Are We There Yet?

Calculating Geographical Distances – S113

By Eric Hutchinson

Getting There: Using a Map

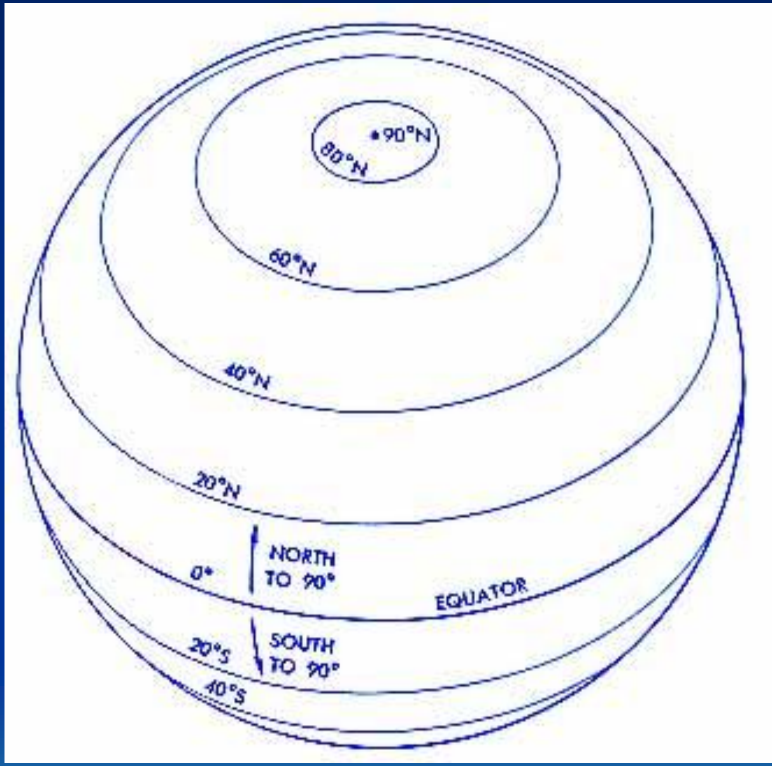
- All flat maps are projections
- There are several types of projections
- All projections have some distortion

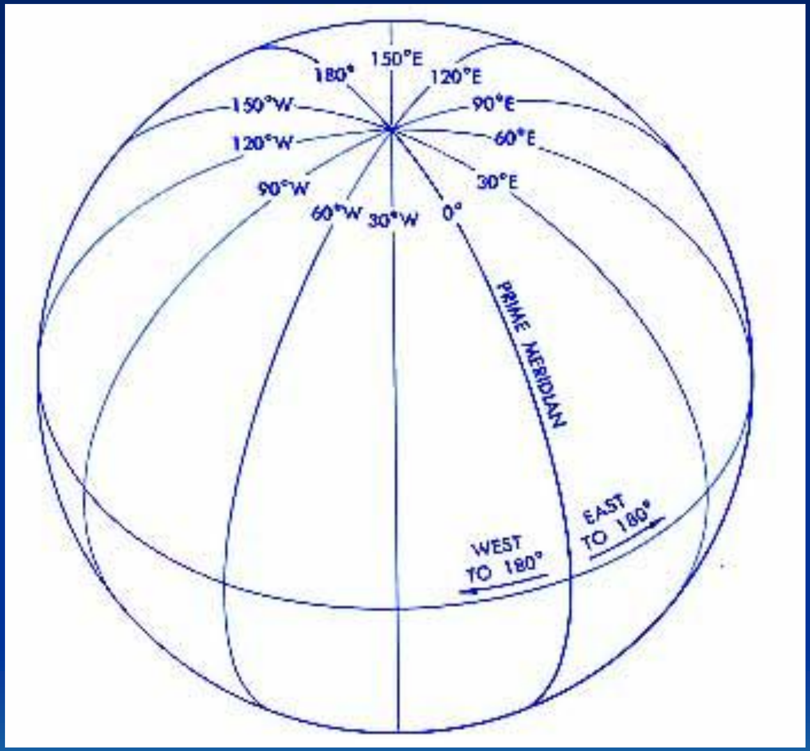
Getting There: Using a Map

- In our example, 0.75 inches equals 200 miles
- Set up a proportion: 0.75/200 = 7.25/x
- Solving gives x = 1933.33 miles

- Fairly accurate over short distances, however larger distances give greater distortion

Latitude and Longitude





Getting There: Equirectangular Approximation

- Projecting a latitude and longitude on a flat surface.
- Also known as Cylindrical Equidistant Projection
- Once projected on a flat surface the Pythagorean Theorem can be used
- Longitudes and Latitudes must be in radians.

