Rate, Ratio, and Average Rate of Change: What Does It All Mean?

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The Hand Squeeze

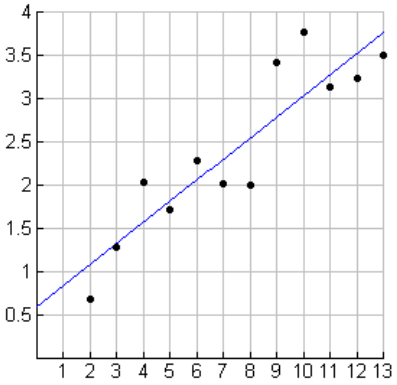
Set Up

Start with two people standing face-to-face holding hands (like at a wedding). A third student can be the time keeper. When the time keeper says go, the designated squeeze starter squeezes one of the hands of the second person. Upon feeling the squeeze, this second person squeezes back with the other hand. The squeeze starter shouts "Stop!" when the squeeze is felt and the time keeper stops the stopwatch. Record the time. Continue this procedure by adding one person to the circle each iteration.

Sample Data

The following data were collected in an Intermediate Algebra class in the Spring of 2004.

People	2	3	4	5	6	7	8	9	10	11	12	13
Time (seconds)	0.69	1.29	2.04	1.72	2.29	2.02	2.00	3.41	3.76	3.13	3.24	3.50



Linear Regression (ax+b)
regEQ(x) = .243462x + .598205
r = .903828
r^2 = .816905

Suggested Questions

1. Determine a linear function that models the data.
2. Describe the meaning of the constant rate of change in this situation.
3. Describe the meaning of the vertical intercept in this situation.
4. Using the linear model, determine the time it would take for the squeeze to travel from beginning to end if there were 30 people involved.
5. If the time keeper recorded a time of 10 seconds for the squeeze to travel from beginning to end, how many people were involved according to the linear model you found?
6. Explain the meaning of the coefficient of determination.
7. Explain the meaning of the correlation coefficient.
8. Since a linear model was used in this situation, we would say that the rate of change is constant. Explain what constant rate of change means in the context of this situation.
9. Suppose each person was precisely 0.20 seconds slower than what was recorded. How could you adjust the linear model found in #1 and write the equation of the linear function that models this new data set?
10. Compare the slope of the linear function found in #1 with the slope of the linear function in #9. What do you notice? Why is this the case?

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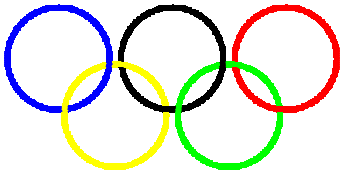
The



Olympic Sprinter

Watch the video showing Florence Griffith-Joyner in the 1988 Olympics 100-meter sprint. She is in Lane 3 (third lane from the right as you view the video). You will probably need to watch it several times as you work to develop a graph of Griffith-Joyner's distance from start as a function of time. The following questions are designed to promote your thinking about your thinking concerning this situation.

1. On your whiteboard, create a graph that could plausibly be Griffith-Joyner's distance from start as a function of time.

2. This graph is to appear as part of an article or report and you are given the task of writing the caption for this graph. Write a concise caption that communicates the information that the graph provides.

3. Choose a particular point on the graph. Describe the meaning of this point.

4. What aspect of the graph do you hope your audience envisions as you describe the meaning of the point in #3 by pointing to it?

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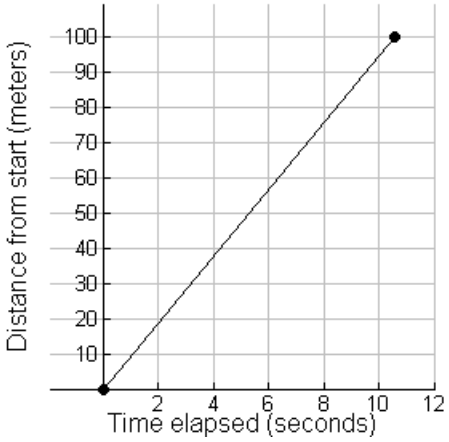
5. Students are asked to create the graph as you were asked to do. A student decides to take the initial point (initially, time is 0 and distance from start is 0) and connect it to the final point (after 10.54 seconds, Griffith-Joyner was 100 meters from start). The graph below shows this graph.
   - Discuss how the student might be thinking while creating such a graph.
   - What aspects of the situation might the student NOT be thinking about?
   - What would you ask the student to promote further thinking?



