A student should be able to . . .
- Research, Analyze, and Report
- Analyze Data by Inspection
- Verify
- Models Get Modified
- Use Models to Support an Argument
- Ambiguous Authentic Situations
- Correct terminology
- Convert Units
- Sift Through Large Amounts of Information
- Mathematics is Truly Useful

Student Quote

Explorations are helpful and introduce new ideas in an easy manner. I like them—easy to follow and they’re short. The class is not hard if you do the homework, projects, and pass all the tests.

Student Quote

Like any other art form, math takes practice and a lot of attention. I guess more like a girlfriend.

Every college graduate should . . .

be able to perform research, analyze the information, and report conclusions.

Modeling an Authentic Situation

Percentages of Americans Who Are Baseball Fans

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>54</td>
</tr>
<tr>
<td>2001</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
</tr>
</tbody>
</table>

Source: The Gallup Organization
Modeling an Authentic Situation

Percentages of Americans Who Are Baseball Fans

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>54</td>
</tr>
<tr>
<td>2001</td>
<td>51</td>
</tr>
<tr>
<td>2003</td>
<td>50</td>
</tr>
<tr>
<td>2005</td>
<td>48</td>
</tr>
<tr>
<td>2007</td>
<td>44</td>
</tr>
<tr>
<td>2008</td>
<td>43</td>
</tr>
</tbody>
</table>

Source: The Gallup Organization

Every college graduate should . . .

be proficient in analyzing data by inspecting a table or graph.

Analyze Data by Inspecting a Table or Graph

Percentages of Americans Who Are Baseball Fans

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>54</td>
</tr>
<tr>
<td>2001</td>
<td>51</td>
</tr>
<tr>
<td>2003</td>
<td>50</td>
</tr>
<tr>
<td>2005</td>
<td>48</td>
</tr>
<tr>
<td>2007</td>
<td>44</td>
</tr>
<tr>
<td>2008</td>
<td>43</td>
</tr>
</tbody>
</table>

Source: The Gallup Organization

By inspecting the table,
- determine the type of relationship.
- make a prediction.

Scattergram of Baseball Data

Finding a Linear Model

$p$: percent
$t = 0$: 1990

Use the “good” points $(9, 54)$ and $(18, 43)$:

$$m = \frac{43 - 18}{18 - 9} = -1.22$$

$$p = -1.22t + b$$

$$54 = -1.22(9) + b$$

$$64.98 = b$$

$$p = -1.22t + 64.98$$

Every college graduate should . . .

know the value of verifying results.
Verify the Fit

\[ p = -1.22t + 64.98 \] good model

Selecting “Good” Points

\[ p = -1.22t + 64.98 \] good model
\[ p = -2t + 78 \] poor model

Meaning of Parameters

\[ p = -1.22t + 64.98 \]

Meaning of Slope  -1.22
Percentage is decreasing by 1.22 percentage points per year.

Meaning of constant 64.98
The percentage was 64.98% in 1990.

Evaluating a Function

\[ p = f(t) = -1.22t + 64.98 \]

Predict the percentage in 2010.
\[ f(20) = -1.22(20) + 64.98 = 40.58 \]
40.58%

Solving an Equation and Evaluating an Inverse Function

\[ p = f(t) = -1.22t + 64.98 \]

Predict when the percentage will be 33.33%.
\[ 33.33 = -1.22t + 64.98 \]
\[ -31.65 = -1.22t \]
\[ t \approx 26 \text{ (2016)} \]

\[ t^{-1}(p) = \frac{p - 64.98}{-1.22} \]
\[ t^{-1}(33.33) = \frac{33.33 - 64.98}{-1.22} \approx 26 \text{ (2016)} \]

Model Breakdown

Find the \( t \)-intercept of the model. What does it mean in this situation?

\[ 0 = -1.22t + 64.98 \]
\[ 1.22t = 64.98 \]
\[ t \approx 53 \]

\( t \)-intercept: (53, 0)

No Americans will be baseball fans in 2043. Model breakdown has occurred. (Or has it?)
Every college graduate should...

understand that a model is the best approximation available that will likely be modified as more data is collected.

Verify the Fit

\[
p = -1.22t + 64.98
\]

Use Models to Support an Argument

The declining baseball fan model might be used to persuade...

- a city to decline to build a new baseball stadium.
- a television station to decline airing baseball on their station.

Modeling Illuminates Concepts

- Functions
- Inverse Functions
- Algebra of Functions
- Graphing
- Meaning of Parameters
- Solving Equations
- Model Breakdown
- Critical Thinking
- Rule of Four
- Technology

Song

The Number Guy
Selecting a Model

Federal Cost of Secrecy

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost (billions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>3.4</td>
</tr>
<tr>
<td>1998</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td></td>
</tr>
</tbody>
</table>

Source: National Security Archive

---

Every college graduate should . . .

be able to develop a reasonable plan even if an authentic situation is ambiguous.