Getting There: Equirectangular Approximation

- X =(long2-long1)*cos[(lat1+lat2)/2]
- Y= lat2 – lat1
- D= R*sqrt(x^2 + y^2)
- R = 3963.1906 miles

- LAS: 0.629716, -2.00979
- JAX: 0.532222, -1.425722

Getting There: Equirectangular Approximation

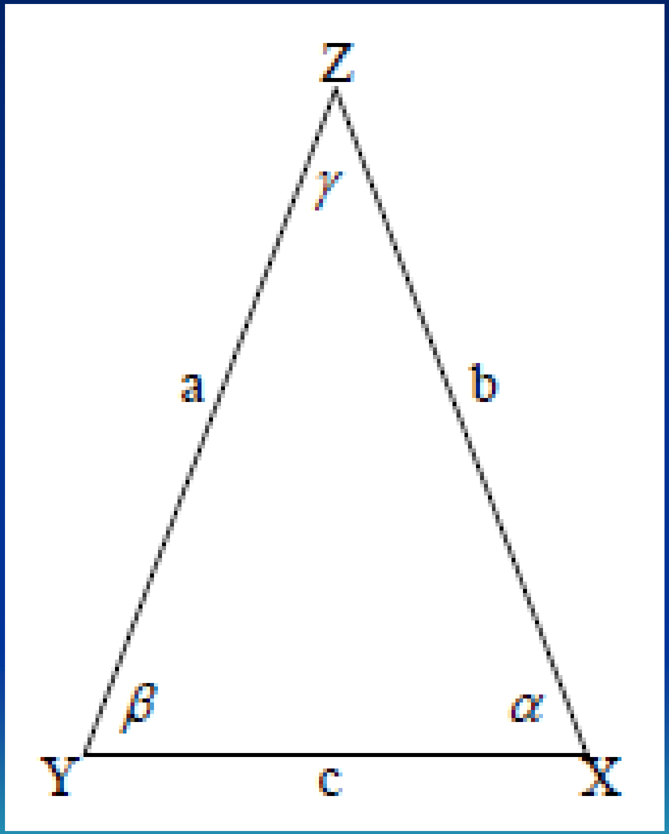
- X = (-1.42+2.01)*cos[(0.62+0.53)/2]=0.48
- Y=0.53 – 0.62 = -0.97
- D = 3963.1906*sqrt[(0.48)^2+(-0.97)^2]
- = 1973.192 miles

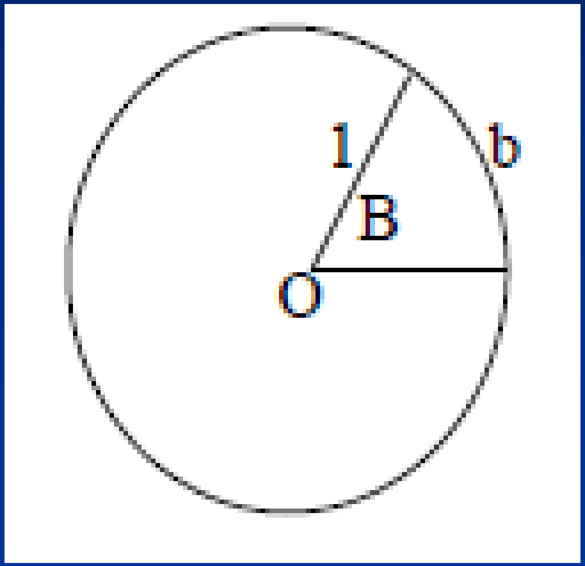
- This method gives greater accuracy over shorter distances

Getting There: Spherical Law of Cosines

- We will consider the Earth as a sphere
- Finding distances on a sphere will involve spherical geometry
- We will develop a formula to find the distance of one of the sides of a triangle on a sphere (Triangle XYZ).
- Will be similar to the Euclidean Law of Cosines.

