Ten Strategies for New (and Experienced) Instructors…Algebra to Calculus

Alice Kaseberg, kaseberg_alice@msn.com **AMATYC S135**

Strategy 1: Set the tone for your course from day one.

a. **Replace calling names for attendance with a Student Information Card**

Getting Acquainted

Your full name:_____

Name you go by: _____ Email: _____

Your Major:_____

Highest math class completed in high school:_____

Last math class taken and when:_____

A personal goal, hobby, or area of interest:_____

_____

Number of credit hours you are taking this term:_____

Number of hours you work each week:_____

From the card:

Placement (Identify students clearly misplaced or potential group leaders from math class data.)

Persistence or motivation (Students who state a major or a goal tend to stick with their courses. Look out for ski/snowboarding enthusiasts during winter months. Interact with these students on how they will balance fun with academics.)

Algebra of Reality (Multiply number of credit hours by 3 and add the number of hours worked = "load" for term; 12 credit hours plus 10 hours work is a load of 46. Compare that to a full time job of 40 hours per week. A student should consider other responsibilities such as child or parent care.)

End of Term Research (Record grades on the card at the end of the term. Is student success from prior math background, from personal motivation, or from your instruction or other classroom factors?)

b. **Replace talking about your syllabus with a take-home syllabus quiz**. This quiz highlights the important aspects of the syllabus by asking "Write out the sentence that describes…". The quiz is collected at the fourth class and not returned to the student. It is filed until the end of the course (plus a few weeks) as verification that the student received and read the information.

c. **Class time may now be spent on a problem-solving or non-intuitive activity in groups.**

For example:

Algebra: A. Alice's grand-mother was x years old in the year x^2. In what year was her grand-mother born? B. The son of John, an instructor at Grand Rapids CC, was born one year off from being x years old in the year x^2. In what two years might the son have been born? [pure reasoning, no algorithm required]

Intermediate or College Algebra: An airplane travels 100 miles at 500 miles per hour and a second 100 miles at 300 miles per hour. What is the average speed for the whole trip? (Ans. 375 mph)

Calculus: A 20-foot ladder is leaning against a wall in a safe ladder ratio of 4 to 1, wall height to distance the base is from a wall. If the top slipped one foot in 1 second, how fast did the bottom of the ladder slip? (Ans. 2.98 feet per second) This non-intuitive problem is one of the fun things about the calculus of related rates and I've used it several times on the first day of calculus.

Strategy 2 Share stories that offer career guidance such as "follow your passion."
Alice's former student, Eric Stillwell, was the dedicated president of Star Fleet … the Star Trek fan club from 1981 to 1984. After graduating in political science from the University of Oregon in 1985, he handed his diploma to his parents and left for Hollywood…and as a studio page, got coffee for people on the Next Generation set. In 1990 he co-wrote *Yesterday's Enterprise*. Avid Trek fans know this time-warp episode. Says Eric: "The greatest gift from working on Trek has been the opportunity to travel around the world and do conventions and meet fellow fans. That's what I love the most, interacting with other fans, visiting new places and exploring other cultures." http://www.startrek.com/article/recalling-yesterdays-enterprise-with-eric-stillwell-part-1 and part 2.

Julie Foudy joins Madeline Albright, Eunice Kennedy Shriver, and Dwight D. Eisenhower in the top 100 Most Influential Student-Athletes celebrated by the NCAA's 100th Anniversary in 2006. These 100 selections made a significant impact or major contribution to society. Foudy, a Stanford University soccer All American, earned two Olympic Gold medals in spite of her parents refusing to let her compete on girls teams until she turned seven. (She competed with the boys instead.) Foudy declined to endorse a new line of Reebok soccer balls until personally inspecting their Pakistani manufacturing process. She and her husband operate both leadership camps and soccer camps.
http://www.uclabruins.com/genrel/031006aab.html; www.juliefoudyleadership.com; and Christina Lessa, *Women Who Win in Sport and in Life*, Universe Publishing, c. 1998

Alice's friend, Peter Hammar, created and curated the AMPAX Museum of Magnetic Recording. Pete was passionate about magnetic recording and spoke fluent German, both of which led him to this opportunity. The museum opened in 1983. The contents of the museum are now housed with the Silicon Valley Archives, Stanford University.