---

Criteria for selecting a model

- The graph of the model should fit the points well.
- The model should make sense within the context of the authentic situation.
- Piece-wise defined functions
Every college graduate should . . .

be able to use terminology such as *linearly*, *exponentially*, *varies directly*, and *varies inversely appropriately*.

---

**Model Breakdown**

- **$C(t)$ (billions of dollars)**
- **$t$ (years since 1990)**

\[
Q(t) = 0.064t^2 - 0.79t + 5.75
\]

**Use Correct Terminology**

Two theories:

For the years before 1997, federal cost of secrecy and time are approximately exponentially related.

From 1997 on, federal cost of secrecy and time are approximately quadratically related.

**Meaning of Parameters**

- **$E(t) = 1.44(1.12)^t$**
  - Meaning of the coefficient 1.44
    - The cost was $1.44$ billion in 1990.
  - Meaning of the base 1.12
    - The cost is increasing by 12% per year.

**Estimations and Predictions**

Estimate when the cost was $2$ billion.

\[
2 = 1.44(1.12)^t
\]

\[
\frac{2}{1.44} = 1.12^t
\]

\[
\log \left( \frac{2}{1.44} \right) = \log (1.12^t)
\]

\[
\log \left( \frac{2}{1.44} \right) = t \log (1.12)
\]

\[
\frac{\log (\frac{2}{1.44})}{\log (1.12)} = t
\]

\[
t \approx 3 \text{ (1993)}
\]

Predict when cost will be $16$ billion.

\[
16 = 0.64t^2 - 0.79t + 5.75
\]

\[
0 = 0.64t^2 - 0.79t - 10.25
\]

\[
t = \frac{-(-0.79) \pm \sqrt{(-0.79)^2 - 4(0.64)(-10.25)}}{2(0.64)}
\]

\[
t \approx -8 \text{ (1982) and } t \approx 20 \text{ (2010)}
\]
Every college graduate should . . .

be fluent in converting units, including units such as thousands, millions, and billions.

Estimations and Predictions

Estimate the cost in 1985.

\[ E(-5) = 0.64(-5)^2 - 0.79(-5) + 5.75 \approx 0.82 \]

$0.82$ billion
or $820$ million

Student Quote

It’s a way that I haven’t seen before. It makes me see math as a part of my life. It makes it easier to me because I’m more interested. It’s very easy to understand because it’s real data. You can’t fake real data.

Student Quote

Mathematics is numbers and trying to figure out why \( x \) should be a number, too, despite the fact that it’s quite content as a letter.

Every college graduate should . . .

be able to sift through large amounts of information to perform a task.

Labs

- Pass out samples to students.
- Collect data through experiments or research.
- Find an appropriate model.
- Use the model to make estimates and predictions.
- Students type a report.
  - Opening paragraph
  - Respond to generic questions.
  - Summary paragraph
- Students turn in labs in two stages.
- Use a rubric to grade labs.
- Benefits
  - Students see big picture.
  - Students see that math is useful.
  - Writing across the curriculum
### Projects
- Consist of three labs: linear, exponential, and quadratic
- Projects are submitted in six stages.
- Students can resubmit projects.

### Oral presentations

### Benefits
- Same as for labs
- Serves as great prep for final

### Project Requirements
- Write-ups must be typed.
- Questions for the different types of models are given in the Topic of Your Choice Labs (linear, exponential, and quadratic).
- When finding an equation of an appropriate model:
  - First show a linear, quadratic, and exponential regression equation for each set of data.
  - Then choose the best type of function to use as a model. Explain why it is the best choice.
  - Then find the equation of your chosen type without using regression. (Do it by hand.)
  - Finally, compare the equation you found by hand with the regression equation.
  - Use the regression equation to solve remaining problems.
- You must show your work for every problem.

### Project Assignment: Linear Part
1. What two quantities did you explore? Define variables for the quantities. Include units in your definitions.
2. Which variable is the dependent variable? Which variable is the independent variable? Explain.
3. Describe how you found your data. If you conducted an experiment, provide a careful description with specific details of how you ran your experiment. If you researched data, state the source of your data.
4. Include a table of your data.
5. Use a graphing calculator to draw a scattergram of your data. (If your data are not approximately linear, find some data that are.)
6. Find an equation of a linear model to describe the data.
7. What is the slope of your linear model? What does it mean in this situation?
8. Does it make sense that your variables are approximately linearly related in terms of the situation you chose to model? Explain.
9. Choose a value for your independent variable. On the basis of that chosen value, use your model to find a value for your dependent variable. Describe what your result means in the situation you are modeling.
10. Choose a value for your dependent variable. On the basis of that chosen value, use your model to find a value for your independent variable. Describe what your result means in the situation you are modeling.
11. Find the intercepts of your linear model. What do they mean in the situation you are modeling? Has model breakdown occurred at the intercepts?
12. Comment on your lab experience. For example, you might address whether the lab was enjoyable, insightful, and so on.
   - Were you surprised by any of your findings? If so, which ones?
   - How would you improve your process for this lab if you were to do it again?
   - How would you improve your process if you had more time and money?
Rubric

