Good afternoon, everyone. Today’s talk is a shorter version of my David L. Underwood memorial lecture award presentation at St. Louis Community College- FV. To be recognized by this award is a great honor on my campus and the recipient then gives a talk to a general audience consisting of faculty, students, staff, and members of the community. This a 46-year traditional at our college in recognition of former Dean of Instruction, David Underwood. These talks generally relate the recipient’s area of expertise to a topic of interest to the campus community.

I am a recently retired prof of mathematics and a travel enthusiast! One of the most amazing opportunities that I experienced as a faculty member is study-abroad travel with our students. In 2004, I traveled overseas for the first time escorting a group of STLCC students to China and continued to travel with different groups of students over the years. I know that this experience is life changing for students and to witness and assist in this transformation is truly a gift. Here is a picture including 3 of my quantitative reasoning students at the Trevi fountain in Rome.

Over the years I have researched the areas we will be visiting and created mathematical travel guides to connect my discipline to art and architecture. Here are a few examples of presentations I have done for our students and community members:

A Math Lovers Guide to Sicily where I explored the mathematics of Greek and Roman theatre design.

A math lovers guide to Northern Italy, where we explored the mathematics of the architecture and sculpture of the region.

And a personal favorite, the mathematics of Brunelleschi’s dome in Florence, Italy. Fillipo Brunelleschi excelled not only as an architect, a goldsmith, a clockmaker but also as a brilliant mathematician who invented the principles of linear perspective with the use of mirrors around 1415.

The beauty of teaching math in the context of our travels is that students point out examples of the mathematics that I have presented during our trips.

So since none of us could travel in 2020, I created a Math Lover’s Guide to Saint Louis, Missouri for my Underwood talk.
The idea behind this was to provide a safe driving tour of 15 mathematically and architecturally significant buildings, monuments, or sculptures around the St. Louis region.

Today, I will highlight six of these locations plus add two locations here in the Phoenix area which are truly worth a visit if you are interested in mid-century modern work.

**Slide #7**
Let’s begin with discussing principles that are central to studies of mathematics, art and architecture.

The principles of:
- Symmetry,
- proportion,
and use of basic two-dimensional and three-dimensional shapes,
are foundational to these studies. And while these three principles have connected our disciplines since ancient times, today, we will focus on works of Mid-Century Modern Architecture which relies heavily on pure geometric forms.

Let’s begin with the idea of **symmetry** and the types of symmetry:

**Slide#8**
An object possesses **reflection (or bilateral) symmetry** if it remains unchanged when reflected over a straight line.

**Rotation (or radial) symmetry** if it remains unchanged after being rotated some measure about a point.

Or **translation symmetry** if it remains the same after being shifted to the left or to the right.

**Slide#9**
Here are some examples of symmetry used in floorplan designs by Frank Lloyd Wright and Le Corbusier.

Another important connecting principle between mathematics and art is **proportion**, a central principle of architectural design.

**Slide #10**
The **Golden Ratio** is an irrational number and like pi, we use a Greek letter, phi (or $\Phi$), to represent it. The approximate value of the Golden Ratio is 1.62, but its exact value is

$$
\Phi = \frac{1 + \sqrt{5}}{2} \approx 1.62
$$
Reference to the Golden Ratio dates back to ancient Greece (500 BC) when philosophers asked the question, “How can a line segment be divided into two pieces that are most pleasing to the eye?” It turns out that this “divine proportion,” as the Greeks called it, was the Golden Ratio.

**Slide #11**

Leonardo Da Vinci’s Vitruvian Man, based on a model of ideal proportions established by the ancient Roman architect Vitruvius, reveals within repeated iterations of the Golden Ratio.

**Slide #12**

In geometry, a golden rectangle is a rectangle whose side lengths are in a golden ratio 1: *phi*. These rectangles are the most aesthetically pleasing to the eye and are commonly used in works of art or architecture.

