Crossing Through and Bouncing Off $\infty$: Graphing Rational Functions

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While rational functions are initially studied in high school, some of their behavior may be difficult to understand. For instance, when a rational function approaches a vertical asymptote, the graph may go to $+\infty$ and then either return from $+\infty$ or return from $-\infty$. But, where does it go and how does this happen? To explain this mathematical phenomenon, we consider topological techniques that wrap the real number line into a circle where $-\infty$ and $+\infty$ meet as one point. This then allows us to investigate principles of limits and local dominance in polynomial and rational functions. We also consider reciprocal relationships between zero and $\infty$ and polynomial and rational parent functions. Altogether, we connect all of these notions to better understanding vertical asymptotic behavior of rational functions.

Keywords: rational functions, polynomial functions, vertical asymptotes, limits, topology, compactification

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