Strategy 3 Be an active learner: Regularly learn something new. This places you in the role of being a student. Observe your motivation level. Observe what knowledge or experience permits you to learn new information. Observe the form (visual, numeric, verbal, symbolic, kinesthetic) of information. Observe what frustrates you and sympathize with your students.
Corollary to Strategy 3: A recent flurry of emails among Project ACCCESS Fellows regarding use of the iPad in the classroom reminded me of the corollary: Try something new in your classroom every term. To learn more about Project ACCCESS, go to www.amatyc.org.

Strategy 4 Read material unrelated to mathematics. This is different than #3 because you are not necessarily seeking to be a dancer, a snowboarder, or an actor but instead you are looking for mention of skill sets that reflect characteristics of a learner or strategies for learning or success that also work in mathematics. Another way to look at it is in transfer of skills between subjects. Consider:
Sport Riding Techniques, page 13, *Riding safely and within your limits takes focus, confidence, practice, patience and self-discipline.* page 14: *As your riding skills grow, rereading the information will spark you to new levels of safety and speed because your newfound skills will allow you to grasp a technique that was unavailable to you earlier.*
Backcountry Snowboarding, page 23 *…I remember the early days. The humbling agony of being a beginner. The days I wanted to quit but forced myself to call a learning experience. The pain of pounding the ground hard.*
Advice for Dancers, page 14, *Strongly motivated dancers have a big advantage over their peers, because they are able to train even in the face of distractions.*
Acting Onstage and Off Mathematics helps with these *Career Capabilities* which are important in any job, page 35: *adapting within structure, changing strategies, balancing multiple tasks, managing time effectively, meeting deadlines, working unsupervised, editing, retaining information, …* (

Share with a partner #1 [Share a first day activity or general strategy you have used.]

Strategy 5 Stress Units of Measure and Unit Analysis

a. Heard on the radio: If the water in Lake Superior were emptied out, it would cover all of North America and South America to a depth of one foot. True or False?
Volume of Lake Superior: 2935 cubic miles, Area of North America: 8,912,454 square miles, Area of South America: 6,731,004 square miles (The World Almanac, 2010) The volume of water is

$$\frac{2935 \text{ cubic miles}}{1} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} = 15,496,800 \text{ square mile} \cdot \text{feet}$$

b. Suppose the growth of a baby is from 21 inches at birth to 5 ft 9 inches at age 18. What is the growth rate in miles per hour?

$$\frac{69 \text{ in.} - 21 \text{ in.}}{18 \text{ yrs}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{1 \text{ yr}}{365 \text{ d}} \cdot \frac{1 \text{ d}}{24 \text{ hr}} \approx 4.805 \times 10^{-9} \text{ miles per hour}$$

Strategy 6 Use a variety of approaches

a. Radical Equations and Extraneous Roots When we solve $\sqrt{2-x} = x+1$ we obtain, to the nearest thousandths, x = 0.303 and x = -3.303. The negative result is extraneous.
The graphs of $Y1 = \sqrt{2-x}$ and $Y2 = x+1$ are in Figure 1. The graphs in Figure 1 intersect at x = 0.303. There is no intersection at x = -3.303.

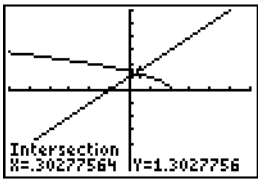

Figure 1: Solution at x = 0.303

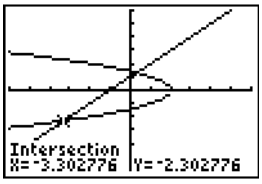

Figure 2: Extraneous root at x = -3.303

However, if we add to the graph the reflection of $Y1 = \sqrt{2-x}$, $Y3 = -\sqrt{2-x}$, Figure 2, then the graphical meaning of the extraneous root becomes apparent. In Figure 2, the graphs of the line, Y2, and the reflection intersect at x = -3.303.

b. Proving a Result Based on the Solution to a Rational Equation

Suppose we solve: $2 + \frac{9}{x-3} = \frac{3x}{x-3}$, $x \neq 3$

Symbolic Solution:

$2 + \frac{9}{x-3} = \frac{3x}{x-3}$ Multiply by (x - 3).

$2(x-3) + (x-3)\frac{9}{x-3} = (x-3)\frac{3x}{x-3}$ Simplify.