**Slide #13**

The Swiss-French architect and artist Le Corbusier, famous for his contributions to modern international style, centered his design philosophy on systems of proportion. Le Corbusier’s faith in the mathematical order of the universe was closely bound to the golden ratio and the Fibonacci Sequence. Many of the architects whose designs we will see today were influenced by Le Corbusier’s work.

**Slide #14**

In our tour, another important principle will be the use of basic two-dimensional and three-dimensional shapes.

**Geometry** is the mathematical study of forms and their order. Geometric figures, forms and transformations build the material of architectural design.* Geometric rules based on the ideas of proportions and symmetries have guided the architectural works we will see today.


**Slide #15**

The use of curves called conic sections in two-dimensions, literally slices of cones, will be evident, including ellipses, parabolas, and hyperbolas. The equations of these shapes are studied in a traditional precalculus course and can be used to solve many important applications.

**Slides #16**

Quadric surfaces are studied by a Calculus student. They are defined as surfaces in three dimensions having the property that the traces of the surface are the conic sections.
A calculus student is taught to find the surface area of a quadric surface and the volume of the space it encloses. A calculus 3 student would also be proficient in sketching these implicit equations by hand and by using a 3-dimensional graphing program.

The use of quadric surfaces became possible in architectural design because of new building techniques. Modern architecture uses geometry in a pure and simple form. These geometric shapes are the dominant motif where the architecture is a concrete expression of mathematical ideas, becoming, in a sense, “visual mathematics”. *

*Nexus Network Journal (NNJ) is a peer-reviewed journal for researchers, professionals and students engaged in the study of the application of mathematical principles to architectural design.

The Modern Movement in architecture can be described as an era of developing a new style that embraced technological advances in materials and building methods, and rejected ornamentation, decoration and references to the past. Modern design focused on simplicity, spatial clarity through the use of pure geometry, and daylight to create healthy living and working spaces. *

*Source Architectural trends, forms, materials and expression important in the St. Louis School of Modern Movement Architecture, c. 1945-1975 by Kristen Minor

So modern architecture was a result of reconciling the principles underlying architectural design with technological advancements and a rapidly modernizing society.

One of the technological advances we will see in this tour is the process of pouring thin shelled concrete over wooden or metal structures pioneered by Italian engineer and architect Pier Luigi Nervi (1891-1979) who explored the use of concrete in a variety of inventive structural projects and who served as a consultant on several of the projects we will see here today.

#1 Lambert-St. Louis International Airport - Main Terminal in STL County was designed by architect Minoru Yamasaki of the firm Hellmuth, Yamasaki & Leinweber in 1956. This was the first building in St. Louis to win a national American Institute of Architects honor award and set the standard for designs of future generations of air travel terminals.

The terminal was built with three halls, each with barrel-vaulted ceilings with a diameter of 120 feet constructed of thin-shell concrete.
The Modern Movement in Architecture was responsible for elevating concrete to a finish material on buildings.

Slide #23
In geometry, a Steinmetz solid is the solid body obtained as the intersection of two cylinders of equal radius at right angles. Each of the curves of the intersection of two cylinders is an ellipse as shown in this animation.

Slide #24
Yamasaki’s innovative design established an international model for modern air travel terminals - vaulted and expandable.*

Slide #25
The terminal was expanded in 1965 and a fourth dome designed by architecture Gyo (Gi-O) Obata, of Hellmuth, Obata and Kassabaum or HOK.

* Source: Architectural trends, forms, materials and expression important in the St. Louis School of Modern Movement Architecture, c. 1945-1975 by Kristen Minor

The next building in the west St. Louis county suburb of Creve Coeur was designed by Gyo (Gi-O) Obata and has been referred to as a Modernist Masterpiece.

#2 Slide 26

The Abbey Church, also known as the Priory of St. Mary and St. Louis, was designed by Obata of Hellmuth, Obata and Kassabaum, HOK, in 1962 with Nervi serving as consultant on the modular construction methods incorporating movable parabolic forms.

Slide 27 The design includes three tiers of thin-set parabolic concrete arches.

