.

- Remedial needs of students before and after program participation. (N=841).

- Pre-Math Start: 11% would have to take developmental Pre-Algebra & Algebra, 89% would have to take developmental Algebra.
- Post-Math Start: 23% no developmental math requirement.

Office of Research, Evaluation & Program Support
CUNY Office of the Senior University Dean for Academic Affairs
Solve mentally:

$10 \times 3 = \ $

$10 \times 13 = $ 

$20 \times 13 = $ 

$30 \times 13 = $ 

$31 \times 13 = $ 

$29 \times 13 = $ 

$22 \times 13 = $ 

$\ $
Example of student work:

\[
\begin{align*}
10 \times 3 &= 30 \\
10 \times 13 &= 130 \\
20 \times 13 &= 86 \\
30 \times 13 &= 120 \\
31 \times 13 &= 123 \\
29 \times 13 &= 116 \\
22 \times 13 &= 92
\end{align*}
\]
Student Views

• “Math is just all these steps.”
• “In math, sometimes you have to just accept that that’s the way it is and there’s no reason behind it.”
• “I don’t think [being good at math] has anything to do with reasoning. It’s all memorization.”
Learning Goals in CUNY Start Math

• Keep students active in their math learning, thereby transforming their views of math and of themselves as math learners
• Deepen students’ math learning
  Conceptual understanding and procedural fluency
  Adaptive reasoning
  Cognitive autonomy
• Improve academic habits
• Reduce or eliminate developmental needs
• Prepare students to do well future classes
A Moment for Mental Math

Attempt the following problem only by using mental math. That means without a pencil and paper, a calculator, or a cell phone.

A teacher buys binders for 8 students. The binders cost $3.10 each. What is the total cost of the binders before any taxes are added?

(Remember — think about how to do this in your head!)
Curriculum Sample 1

A Moment for Mental Math
3 Scenarios and the Distributive Property

Scenario #1 — Multiplication “in parts”: 15 (1.03)
\[15(1.00 + .03) = 15 \cdot 1.00 + 15 \cdot .03\]

Scenario #2 — Writing out groups: 5 \((x+2)\)
\[5(x + 2) = (x + 2) + (x + 2) + (x + 2) + (x + 2) + (x + 2)\]
\[= x + 2 + x + 2 + x + 2 + x + 2 + x + 2\]
\[= 5x + 10\]

Scenario #3 — Rectangle area:

**Total Area** = 8\((x + 2)\)

**Total Area** = 8\(x + 16\)
How many terms do we end up with when we multiply two binomials and simplify?
Curriculum Sample 2

• When do we end up with just two terms after multiplying two binomials and simplifying?
Curriculum Sample 2

• When the product of two binomials is a binomial, what do you notice about the resulting binomial (the “answer”)?
Curriculum Sample 2

• Without actually multiplying any binomials, can you write a binomial that is the product of two binomials?
Curriculum Sample 2

- Factor:

\[4x^2 - 121\] \[16x^2 - 75\]
Some underlying pedagogical values

- Students learn most effectively when they are active in the classroom.
- Student talk is the most important talk.
- Rules can be the pedagogical endpoint, not the starting point.
- Number and realistic contexts can be entry points into more abstract ideas and formal representations.
- Multiple solution methods that students use can be encouraged and explored.
More underlying pedagogical values

• Students can learn to think and communicate like mathematicians.

• “Relentless” questioning to develop new ideas and for assessment

• Expecting and respecting errors

• Some student struggle is important – teachers should resist being “too helpful”.

Professional Development

➢ Apprenticeship Semester as Cooperating Teacher
  ❖ Observing, circulating, and tutoring
  ❖ Participating in “preview” and “reflection” PD meetings
  ❖ Teaching lessons and receiving feedback

➢ Ongoing Professional Development
  ❖ Observations & coaching
  ❖ Participating in regular PD meetings
Effective Questioning

Decide if each question below is effective or ineffective.

1. The slope of a vertical line is undefined, right?

2. How is this problem similar to others that we’ve looked at? How is it different…?

3. This is an easy one — what is the probability of rolling a 2 on a single die?

4. Everyone write down what you would do next.
Building a New Course: CUNY Start-Quantitative Reasoning

• A team of 4 curriculum developers drafting and revising new lessons over the course of a semester

• A pilot semester with 2 instructors leading class and 5 other instructors observing and offering feedback

• Continuing revisions in subsequent semesters, with new instructors observing to learn the curriculum
Questions
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Effective Questioning

Decide if or when each question below is effective or ineffective. If you decide any are ineffective, edit them to make them effective, or explain how you might use them in an effective way.

1. The slope of a vertical line is undefined, right?

2. How is this problem similar to others that we’ve looked at? How is it different…?

3. This is an easy one — what is the probability of rolling a 2 on a single die?

4. Everyone write down what you would do next.

5. Hamza [who did not volunteer], how did you do it?

\[3x^4 + 4 = y^2\]

6. A student’s written work for an implicit differentiation exercise: \[12x^3 = 2y\]
   \[y = 6x^3\]
   
   Instructor: What are you forgetting here?

7. [After a student answers a question] What do other people think of what she said?

8. What are the factors of \(x^2 + 7x + 10\)?

9. What does it mean to factor?

10. Raise your hand if you think this experiment will give us a left-tailed distribution. (Count.) Now, raise your hand if you think it will be right-tailed? (Count.) Uniform? (Count.) Bell-shaped? (Count.) Raise your hand if you’re not sure. (Count.)
11. **Student written work**: \[\sqrt[3]{x} + 2x = \sqrt[3]{27} + 2(27) = 3 + 54 = 57\]

   **Student**: I did the square root of 27, which is 3...
   **Instructor**: Don’t you mean cube root?

12. Any questions?

13. Okay, what are your questions?

14. [After students agree that 98 is the maximum of the following test scores and not the mode: 79, 85, 85, 85, 88, 90, 90, 96, 98] Can you come up with a data set that would have a mode of 98?

15. How can we be sure that’s right?

16. Is this always true?

17. What do you notice when you look at the areas under these lines on these intervals?

![Graphs showing areas under lines](image)