Cultural Contexts for Two-Year College Mathematics

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What do I mean by *cultural contexts*

• Mathematics doesn’t happen in a vacuum
  – It arises from and inspires
    • art and storytelling
    • governmental policies
    • language
    • beliefs and superstitions
    • social structures
Where do these ideas come from

• Ethnomathematics
  – ISGEm and NASGEm Study Groups
• “Humanistic Mathematics”
Why study within these contexts

• Growing multiculturalism in the classroom
• Immediate relevance to other coursework
• Students are interested
• Creates depth of conceptual understanding
• Generates desire for (Western) common notations
• Decreases math intimidation
How can we use cultural context

• Stand-alone activities introduce, reinforce, or deepen topics
• Course focused on math in cultural context
  – Barbra Gregory (Steinhurst) at St. Mary’s College of Maryland, Spring 2007
  – Phil Straffin at Beloit College
  – Rachel Hall at Saint Joseph’s University
What will we learn today

• Egyptian fractions
• Sand drawings
  – graph theory
  – Euclidean Algorithm
  – symmetry groups
• The Game of Dish (probability)
Egyptian number system

• It’s like counting change
• Each symbol has a value
• Add up all the symbol values to get the total
Parts of the whole

• The Egyptians knew
  - a unit can be divided into parts.
• They added a symbol to represent this.
• But this way they were limited to…

fractions of the form $\frac{1}{n}$
Multiple parts of a whole

• 3/5 might be written
  \[ \frac{1}{2} + \frac{1}{10} \]

• And 4/7 could be
  \[ \frac{1}{2} + \frac{1}{14} \]
Why bother

If you *really* like pumpkin pie, would you rather share 3 pies among five family members or 4 pies among seven?

How do you split up the pies evenly?
Answers, Part 1

• Which gives you a bigger portion?
  \[ \frac{3}{5} = \frac{1}{2} + \frac{1}{10} \]
  \[ \frac{4}{7} = \frac{1}{2} + \frac{1}{14} \]

(Have you heard the one about the minister whose congregation was moaning about tithing a tenth of their income?)
Answer, Part 2

- We can either give each person $\frac{3}{5}$ of a pie,
- or we can give each $\frac{1}{2}$ plus $\frac{1}{10}$ of a pie.
- Which would you rather slice?
How can I extend this activity

• Find several decompositions of a fraction*
• Develop the Greedy Algorithm (Fibonacci’s Method)
• Work the problems backward, finding common denominators
• Explore the Rhind Papyrus
  – includes a table of $\frac{2}{n}$ fractions

* There are infinitely many to choose among.
Story Time!

- The story of creation according to the Tschokwe people of Africa (Zaire, Dem. Rep. of Congo, and Angola)
Where are these drawings found

• Bushoong (sub-Saharan Africa)
  – decoration, political symbolism
• Tschokwe *sona* (sub-Saharan Africa)
  – storytelling
• *Kolam* (Tamil Nadu, India)
  – ritualism, symbolic of order and beauty
• Malekula (Vanuatu, Pacific Ocean)
  – mythos and ritualism
• Celtic knots are closely related
How is it studied today

- The *Kolam* inspired the development of some array languages in number theory.
Yes, but you promised classroom uses

- Graph Theory
- Euclidean Algorithm and GCFs
- Symmetry groups

http://math.beloit.edu/chavey/HOM/
http://www.vanuatuculture.org/sand/050627_sanddrawing.shtml
Graph Theory and the Bushoong

• Children’s game: draw each figure as one continuous curve
• Worksheet 1
  – Which figures can be traced in a continuous curve?
  – In which ones do you end where you started?
  – Can you find a rule to answer the above questions without tracing?