Slide 28 The bottom two tiers are each composed of 20 perfectly proportioned arches, and the top level with 10 very steep parabolics that form the bell tower and lantern.

Slide 29 The design is striking in its radial symmetry. The arches support large fiberglass double paneled windows, providing a 360-degree view of the rolling hills. The interior appears to be free from vertical supports while the steel structure is concealed in the white thin set concrete.

Slide 30
This work is viewed as a modernist triumph worldwide and propelled the HOK architecture firm to international recognition*.

* Gyo Obata's Abbey Church: A Modernist Masterpiece in West St. Louis County, Riverfront Times, Chris Naffziger, 2014

Slide 31
Today, HOK or Hellmuth, Obata and Kassabaum, is the largest US based architectural-engineering firm and its designs include The National Air and Space Museum and its extension in Washington DC. (The Steven F. Udvar-Hazy Center is an extension of the
Smithsonian’s original National Air and Space Museum designed by HOK on the National Mall and is located Washington Dulles International Airport)

#3 Slide 32
Next let’s visit the Climatron Greenhouse in Missouri Botanical Gardens. Founded in 1859, the Missouri Botanical Garden is the nation’s oldest botanical garden in continuous operation and a National Historic Landmark. Architects Wayne Mackey Sr. and Joseph Murphey designed the greenhouse based on Richard Buckminster Fuller’s Geodesic Dome designs.

Slide 33 The Climatron is said to be the first geodesic dome ever used as a conservatory. In 1976, it was named one of the 100 most significant architectural achievements in United States history.

Slide 34
Slide 35 The Climatron dome is 175 feet in radius, consists of 3,625 glass panes of 72 different shaped triangular pieces. * It encloses 24,000 square feet of floor space and over 1 million cubic feet in volume.

The American Institute of Architects calls the geodesic dome “the strongest, lightest and most efficient means of enclosing space known to man.” Geodesic Domes are composed of interconnected triangles that give them unparalleled strength.

* Source: Building Big Databank, PBS Online

Let’s move on to another of my favorites of the buildings highlighted today the James S. McDonnell Planetarium.

#4 Slide 36
An example of Space-age or futurist design, the James S. McDonnell Planetarium in Forest Park is now part of the St. Louis Science Center. The planetarium was constructed was designed by HOK and completed in 1963 with Gyo (Gi-O) Obata serving as lead designer.

Slide 37 The building is striking in its hyperboloid design and built using thin shell concrete.

Slide 38 The hyperboloid rises to a height of 23 meters with a diameter of nearly 50 meters but its thin-set concrete shell has an average thickness only 7.5 cm.

Architect Obata is quoted as saying the hyperboloid shape was the result of form following function.
A spherical domed auditorium was planned for the building’s center, and included a staircase that would lead from there to the roof so aspiring astronomers could quickly go from learning about stars to actually viewing them.

The flare shape at the top shields the building from the city’s lights to aid in star gazing.

The proportion and radial symmetry of the space always felt magical to me on field trips attending star shows and later as a teen seeing laser light shows. This hyperboloid of one sheet design has some fascinating properties. Despite its curvaceous design, they can be constructed using straight rods as shown in this animation and are incredibly strong while using relatively little material.

Council Plaza’s Flying Saucer Building was designed by architect: Richard Henmi of the firm Schwarz & Van Hoefen in 1967, a firm he would become partner in in 1968. The “flying saucer” building opened in 1967 as a gas station, part of the Henmi-designed Council Plaza, a mostly residential high-rise development. The round building served an important purpose as the focal point of the otherwise rectilinear complex and the development is included in the National Register of Historic Places. The UFO-inspired nickname came from the station’s roof, technically an elliptic paraboloid supported by four posts and made of thin-shell concrete.

The roof is only five-and-a-half-inch thick concrete, but it is 120 feet in diameter. The strength of its design allows for a very thin structure that spans a very long distance. *

* Take five: Architect Richard Henmi designed Del Taco building, St. Louis Public Radio by Alex Sciuto.