$2x - 6 + 9 = 3x$ Solve.

$x = 3$

The solution $x = 3$ is an extraneous root because it gives a zero in the denominator in the original equation.

Graphical Solution: The convenience of a calculator graph is important here. We enter the left and right sides of $2+\frac{9}{x-3}=\frac{3x}{x-3}$ as separate equations $y_1=2+\frac{9}{x-3}$ and $y_2=\frac{3x}{x-3}$

We graph with the standard window. Use the up or down curser to swap between the two graphs. The x-intercepts are (0, -1) and (0, 0). Move to the right and observe that the curves continue to be 1 unit apart. The graphs shown in Figure 3 do not intersect. At $x = 3$, both sides of the equation have a zero denominator. Both curves approach the asymptote at x = 3.

The tables in Figures 4 and 5 also suggest that the two graphs are 1 unit apart. We can prove this separation by showing $y_2 - y_1 = 1$, thus applying the subtraction of rational expressions.

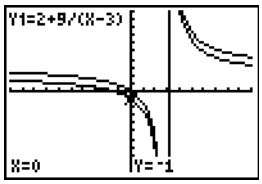

Figure 3 Graph

X	Y1	Y2
0	-1	0
1	-2.5	-1.5
2	-7	-6
3	ERR:	ERR:
4	11	12
5	6.5	7.5
6	5	6

X=0

Figure 4 Table

X	Y1	Y2
2.7	-28	-27
2.8	-43	-42
2.9	-88	-87
3	ERR:	ERR:
3.1	92	93
3.2	47	48
3.3	32	33

X=2.7

Figure 5 Zoomed table

Strategy 7. Offer student self-appraisal, a 1954 National Council of Teachers of Mathematics suggestion on evaluation.

Some of the following fifty statements will make you smile but others are still valid and still others can be modified to create a Self-Appraisal for your classroom use.

Student Self-Appraisal Check list in Mathematics*

This check list is intended to help you discover your strengths and how you can improve. Check each statement truthfully and thoughtfully so that the best kind of activities can be planned for you. Use the code below in describing how you think, feel, or act with respect to the statement given.

Always 5 Usually 4 Sometimes 3 Seldom 2 Never 1

1. I complete the required assignments on time.
2. I complete the optional assignments.
3. I complete my assignments independently.
4. I check my work to find my errors.
5. I understand the processes I use to solve problems.
6. I understand the mathematical words used in my textbook.
7. I understand the mathematical words used by my instructor.
8. I add, subtract, multiply, and divide accurately.
9. I can use mathematical words to express myself in writing.
10. I can use mathematical words to express myself in discussions.
11. I bring applications of mathematics to class.
12. I can interpret graphs which I find in newspapers and magazines.
13. I can read tables of data which I find in newspapers and magazines.
14. I find mathematics used in the newspapers, books, or magazines which I read outside of class.
15. I use the mathematical facts I learn in this class in my other classes.
16. I use the mathematical facts I learn in this class at home, in recreations, or at work.

17. I discover principles and generalizations before I read them or hear them stated by my instructor or fellow students.
18. I use materials other than my textbook to learn mathematics that is not required in this course.
19. I listen carefully to the explanations of my instructor.
20. I do original, creative work in drawing designs, giving reports, or completing projects rather than repeating what someone else has said or done.
21. I keep a list of important ideas or rules studied.
22. I understand the explanation of my instructor.
23. I use the methods of analysis I earn in this class in other classes.
24. I use the methods of analysis I learn in this class at home, in recreation, or at work.
25. I learn to reduce my errors by checking my work.
26. I use the statements of the textbook or instructor rather than my own words when I recite.
27. I participate in classroom discussions.
28. I look for assumptions behind controversial statements which my classmates make.
29. I can locate useless or unimportant material in a problem or in an argument.
30. I can translate relationships expressed in words or sentences into equations.
31. I can visualize statements or descriptions of stated problems so that they become realistic.
32. I can measure accurately with measuring instruments such as a ruler, protractor, scale.
33. I can multiply and divide with a slide rule.
34. I can use a compass and ruler to draw accurately.
35. I can represent a three-dimensional figure on a plane surface.
36. I use short cuts such as “transpose” or “cancel” without knowing the reason the short-cut works.
37. I solve practice exercises (such as equations) mentally rather than writing down steps in the solution.
38. Drawings in the text, by the instructor or by myself, help me find the solutions to problems.
39. I can memorize rules better than I can understand how the rule is to be used.
40. I can read my textbook with understanding.
41. I can learn how to work exercises by going over the textbook illustrations.
42. I know how to get information about a mathematical topic in the library.
43. I ask questions about class work I don’t understand.
44. I label and organize my work so that someone else looking at it can closely see what I have done.
45. I get most help in understanding my work from my friends.
46. I get most help in understanding my work from my parents.
47. I could learn much more mathematics if I tried harder.
48. I review the important ideas even when I am not preparing for a particular test.
49. I look up or ask about words when I am not sure of their meaning.
50. Most of my errors are due to a lack of understanding of the process to use.

