Perplexing Percentages and Relative Risks

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Take-home messages

- Percentages can be perplexing
  - Percentage *differences* and percentage *changes* can be *very* perplexing
- Give students lots of practice with calculating and interpreting these, based on real data
  - Sneak in topic of relative risk
Examples

1. Seat belt usage
2. Malaria vaccine
3. House prices
4. Inflation
5. Oddities
6. Blog post titles
Example 1: Seat belt usage

- 2017 Youth Risk Behavior Surveillance Survey

<table>
<thead>
<tr>
<th></th>
<th>Arizona</th>
<th>California</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rarely or never</td>
<td>173</td>
<td>103</td>
</tr>
<tr>
<td>Sometimes, most of the time, or always</td>
<td>1966</td>
<td>1675</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2139</strong></td>
<td><strong>1778</strong></td>
</tr>
</tbody>
</table>

- For each state, calculate percentage who rarely or never use seat belts.
  - AZ: 173 / 2139 ≈ 0.081, or 8.1%
  - CA: 103 / 1778 ≈ 0.058, or 5.8%
Does this mean that the percentage of teens who rarely or never wear a seat belt was 8.1% - 5.8% = 2.3% greater in AZ than in CA?

- No! The difference is 2.3 percentage points, but that’s not the percentage difference.

Calculate the percentage difference.

- \((A - B) / B \times 100\%\)
- \((8.1 - 5.8) / 5.8 \approx 0.396\)
- So this is a 0.396 \times 100\% = 39.6\% difference
Example 1: Seat belt usage

- Write a sentence to interpret this percentage difference in context.
  - The percentage of AZ teens who said that they rarely or never use a seat belt is 39.6% greater than the percentage of CA teens who said that they rarely or never use a seat belt.
Example 1: Seat belt usage

- How many times greater is the AZ percentage than the CA percentage?
  - Ratio: $8.1 / 5.8 \approx 1.396$

- Use this number in a sentence.
  - The number that I just calculated is 1.396.
Example 1: Seat belt usage

- How many times greater is the AZ percentage than the CA percentage?
  - Ratio: $8.1 / 5.8 \approx 1.396$

- Use this number in a sentence.
  - The number that I just calculated is 1.396.

- Write a sentence to interpret this number in context.
  - AZ teens were 1.396 times more likely than CA teens to say that they rarely or never use a seat belt.
Example 1: Seat belt usage

- Relative risk: ratio of “success” proportions between two groups
  - Relative risk of rarely or never wearing a seat belt = 1.396 between AZ and CA teens

- Does this number 1.396 bear some resemblance to an earlier calculation?
  - Percentage difference (as decimal) = 0.396
  - Percentage difference = (relative risk – 1) × 100%

  - Short-cut for calculating percentage difference
Example 2: Malaria vaccine

Randomized experiment conducted in Burkina Faso, reported in *Lancet* (May 15, 2021):

- High dose: 38 of 146 children aged 5-17 months contracted malaria within 6 months
- Low dose: 43 of 146 contracted malaria
- Placebo: 105 of 147 contracted malaria

For each group, calculate percentage who contracted malaria

- High: 26.0%, Low: 29.5%, Placebo: 71.4%
Example 2: Malaria vaccine

- Calculate relative risks to “high dose” group
  - Low dose: \( \frac{29.5}{26.0} \approx 1.132 \)
  - Placebo: \( \frac{71.4}{26.0} \approx 2.744 \)

- Write a sentence interpreting these in context.
  - Children who received low dose were 1.132 times more likely to contract malaria than those who received high dose.
  - Children who received placebo were 2.744 times more likely to contract malaria than those who received high dose.
Example 2: Malaria vaccine

- Use relative risks to calculate percentage differences, then interpret them.
  - Rel risk = 1.132: Risk of contracting malaria was 13.2% higher with low dose than with high dose.
  - Rel risk = 2.744: Risk of contracting malaria was 174.4% higher with placebo than with high dose.

- Wait, what – can that be right?!?
  - Sure: \((2.744 - 1) \times 100\% = 174.4\%\)
  - 100% increase would mean doubling
    - 71.4% with malaria is more than twice 26.0%
Examples 1 and 2: Seat belts and malaria vaccine

 Were these examples based on observational studies or randomized experiments?

 - Example 1: observational study
 - Example 2: randomized experiment
Example 3: House prices

- Predict house price ($K) from size (sq ft):

![House price vs size graph](image-url)
Example 3: House prices

- Sum of squared residuals from \((y = \text{mean})\) line = 120,997.32
- Sum of squared residuals from least squares line = 47,388.11

By what percentage does the least squares line reduce the sum of squared residuals, as compared to the \((y = \text{mean})\) line?

1. \((120,997.32 - 47,388.11)/120,997.32 \times 100\% \approx 60.8\%
2. This percentage reduction is what \(r^2\) measures
Example 4: Inflation

- One of my favorite topics to teach concerns CPI (Consumer Price Index) and adjusting monetary values for inflation over time
- CPI in September 2020: 260.28
- CPI in September 2021: 274.31
- Calculate percentage increase in CPI over this one-year period
  - \((274.31 - 260.28) / 260.28 \times 100\% \approx 5.4\%\)
  - This is the inflation rate from Sept 20 – Sept 21
Example 5: Oddities

- If a stock increases in value by 20% in one year and then decreases in value by 20% in the next year, has it returned to its original value?
  - No! For example:
    - $100 stock: worth $100 + $20 = $120 after year 1
    - $120 stock: worth $120 - $24 = $96 after year 2
  - Why? The 20% reduction is based on a larger baseline value than the 20% increase
Example 5: Oddities

If Bella earns 50% less than Janelle, and Kelly earns 50% more than Janelle, does Kelly earn 100% more than Bella?

- No! For example:
  - If Janelle earns $20/hour, then
  - Bella earns $10/hour and Kelly earns $30/hour

Kelly actually earns ___ % more than Bella

- \((30 - 10) / 10 \times 100\% = 200\%\)
Example 6: Blog post title

- Draft title: *A persnickety post that preaches about a pervasive, persistent, and pernicious pet peeve concerning percentages* (60% P-words)
- Actual title: *A pervasive pet peeve* (75%)
  - So, is this a 15% increase in % of P-words?
  - No, that’s the pet peeve!
  - This is a 15 *percentage point* increase and a 25% increase in the % of P-words in the title
Take-home messages

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- Give students lots of practice with calculating and interpreting these, based on real data
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Thanks very much!

- [https://askgoodquestions.blog/posts](https://askgoodquestions.blog/posts)
  - Post #28 discusses this topic

- I will repeat this presentation on Sat Nov 6 during Virtual Days at 1pm ET, 10am PT

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