Many iconic mid-century modern buildings have been destroyed or are in danger of demolition. In 2011, when word got out that the building was to be demolished, St. Louis University students and preservationists came together and eventually saved the building which now holds a Starbucks.

The Jefferson National Expansion Memorial, more commonly referred to as the St. Louis Gateway Arch, stands along the banks of the Mississippi River. The sculptural monument was built to commemorate President Thomas Jefferson and the westward expansion of the U.S. and is the tallest memorial in the US and the tallest stainless-steel monument in the world.
The design of the monument, created by Finnish architect Eero Saarinen, was selected unanimously in 1948 by a panel of internationally known architects in a National Park Service major design competition.

**Slide 48:** Saarinen chose an elegant and modern inverted catenary curve for his design to symbolize a gateway. A *catenary* is the shape created when a chain hangs loosely between two points. Saarinen considered the catenary to be perfect in its form and its symbolism for the project.

The arch was constructed from 1963 to 1965 at a cost of 13 million dollars.

**Slide 49:** American Mathematician, Robert Osserman, of Stanford University published a paper on the “Mathematics of the Gateway Arch” in 2010 examining the use of the term catenary to describe our Arch.

Osserman writes: It was Robert Hooke who in 1675 made the connection between the ideal shape of an arch and that of a hanging chain and wrote, “As hangs the chain, so stands the arch.” In other words, the geometry of a standing arch should mirror that of a hanging chain.

Because the thickness of the legs decreases as we move higher, mathematicians agree it was designed around the idea of a catenary. Osserman goes on to analyze the design mathematically using calculus, landing on the term “flattened or weighted catenary” to describe its shape. *


**Slide 50**
The legs of the arch are made up of sections of equilateral triangles with sides ranging in length from 54 feet to 17 feet. The perfectly proportioned arch legs taper gracefully to the top of the monument giving the illusion that the arch is much taller than wide when in fact both measures are 630 feet.

**Slide 51**
Next let’s move to a couple of locations here in Phoenix and the work of Frank Lloyd Wright, who is considered the father of American architecture and modern design.

**Slide 52**
Taliesin West is a [UNESCO World Heritage site](https://whc.unesco.org/en/list/405) and National Historic Landmark nestled in the desert foothills of the McDowell Mountains in Scottsdale, Arizona.

Wright’s beloved winter home and desert laboratory was established in 1937 and diligently handcrafted over many years into a world unto itself. Deeply connected to the desert from which it was forged, Taliesin West possesses an almost prehistoric grandeur. It was built and maintained almost entirely by Wright and his apprentices, making it among the most personal of the architect’s creations.
Wright told the apprentices that the desert was like a revelation to him: “I was struck by the beauty of the desert, by the dry, clear sun-drenched air, by the stark geometry of the mountains, the entire region was an inspiration in strong contrast to the lush, pastoral landscape of my native Wisconsin. And out of that experience, a revelation is what I guess you might call it, came the design for these buildings.”
Source: franklloydwright.org

Slide 53
In their published work, The Geometry of Wright, architects Linda and Mark Keane, investigate the connections between Wright's designs and geometry.
“The square, rectangle, octagon, triangle, hexagon, parallelogram, circle, spiral, and arc are keys to the consistent and systematic quality underlying all of Wright’s work. Using nature as his mentor and geometry as his tool he developed what he called organic architecture. He used number, geometry, proportion, pattern, hierarchy and orientation in all of his work. Wright used geometry as a formative idea with the concepts of plane and solid geometry determining the built form.

Slide 54
Besides basic geometries, Wright used combinations, multiples, derivatives, and manipulations. This structural vocabulary consists of a three-dimensional field of lines through which the solid elements of the building are located enabling the voids to be integral to the whole and equally meaningful. Architecture is, after all, the space.

Slide 55
The primary quality in Wright’s design is a union of abstraction and geometry. Describing his sources of design, Wright said that exploring the geometric blocks of the Froebel gifts as a child taught him to consider these shapes as a geometric lexicon form with which to interpret nature.