- 5% _____ Each data set contains at least five points.
- 5% _____ Each data set is modeled well by a function (there is good fit).
- 5% _____ For each data set, a source is provided.
- 10% _____ For each data set, scattergrams and three models are shown and there is a complete explanation of which model is best.
- 60% _____ Responses to all questions are correct. All work is shown.
- 10% _____ Project is typed, well-organized, precise graphs, cool cover
- 5% _____ Opening paragraphs and summary paragraphs are thoughtful and complete.

Student Quote

This is without a doubt the best math course I've taken. I've taken this very course three times and have either dropped or failed. This is the only class that I've seen all the information so clearly.

Linear Modeling

Table: Ford's U.S. Market Shares

<table>
<thead>
<tr>
<th>Year</th>
<th>Market Share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>26</td>
</tr>
<tr>
<td>2000</td>
<td>23</td>
</tr>
<tr>
<td>2002</td>
<td>20</td>
</tr>
<tr>
<td>2004</td>
<td>19</td>
</tr>
<tr>
<td>2006</td>
<td>15</td>
</tr>
<tr>
<td>2008</td>
<td>14</td>
</tr>
</tbody>
</table>

Source: Ward's AutoInfoBank

Every college graduate should . . .

know that mathematics is truly useful in the “real world.”

Linear Modeling

Table: Percentages of Americans Who Think That the Press Has Too Much Freedom

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>51</td>
</tr>
<tr>
<td>2001</td>
<td>46</td>
</tr>
<tr>
<td>2002</td>
<td>43</td>
</tr>
<tr>
<td>2003</td>
<td>46</td>
</tr>
<tr>
<td>2004</td>
<td>42</td>
</tr>
<tr>
<td>2005</td>
<td>39</td>
</tr>
<tr>
<td>2006</td>
<td>40</td>
</tr>
<tr>
<td>2007</td>
<td>34</td>
</tr>
</tbody>
</table>

Source: First Amendment Center

Linear Modeling

Table: Percentages of Army Recruits Who Needed to Get Conduct Waivers Because of Criminal Records or Other Past Misconducts

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>4.6</td>
</tr>
<tr>
<td>2005</td>
<td>6.0</td>
</tr>
<tr>
<td>2006</td>
<td>7.9</td>
</tr>
<tr>
<td>2007</td>
<td>11.2</td>
</tr>
<tr>
<td>2008</td>
<td>13.0</td>
</tr>
</tbody>
</table>

Source: U.S. Army Recruiting Command
### Linear System Modeling

**Table: Chrysler and Dodge Sales**

<table>
<thead>
<tr>
<th>Year</th>
<th>Chrysler (millions)</th>
<th>Dodge (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>0.31</td>
<td>1.44</td>
</tr>
<tr>
<td>2000</td>
<td>0.47</td>
<td>1.50</td>
</tr>
<tr>
<td>2002</td>
<td>0.47</td>
<td>1.26</td>
</tr>
<tr>
<td>2004</td>
<td>0.60</td>
<td>1.20</td>
</tr>
<tr>
<td>2006</td>
<td>0.61</td>
<td>1.08</td>
</tr>
</tbody>
</table>

*Source: Autodata*

### Exponential Modeling

**Table: Average Prices of Flat-Panel Plasma Televisions**

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Price (thousands of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>9.8</td>
</tr>
<tr>
<td>2001</td>
<td>6.8</td>
</tr>
<tr>
<td>2003</td>
<td>4.6</td>
</tr>
<tr>
<td>2005</td>
<td>2.5</td>
</tr>
<tr>
<td>2006</td>
<td>1.7</td>
</tr>
<tr>
<td>2007</td>
<td>1.3</td>
</tr>
</tbody>
</table>

*Source: DisplaySearch*

### Linear System Modeling

**Table: Percentages of American College Students Who are Minorities**

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976</td>
<td>15</td>
</tr>
<tr>
<td>1980</td>
<td>16</td>
</tr>
<tr>
<td>1990</td>
<td>20</td>
</tr>
<tr>
<td>2000</td>
<td>28</td>
</tr>
<tr>
<td>2005</td>
<td>31</td>
</tr>
</tbody>
</table>