*From p; 403-405, High School Self-Appraisal Form, *Emerging Practices in Mathematics Education,* Twenty-Second Yearbook, The National Council of Teachers of Mathematics, Washington, D.C., 1954.

Share with a partner #2 [Share a modern student self-appraisal statement.]

Strategy 8 Take control of your time

a. **Set goals and objectives:** List your goals for your preparation time, for your day, and for the week, in general. Keep them visible. Write tests based on objectives for the course. This permits you to write a test well in advance and, after printing, write the answer key and scoring rubric (surely you do have expected

responses in mind) to check for errors and length of time it will take students to do the test. Insist that department meetings have a written agenda and a fixed length.

b. **Know yourself:** What time of the day are you most productive? What time, least productive? Set tasks accordingly. During most productive do lesson planning. During least productive, do email.

c. **Be prepared:** Don't leave for the weekend without having upcoming tests written, Monday's lessons planned, and everything in place on your desk. Grading tests and written work and recording scores in your electronic grade book are more portable than planning and test writing.

d. **Containerize:** Have one tote bag for each course you teach (book, tests, special equipment)

e. **Stay in motion**: Stand when you talk on the phone. Follow "Hello" with "What can I do for you?" If a student or colleague comes to your office just as you have to leave for class or a meeting, stand up to greet them, and set an appointment for later or let them talk as you walk away together. After class, talk with an individual as you pack up and again, set an appointment as you leave the classroom if more time is needed.

Read about time management. Retired consultant Ann Reeves is a favorite:
http://www.3dman.com/SMC/sidenotes2.htm

Strategy 9 Share things that are fun to you

Geometry and Stew (Play with your food!): I observed that the cross section of many potatoes appear elliptical. Suppose the short axis of the potato in Figure 1 is $2a$ and the long axis of the potato is $2b$. Suppose we rearrange the four pieces of potato in Figure 1 to give a larger area inside the same circumference. Answer is shown in Figure 2. (The rearrangement is not as trivial as it may appear.) What is the area of the inner square in Figure 2? **Calculus**: What happens to the area of the square in Figure 2 as a approaches b? What happens to the shape of the circumference?

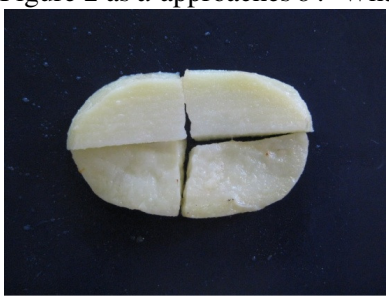

Figure 1

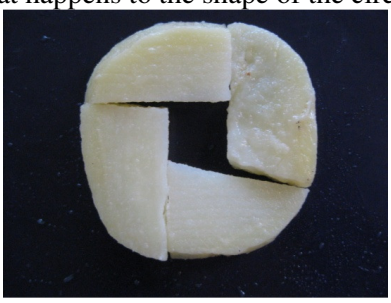

Figure 2

Strategy 10 Point out surprising applications of mathematics seen everyday

The TV satellite dish is a common paraboloid. The railing on a circular staircase forms a spiral helix. Imagination was the key for F. Tamburini, University of Padua, Italy, when in June, 2011, he combined the paraboloid with a helical spiral to make a twisted parabolic antenna. He thus encoded many channels in the same frequency through a radio vortex. [Search for F. Tamburini on the web for photos.]

Thank you for attending and sharing!