Getting There: Spherical Law of Cosines

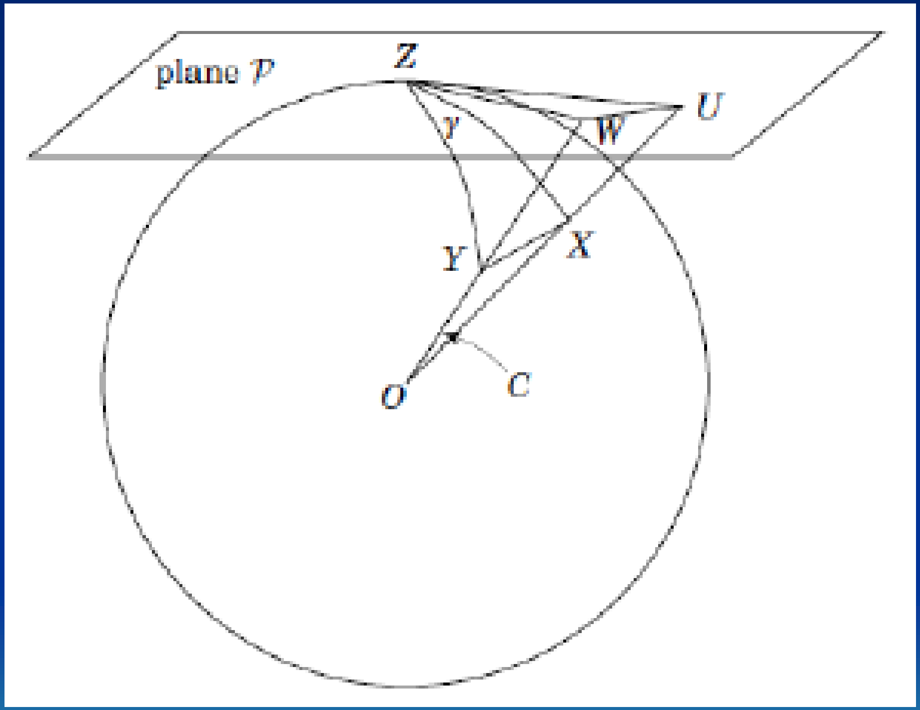


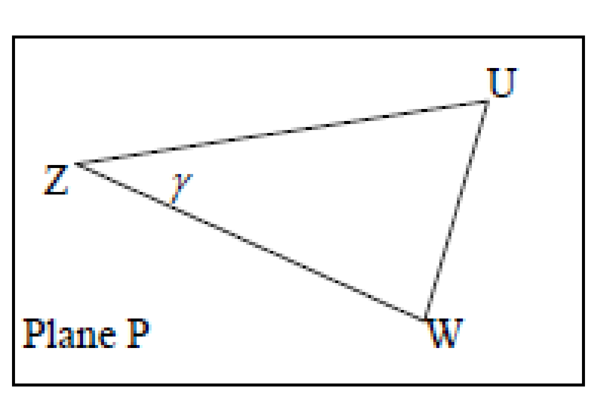


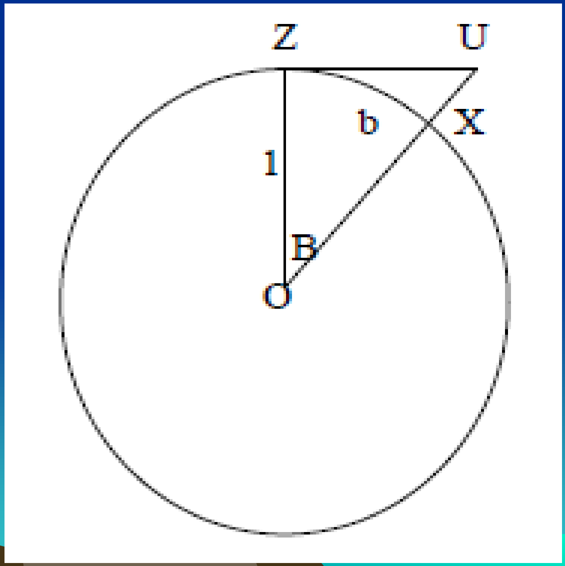
Getting There: Spherical Law of Cosines

- Place point Z at the north pole
- Plane P tangent to the sphere at point Z
- Extend a ray from O through X to P which intersects P at U
- Extend a ray from O through Y to P which intersects P at W