*Source: Department of Education*

### Exponential Modeling

**Table: Amounts of Digital Data Stored in a U.S. Household**

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount of Digital Data Stored on Devices in a Typical U.S. Household (terabytes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>0.4</td>
</tr>
<tr>
<td>2005</td>
<td>0.5</td>
</tr>
<tr>
<td>2006</td>
<td>1</td>
</tr>
<tr>
<td>2007</td>
<td>1.6</td>
</tr>
<tr>
<td>2008</td>
<td>2.5</td>
</tr>
</tbody>
</table>

*Source: Coughlin Associates*

### Quadratic Modeling

**Table: Numbers of International Adoptions in the United States**

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Adoptions (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>19.3</td>
</tr>
<tr>
<td>2002</td>
<td>21.5</td>
</tr>
<tr>
<td>2003</td>
<td>21.5</td>
</tr>
<tr>
<td>2004</td>
<td>23.0</td>
</tr>
<tr>
<td>2005</td>
<td>22.8</td>
</tr>
<tr>
<td>2006</td>
<td>20.5</td>
</tr>
<tr>
<td>2007</td>
<td>19.3</td>
</tr>
</tbody>
</table>

*Source: U.S. Department of State*

### Exponential Modeling

**Table: Percentages of Police Officers Who Are Women**

<table>
<thead>
<tr>
<th>City Size Represent City Size (in thousands)</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-9.999</td>
<td>8.3</td>
</tr>
<tr>
<td>10-24.999</td>
<td>7.5</td>
</tr>
<tr>
<td>25-49.999</td>
<td>8.5</td>
</tr>
<tr>
<td>50-99.999</td>
<td>9.4</td>
</tr>
<tr>
<td>100-249.999</td>
<td>11.7</td>
</tr>
<tr>
<td>250 or more</td>
<td>17.0</td>
</tr>
</tbody>
</table>

*Source: FBI Uniform Crime Report*
Quadratic Modeling

Table: Fatality Rate Per 100 Tornadoes

<table>
<thead>
<tr>
<th>Year</th>
<th>Fatality Rate (Number of fatalities per 100 Tornadoes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>18</td>
</tr>
<tr>
<td>1999</td>
<td>11</td>
</tr>
<tr>
<td>2000</td>
<td>5</td>
</tr>
<tr>
<td>2002</td>
<td>3</td>
</tr>
<tr>
<td>2004</td>
<td>2</td>
</tr>
<tr>
<td>2006</td>
<td>9</td>
</tr>
<tr>
<td>2007</td>
<td>11</td>
</tr>
</tbody>
</table>

Source: National Oceanic and Atmospheric Administration

Square Root Modeling

Table: Percentages of Foundations That Compensate All of Their Board Members

<table>
<thead>
<tr>
<th>Asset Group (millions of dollars)</th>
<th>Asset Used to Represent Asset Group (millions of dollars)</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–5</td>
<td>2.5</td>
<td>4</td>
</tr>
<tr>
<td>5–10</td>
<td>7.5</td>
<td>8</td>
</tr>
<tr>
<td>10–25</td>
<td>17.5</td>
<td>10</td>
</tr>
<tr>
<td>25–50</td>
<td>37.5</td>
<td>15</td>
</tr>
<tr>
<td>50–100</td>
<td>75</td>
<td>20</td>
</tr>
<tr>
<td>100–250</td>
<td>175</td>
<td>21</td>
</tr>
<tr>
<td>250–500</td>
<td>375</td>
<td>32</td>
</tr>
</tbody>
</table>

Source: New York Times

Balancing Concepts, Skills, and Curve Fitting

- Are there skills or concepts in intermediate algebra that aren’t needed for subsequent courses?
- Are there any skills or concepts in intermediate algebra that might be better addressed in subsequent courses?
- Trade some (or all) traditional word problems for some curve fitting.

Elementary Algebra and College Algebra

Curve fitting can be extended to elementary algebra and college algebra.

- Elementary algebra: linear and quadratic models
- Intermediate algebra: linear, exponential, quadratic, rational, and radical models
- College algebra: determining which model to use; piecewise defined models

Student Quote

There is actually no doubt that using actual data not only broadens our understanding of the real world, but it also helps us turn mathematical concepts into tangible and concrete ideas.
A student should be able to . . .

- Research, Analyze, and Report
- Analyze Data by Inspection
- Verify
- Models Get Modified
- Use Models to Support an Argument
- Ambiguous Authentic Situations
- Correct terminology
- Convert Units
- Sift Through Large Amounts of Information
- Mathematics is Truly Useful

Top Ten Things Every College Should Know and How This Can Be Achieved By Curve Fitting

Jay Lehmann
College of San Mateo
MathnerdJay@aol.com
prenhall.com/lehmanninfo