STOKING THE CREATIVE FIRE

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OUTRAGEOUS CLAIM #1:

• THEOREM (FORMAL): For every continuous map \( f: S^n \to \mathbb{R}^n \), where \( S^n \) is the boundary of \( B^{n+1} \), \( \exists x \in S^n \) such that \( f(x) = f(-x) \)

• THEOREM (FUN): At any moment, there are two antipodes on Earth with the exact same temp & atmospheric pressure

Reference: (2) Arora, S. (2002); Figure: (1) Topology Earth
BORSUK-ULAM THEOREM

- Choose two antipodes
  - If they have the same temp, you’re done
  - Else, we can create a continuous antipodal path/loop from one antipode to the other
- At some point on this loop, they have the same temperature

Reference: (3) Stevens, 2016 [YouTube Video]; Figure: (2) Borsuk Ulam World
The antipodal points of equal temperature form an antipodal path/loop themselves.
- Select two antipodal points on this loop.
- If they have the same pressure, we’re done.
- Else, repeat the same process as with temp.

Reference: (4) Stevens, 2016 [YouTube Video]; Figure: (7) Borsuk Ulam World
OUTRAGEOUS CLAIM #2:

• Every continuous endomorphism on a convex, compact metric space has a fixed point

  OR

• No matter how much you stir a cup of iced tea, some point will always return to its initial position

Figure: (3) Just Stirring GIF
SPERNER’S LEMMA

TRIANGLE TRI-COLOURING

• Draw a “triangulation”
• Colour the corner vertices 3 different colours
• Colour the edge nodes either of the colours of the vertices they connect
• Colour the interior nodes any of the 3 colours
• LEMMA: There is an odd number of “3-coloured” triangles

Reference: (4) Fox, 2009, p. 1-2; Figure: (4) Sperner’s 2-D Simplex
BROUWER’S FIXED POINT

• Given \( f : B^n \rightarrow B^n \) (\( n \)-dim ball)
  • Embed in \( \mathbb{R}^{n+1} \) on hyperplane \( \sum_{i=1}^{n+1} x_i = 1 \)
  • Let \( \Delta^n \) be an \( n \)-dim simplex (i.e. triangle)
  • Transform \( B^n \) to \( \Delta^n \) by homeomorphism

• Use Sperner’s Lemma (SL) & Bolzano-Weierstrass Theorem (BW)
  • Colour \( x \) with \( i \in [n + 1] \) if and only if \( i \) is the minimal value such that \( f(x_i) < x_i \)

Reference: (4) Fox, 2009, p. 2-3
**BROUWER’S FIXED POINT**

- Subdivide (subtriangulate) $\Delta^n$ into smaller simplices $\Delta^n_j, j = 0, 1, 2, 3, \ldots$
  - $j = \text{number of subdivisions in } \Delta^n_j$
- $\text{SL } \rightarrow \forall \Delta^n_j \exists [n + 1]$-coloured sub-simplex
- $\text{BW } \rightarrow \forall i \in [n + 1] \exists \text{ conv subsequence}$
- As $j$ increases, $\Delta^n_j$ decrease in size
- Limit point $x : f(x_i) \leq x_i \ \forall i \in [n + 1]$
- $\sum_{i=1}^{n+1} f(x_i) = 1 \rightarrow f(x_i) = x_i \ \forall i \in [n + 1]$

Reference: (4) Fox, 2009, p. 2-3
WHERE DO WE GO FROM HERE?

• Find and share your passion
  • We are inspired by passion
  • Doesn’t need to be practical
  • Your excitement is contagious

• Use story-telling and narrative
  • Facts don’t engage like stories do!

• Spark curiosity and ignite passion
  • “Always leave them wanting more”
OUTRAGEOUS CLAIM #3:

Figures: (5) Ham Sandwich, (6) Ham Sandwich Theorem
HAM SANDWICH THEOREM

• Let the ham, cheese and bread be 3D chunks represented by $A_1$, $A_2$, and $A_3$

• Given point $p$ on the sphere $S^2$, let $P_i$ be a plane tangent to $p$ bisecting chunk $A_i$

• Let $(d_1, d_2) = \text{dist}_p(P_3 - P_1, P_3 - P_2)$
  • $f: S^2 \to \mathbb{R}^2, f(p) = (d_1, d_2)$ is continuous

• By Borsuk-Ulam, $\exists p: f(p) = f(-p)$
  • i.e. $(d_1, d_2) = (-d_1, -d_2) = (0,0)$
  • $\therefore P = P_1 = P_2 = P_3$ cuts all 3 chunks in half

Reference: (5) Dougherty, 2017; (6) SeriousMathsAndUnicorns, 2015 [Blog]
AND WITH THAT...

... It’s Time For Lunch!

(THE END)
FOR MORE INFORMATION

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REFERENCES


FIGURES


3) Just Stirring GIF: https://tenor.com/view/just-stirring-my-tea-gif-5672805

4) Sperner’s 2-D Simplex: https://www.lesswrong.com/posts/svE3S6NKdPYoGepzq/topological-fixed-point-exercises

5) Ham Sandwich: https://farm5.static.flickr.com/4090/4970848133_dd41214ec6.jpg

6) Ham Sandwich Theorem: https://curiosamathematica.tumblr.com/image/53344367846