Getting There: Spherical Law of Cosines







Getting There:
Vincenty Formula

- Earth is an Ellipsoid
- Project points from the Ellipsoid onto a Sphere
- Most accurate
- Complicated mathematics
- Elliptical Integrals solved by power series until convergence

Getting There: Vincenty Formula

a	length of semi-major axis of the ellipsoid (radius at equator);	3963.1906
f	flattening of the ellipsoid;	(1/298.257223563)
$b = (1 - f)\, a$	length of semi-minor axis of the ellipsoid (radius at the poles);	3949.9028
φ_1, φ_2	latitude of the points;	
$U_1 = \arctan[(1 - f) \tan \varphi_1]$, $U_2 = \arctan[(1 - f) \tan \varphi_2]$	reduced latitude (latitude on the auxiliary sphere)	
$L = L_2 - L_1$	difference in longitude of two points;	
λ_1, λ_2	longitude of the points on the auxiliary sphere;	
α_1, α_2	forward azimuths at the points;	
α	azimuth at the equator;	
s	ellipsoidal distance between the two points;	
σ	arc length between points on the auxiliary sphere;	

Getting There: Vincenty Formula

$$\sin\sigma = \sqrt{(\cos U_2 \sin\lambda)^2 + (\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos\lambda)^2}$$

$$\cos\sigma = \sin U_1 \sin U_2 + \cos U_1 \cos U_2 \cos\lambda$$

$$\sigma = \arctan\frac{\sin\sigma}{\cos\sigma}$$

$$\sin\alpha = \frac{\cos U_1 \cos U_2 \sin\lambda}{\sin\sigma}$$

$$\cos^2\alpha = 1 - \sin^2\alpha$$

$$\cos(2\sigma_m) = \cos\sigma - \frac{2\sin U_1 \sin U_2}{\cos^2\alpha}$$

$$C = \frac{f}{16}\cos^2\alpha[4 + f(4 - 3\cos^2\alpha)]$$

$$\lambda = L + (1-C) f \sin\alpha \left\{\sigma + C\sin\sigma\left[\cos(2\sigma_m) + C\cos\sigma(-1 + 2\cos^2(2\sigma_m))\right]\right\}$$

Getting There: Vincenty Formula

- After doing computations below, you will get the distance (s) to be 1964.77 miles

$$u^2 = \cos^2\alpha \frac{a^2 - b^2}{b^2}$$

$$A = 1 + \frac{u^2}{16384}\left\{4096 + u^2\left[-768 + u^2(320 - 175u^2)\right]\right\}$$

$$B = \frac{u^2}{1024}\left\{256 + u^2\left[-128 + u^2(74 - 47u^2)\right]\right\}$$

$$\Delta\sigma = B\sin\sigma\left\{\cos(2\sigma_m) + \tfrac{1}{4}B\left[\cos\sigma\left(-1 + 2\cos^2(2\sigma_m)\right) - \tfrac{1}{6}B\cos(2\sigma_m)\left(-3 + 4\sin^2\sigma\right)\left(-3 + 4\cos^2(2\sigma_m)\right)\right]\right\}$$

$$s = bA(\sigma - \Delta\sigma)$$

Getting There: Computing Distances on Calculator

- You can use a graphing calculator to compute these distances, especially the Vincenty formula.
- Calculator will do iterations until convergence
- Let’s try some examples

Web Resources Used

- http://www.math.unl.edu/~shartke2/teaching/2011m896/SphericalLawOfCosines.pdf
- http://www.movable-type.co.uk/scripts/latlong.html
- http://www.movable-type.co.uk/scripts/latlong-vincenty.html

Thank you!

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