Slide 56
Wright’s innovation of the use of geometry in design was twofold. He could discern what he called the essential character in nature and translate it into a pattern from which he generated new forms of design.

Source: GEOMETRY, ABSTRACTION & THE REFLEX by ARIS GEORGES, NOV 8, 2018

Slide 57
Next, let’s visit Arizona State University Campus in Tempe AZ

Slide 58
The ASU campus is a treasure of midcentury modern and contemporary modern architecture.
Source: modernphoenix.net by Allison King
Gammage Auditorium, is a well known local and national historic landmark and represents the last American architectural masterpiece of Wright (1867-1959). It is the only public building in Arizona designed by Wright.

The design is a complex combination of geometric shapes, with heavy emphasis on circular features.

Wright was a close friend of Grady Gammage, who as President of ASU commissioned him to work on the project. The construction site had previously been the location of an athletic field; Wright chose the site during a tour of the campus with Gammage. He is said to have proclaimed, “I believe this is the site. The structure should be circular in design...with outstretched arms saying, ‘Welcome to ASU!’”

Conceived in the Usonian architectural style, the auditorium itself is 300 feet long, 250 feet wide and 80 feet tall. Fifty concrete columns support the round roof with its pattern of interlocking circles. It features two long ramps extending from opposite sides of the main building, each of which measures 200 feet. These ramps, or “flying buttresses,” were designed to symbolize outreached arms in a welcoming gesture to the visitor.

**Slide 64**
The stage can be adapted for grand opera, Broadway musicals, dramatic productions, solo productions, orchestra recitals and lectures.
Source: National Register Listing 09/11/1985

http://www.asugammage.com/about

**Slide 66**
In conclusion:
As faculty let’s reach across our disciplines to form connections. Students love to see connections between their studies. Even in science, technology, engineering and math we’ve realized art is integral to its understanding and visualization and expanded STEM initiatives to STEAM initiatives. Let’s continue to provide interdisciplinary opportunities and travel to our students.

Sources: slide 67
The Art of Mathematics
Connecting the Disciplines

2021 AMATYC Presentation S068
Sharon North, Professor Emeritus of Mathematics
St. Louis Community College
Climatron Greenhouse
Missouri Botanical Gardens
Architects:
Wayne Mackey Sr.
Joseph Murphey
TC Howard, Synergetics
1959

Design based on Richard Buckminster Fuller’s Geodesic Dome
Aluminum tubes create interlocking hexagons, six sided structures, to form a 70-foot-high dome that needs no interior vertical supports.

Inspired by R. Buckminster Fuller’s “Geodesic Dome”, a geodesic polyhedron.

Most efficient way to enclose a large space with the least amount of material. A way to approximate a curved surface using 3625 glass flat panels. Greater strength and space with minimum weight.

Fuller saw potential in housing.

The triangular elements of the dome are structurally rigid and distribute the structural stress throughout the structure, making geodesic domes able to withstand very heavy loads for their size. Triangles are the strongest polyhedron.
James S. McDonnell Planetarium
St. Louis Science Center
Forest Park - Oakland Ave.
Architect: Gyo Obata (HOK)
1963
(hyperboloid design)
The traces parallel to the $xy$-plane are ellipses and the traces parallel to the $xz$- and $yz$-planes are hyperbolas.
A sphere is the set of all points in space equidistant from a fixed point, the center of the sphere, just as the set of all points in a plane that are equidistant from the center represents a circle.
The St Louis Science Center McDonnell Planetarium (HOK, 1963) illustrates the organic, expressive, mathematically-derived forms that were made possible by engineering and technology.
Council Plaza’s Flying Saucer Building
Architect: Richard Henmi (Schwarz & Van Hoefen)
1967
212S. Grand Blvd.
St. Louis, MO
Many quadric surfaces have traces that are different kinds of conic sections, and this is usually indicated by the name of the surface. For example, if a surface can be described by an equation of the form \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c} \]
then we call that surface an **elliptic paraboloid**.
Signature piece of an urban renewal project is this “strong shape to emphasize the location”

The overhang is a elliptic paraboloid 120 feet in diameter constructed of thin shelled concrete held in place by beams based at angled piers at the buildings center.