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The

Skateboarder (Group Activity)

Watch the video showing the skateboarder performance on the half-pipe. You will probably need to watch it several times as you work to develop a graph of the skateboarder's *horizontal distance* from start (the left edge of the half-pipe) as a function of time. The following questions are designed to promote your thinking about your thinking concerning this situation.

1. On your whiteboard, create a graph of the skateboarder's horizontal distance from start (the left edge of the half-pipe) as a function of time since the video began.

2. This graph is to appear as part of an article or report and you are given the task of writing the caption for this graph. Write a concise caption that communicates the information that the graph provides.

3. Choose a particular point on the graph. Describe the meaning of this point.

4. What aspect of the graph do you hope your audience envisions as you describe the meaning of the point in #3 by pointing to it?

5. Students are asked to create the graph as you were asked to do. A student creates the graph shown in Figure 1. The student explains that the graph shows how the skateboarder first "goes down", then travels across the bottom of the half-pipe, then goes "back up", etc.
   - Discuss how the student might be thinking as they create such a graph.
   - What aspects of the situation might the student NOT be thinking about?
   - What would you first say to or ask this student?
   - Why?

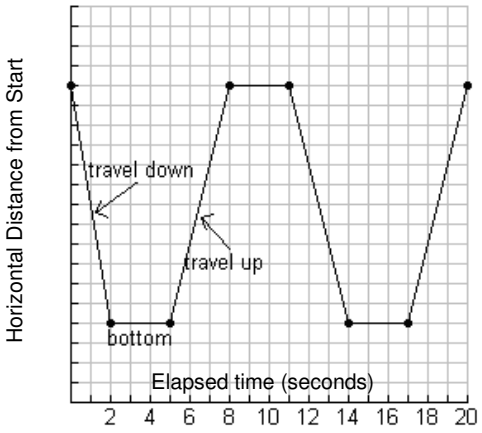

Figure 1

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Bungee Jumping (Group Activity)

Watch the video showing the bungee jumper. You will probably need to watch it several times as you work to develop a graph of the bungee jumper's vertical distance from the top of the bridge (start) as a function of time. The following questions are designed to promote your thinking about your thinking concerning this situation.

1. On your whiteboard, create a graph of the bungee jumper's distance from the top of the bridge as a function of elapsed time.

2. This graph is to appear as part of an article or report and you are given the task of writing the caption for this graph. Write a concise caption that communicates the information that the graph provides.

3. Choose a particular point on the graph. Describe the meaning of this point.

4. How would a student interpret the graph you created? What would they say?

5. Think about your thinking as you worked to create the graph in this situation. What strategies were helpful? How could you prepare students to successfully create a graph of this situation? The strategy, "I will show them how to do it and then have them practice over and over again" is not a valid strategy.

6. What issues or misconceptions need to be addressed in preparing students to create a graph such as the graph from this situation?

7. A student indicates that the graph doesn't make sense because the graph "goes up" when the bungee jumper "goes down". How are students thinking if they make such a comment? What might you say or do to help students with this issue?

Jump!

Watch the video showing the commercial where a car is launched straight up into the air after being jump started. You will probably need to watch it several times as you work to develop a graph of the height of the car as a function of the total vertical distance the car travels. Certainly, you will have to either make up a scale for each axis or use qualitative benchmarks for each axis. The following questions are designed to promote your thinking about your thinking concerning this situation.

1. On your whiteboard, create a graph of the car's distance from the ground as a function of the total vertical distance the car travels.

2. This graph is to appear as part of an article or report and you are given the task of writing the caption for this graph. Write a concise caption that communicates the information that the graph provides.

3. Choose a particular point on the graph. Describe the meaning of this point.

4. It is possible that, while explaining the meaning of the particular point chosen in #3, you pointed to the graph you created. If not, suppose that you did. When pointing to the graph, what might be the purpose? Specifically, what aspect of the graph do you hope your audience envision as you describe the meaning of the point in #3 by pointing to it?

5. Imagine that a student was presented with the graph that you created. How would they interpret what you created? What would they say?