This topic can be found in more detail in *Ethnomathematics* by Marcia Ascher, published by Chapman & Hall in 1991, reprinted in 1998.
What are these good for

• Graph theory definitions
  – vertices, edges, degree, connected graphs, Eulerian paths
• Additional illustrations include the Bridges of Königsberg and various party puzzles
How can I extend the ideas

- Tschokwe *sona* can illustrate
  - Jordan Curve Theorem
- A closed planar curve that does not cross itself separates the plane into two components
  - isomorphic graphs
The *sona* and the Euclidean Algorithm

- Points in the sand guide the storyteller.
- The points are then isolated or grouped by a curve.
- Worksheet 2: the “billiard ball method”
  - Maintaining $45^\circ$ diagonals to all walls, isolate each dot
  - Use straight lines until you hit a wall, then bounce off as a billiard ball would

This topic can be found in more detail in *Geometry from Africa* by Paulus Gerdes, published by the MAA in 1999. The “billiard ball method” of drawing *sona* is defined in more detail by Phil Straffin of Beloit College.
Drawing *sona*, continued

• The number of lines you need using this method determines the *lineality* of the *sona*.

• Only 1-lineal graphs are legitimate *sona*.
  – Which of the graphs you drew could be Tschokwe figures?
Let’s get to the GCF already

*plaited mat sona* can be drawn this way without any interior walls.

Note: These are similar to simple Celtic knot designs.
Lineality and Concatenation

- Turns out...

...adding a perfect square of dots to a plaited mat grid **doesn’t change the lineality**

(as long as the square touches the figure along exactly one side of the square)

This information came from a personal communication with Philip Straffin at Beloit College.
Get to the point

• This can give us a great visual of the Euclidean Algorithm!

• First, let’s draw some more and generate a hypothesis about lineality (Worksheet 3)
Hypothesis

• The lineality of a rectangular plaited matsona is the GCF of the length of the sides
Now we get to the punchline

Euclidean Algorithm

gcf(21,15)

Amputate Squares
Now we get to the punchline

Euclidean Algorithm

gcf(21, 15)

21 ÷ 15 = 1 R 6
Now we get to the punchline

Euclidean Algorithm

gcd(21, 15)

21 ÷ 15 = 1 R 6

15 ÷ 6 = 2 R 3
Now we get to the punchline

**Euclidean Algorithm**

\[ \text{gcd}(21,15) \]

21\( ÷ \)15 = 1 R 6

15\( ÷ \)6 = 2 R 3

6\( ÷ \)3 = 2 R 0

\[ \text{gcd}(21,15) = 3 \]
Drawing with symmetry: Malekula

http://www.vanuatuculture.org/sand/050627_sanddrawing.shtml
Symmetry Groups

- The Malekula begin with one, two, or three shapes.
Symmetry Groups

- They transform them using reflections and rotations.

$A_{90}$
Symmetry Groups

• Then they concatenate the original shape(s) and transformations.

$AA_{90}$
What can I do with this

- Introduce or review
  - symmetries
  - transformational geometry
  - composition of actions
  - group theory
Probability and the Game of Dish

- A Native American game
- Different clans, different materials, similar rules
- Played at community ceremonies
- Played to restore health to ill person
- Always played in community

Rules

• Strike or shake the dish to cause the markers to jump and resettle.
• Assign points according to the table below. (Keep track at each turn!)
• If you get zero points, pass dish to next person. Otherwise, repeat above steps.

<table>
<thead>
<tr>
<th>6H,0T</th>
<th>5H,1T</th>
<th>4H,2T</th>
<th>3H,3T</th>
<th>2H,4T</th>
<th>1H,5T</th>
<th>0H,6T</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>
Questions

• What’s the probability of getting points on a throw?
• How do these probabilities compare to what happened in your game?
Going deeper

• What is the expected number of points on a given throw?
• Draw a tree diagram of what might happen during a given turn (up to three tosses).
Other activities

- Create a new game with similar rules, but a different number of counters. Try to devise a “fair” strategy for awarding points.
- Discuss the Law of Large Numbers and the nonexistent but often-quoted “Law of Averages”
Mathematics in *U.S. Cultural Context*

- Is a good hitter in baseball who has truck out the last six times due for a hit?
- Proving Pythagorean Theorem to become a Master Mason
- Generating art and music using fractals
Mathematics in *U.S. Cultural Context*

- Reforming mathematics education
- “Lucky” and “unlucky” numbers
- Voting theory
- The list goes on…
Thank you

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