The STL public came together to save the building and it received a 2013 Preserve Missouri Award.
1947 open competition for the design of a “Jefferson National Expansion Memorial”

St. Louis Gateway Arch
Downtown St. Louis Riverfront
Architect: Eero Saarinen
Designed in 1947
Construction began in 1963
Inverted catenary, a hyperbolic cosine function
Weighted catenary - the shape assumed by a hanging chain

Hyperbolic cosine function

\[ y = \cosh x = \frac{1}{2} (e^x + e^{-x}) \]
THE ARCH’S BASIC BUILDING BLOCKS

Because the Arch has no structural skeleton, it relies on its own skin of stainless steel and carbon steel for support.

The Arch is made up of a series of double-walled triangular sections stacked one on top of another, gradually decreasing in size as they rise.

- **Carbon steel plates** make up interior wall. There is a 3-foot space between the inner and outer wall in the largest piece, and narrows to less than 8 inches on the smallest section.

- **Concrete fill** was pumped between the carbon and stainless steel walls after the section was set in place.

- **Post-tensioning rods**, embedded in the concrete, kept the sections pulled tightly together during construction.

- **Strengthening rods**

- **Stainless steel plates** make up the exterior wall.

SOURCE: National Park Service

SOURCE: Rich Bolicki | Post-Dispatch
Mid Century Modern near Phoenix AZ

**Taliesin West**, by Frank Lloyd Wright, in Scottsdale, Arizona, 1937

Taliesin West is a UNESCO World Heritage site and National Historic Landmark nestled in the desert foothills of the McDowell Mountains in Scottsdale, Arizona. Wright’s beloved winter home and desert laboratory was established in 1937 and diligently handcrafted over many years into a world unto itself. Deeply connected to the desert from which it was forged, Taliesin West possesses an almost prehistoric grandeur. It was built and maintained almost entirely by Wright and his apprentices, making it among the most personal of the architect’s creations.
Whirling Arrows was a design Frank Lloyd Wright created that was inspired by petroglyphs that Wright found near his desert home. He said the design reminded him of two hands clasped in friendship.
Arizona State University, Tempe Campus

The ASU campus is a virtual hotbed of midcentury modern and contemporary modern architecture. Home to landmark structures by Frank Lloyd Wright, Ed Varney and Antoine Predock, the campus architecture provides constant stimulation and relief through form, texture and materials. Recurring themes are red brick, courtyards, textile blocks, precast concrete, steel and glass. The grounds are open to the public and are a must-see stop for any serious student of architecture.

• Source: Modern Phoenix LLC
ASU Music Building by Wesley Peters, built in 1970
Connections...

• Architecture is a concrete expression of mathematical ideas, becoming, in a sense, “visual mathematics”.
• Mathematics is everywhere, appreciate its beauty.
• STEM to STEAM initiatives.
• Interdisciplinary opportunities for our students.
• Connecting to young people in our community, our future students.
Sources (see also sources cited in script):

Built St. Louis Website builtstlouis.net. Photos and facts from Robert Powers

OpenStax Calculus Volume 3, OpenStax is a nonprofit based at Rice University

How the Gateway Arch Got its Shape, Robert Osserman, 2010

THE MATHEMATICS AND ARCHITECTURE OF THE SAINT LOUIS ARCH by William V. Thayer

Mid-Century Modern Church Survey: Religious Structures 1940-1970 in St. Louis County Compiled by Esley Hamilton and Catie Myers 2009-2010

Missouri History Museum Photo Archives

St. Louis Post Dispatch Photo Archives

MID-CENTURY MODERN ARCHITECTURE IN ST. LOUIS COUNTY: OUTSTANDING EXAMPLES WORTHY OF PRESERVATION St. Louis County Historic Buildings Commission June 2007

Thematic Survey of Modern Movement Non-Residential Architecture, 1945 - 1975, in St. Louis City