6. Think about your thinking as you worked to create the graph in this situation. What strategies were helpful? How could students be prepared to successfully create a graph of this situation? The strategy, "I will show them how to do it and then have them practice over and over again" is not a valid strategy.

7. What issues or misconceptions need to be addressed in preparing students to create a graph such as the graph from this situation?

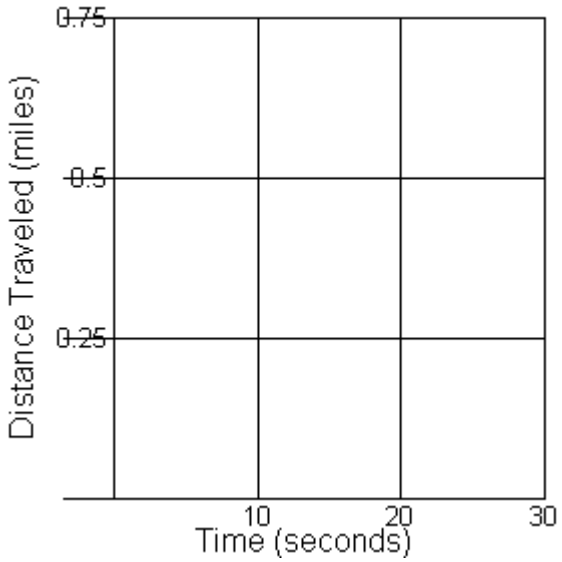
8. A student indicates that the graph should be "steeper at the beginning and at the end" of the event since that is when the car must be traveling at the fastest speed. How are students thinking if they make such a comment? What might you say or do to help students with this issue?

In Pursuit of Justice

1. At _____ time the truck passed the first checkpoint and at _____ time the truck passed the second checkpoint.

2. The distance traveled between safety checkpoints according to Officer Justice was _____ miles.

3. Plot the position of the truck at the two check points below:



4. Connect the points and calculate the slope of the line.

5 a. Interpret the meaning of the slope in question 4.

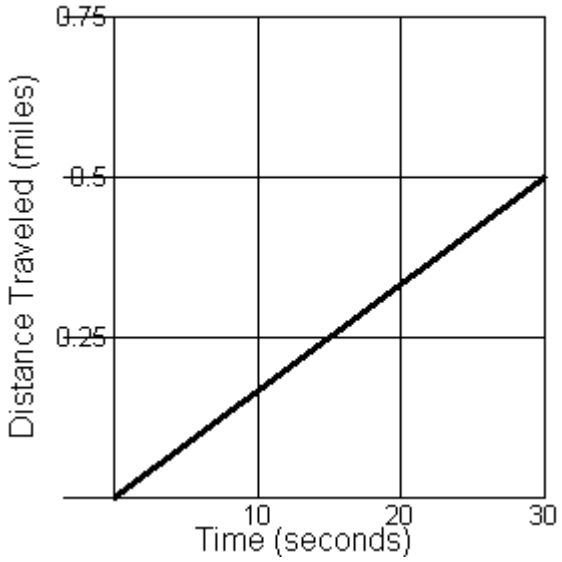
b. What is the slope's unit of measure?

c. Convert your answer to miles per hour.

6. If the posted speed limit on the road was 45 miles per hour, how does the truck's speed compare? Explain.

7. In question 4, you drew a line segment connecting the two points. Do you think that the truck's distance actually increased in *a linear fashion* from the first checkpoint to the second checkpoint? Explain why or why not.

8. Sketch a graph of distance versus time for the truck using the ideas you used in questions 4 - 7. Put the graph on the grid below along with the linear graph from question 3. (Make one graph a bit darker than the other in order to distinguish between the two.)



9. On the graph you drew in question #8, sketch a tangent line (a line that touches the curve in only that place locally) to the curve that is parallel to the original line in question #3.

10 a. The secant line represents the _____ rate of change of the truck.

b. The tangent line represents the _____ rate of change of the truck.

11. In a few sentences describe the speed of the truck between the checkpoints according to how you drew your graph in question #8.

12. Explain how this situation is exemplifies the following theorem:

Theorem 3.14159…: THE MEAN VALUE THEOREM

If a function f is continuous on a closed interval $[a,b]$ and is differentiable on the open interval (a,b), then there exists a number c in (a,b) such that

$$\frac{f(b)-f(a)}{b-a}=f'(c).$$