Activate Your Calculus Students!

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Activities included in this packet:

Inside the box

Related Rate Activity

3-D Model

Graphing Activity

Inside the Box

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Materials Needed for each student:

Graphing Calculator
Standard sheet of paper
Ruler with inch and centimeter markings
Scissors
Tape
Activity sheet for recording
Optional: Candy for the students with the highest volume in each group and overall.

Background

Optimization is a topic that many calculus students struggle with. Many of the same students who were successful finding the absolute extrema of a continuous function on a closed interval, express their difficulty with being able to form the function that represents the situation. Another area of difficulty arises when the students are required to translate their findings to what the results mean to the actual problem at hand. This activity focuses on making a model for the commonly presented problem of finding the dimensions that will produce the maximum volume of an open box formed by cutting identical squares from the corners of a sheet of material and folding up the sides.

The Activity

Divide the class into groups of 3 or 4 students per group. Instruct the class to measure the dimensions of their paper using centimeters and record the dimensions on the activity sheet. Next they should individually decide on the length of the sides of identical squares to cut out of each corner of their paper. The object is to create a box by folding up the sides and forming an open box that will have the largest volume in their group. Each student should record their length on their activity sheet in the first line under the L1 heading. At this point, instruct the students to cut the squares out of the corners of their paper, fold up the sides and tape them to form the open box. Once this has been done, have the students measure the length, width and height of their box and calculate the volume, recording the volume on the activity sheet in the column with the L2 heading.

Compare Results in the Group

Within each group have the students compare their choices for the length of the sides of the squares they cut out and the calculated volume.

Compare Results in the Class

Bring the class back together and ask each group to report the information about the box with the largest volume in their group. Record the information on the board and compare the results. Call the students' attention to any cases that may have the same size square cut out of the corner but a different calculated volume. Discuss what can cause the discrepancy and correct it. Show the class the boxes with the largest volume.

Record the Rest of the Measurements on the Activity Sheet

Write the length of the square and the volume of the boxes made by each student on the board and have them copy the values on their activity sheet.

Use the Graphing Calculator to Find a Regression Model for the Volume

Once the measurements have been recorded, ask the students to enter the data in L1 and L2 of their graphing calculator and find the regression models that are listed on the activity sheet. If a student's calculator doesn't give the correlation coefficient, on the TI83/TI84 models they can access the catalog, choose diagnostics on and then recalculate the regression models. The coefficient of determination, r^2, should then be visible. This is a great time to discuss the different models, compare the regression equations and how the coefficient of determination can aid in deciding the best model to use. Record the best model found on the board.

Develop the Theoretical Model

Now that the students have had experience with making the open boxes, lead them through the thought process to create the volume function with the independent variable being the length of the side of the square cut from the corners of the paper. Compare the theoretical model with the regression model found earlier. Your results for the class will vary depending on the number of cases developed in the class and the accuracy of the measuring done by the students.

Apply the Techniques from Calculus

Here is a great chance to discuss the domain of the length of the side of identical squares to be cut from the corners of the paper. Then walk the students through the techniques from calculus used to find the length of the side of the square that will create the open box with the largest volume. Compare these findings with the physical models made in class by the students.

Overview

This activity provides the students with hands on experience interpreting the written model to aid them in production of a physical model. With materials that are readily available, it serves as a great review for using their graphing calculator to find regression equations from data and opens the door for discussion on how to choose the best model for the given situation. Friendly competition can also be incorporated by offering prizes for the boxes with the largest volume. I hope your classes will find this activity as engaging as mine have.

Optimizing Volume Worksheet

Dimensions of Material:

Measure of side for square cut from corner	Width of Box	Length of Box	Height of Box	Volume of Box

Activity Sheet

Overall dimensions of the sheet of material:

Length of the side of the square cut from the corners of the paper, measured in centimeters L1	Calculated volume of the box in cubic centimeters L2

Regression Models — Coefficient of Determination, r^2

Linear

Quadratic

Cubic

Theoretical Model

Length of square cut to create maximum volume

Maximum volume value

Maximum volume attained from class models

Related Rate Activity

Materials needed for each student:

Triangular Prism model

Scissors

Tape Ruler with centimeter markings

The Activity

This activity can be done individually or in groups. It fits well right after related rate problems have been introduced with examples.

Instruct the students to cut out the model of an open trough with ends that are equilateral triangles and tape to form the trough.

Statement of the problem:

Water is dripping into the trough at a rate of 1 cm^3/sec. How fast is the water level changing when the water is 1 cm deep?

Guiding Questions:

What is the formula for the volume of the water in the trough at any given time?

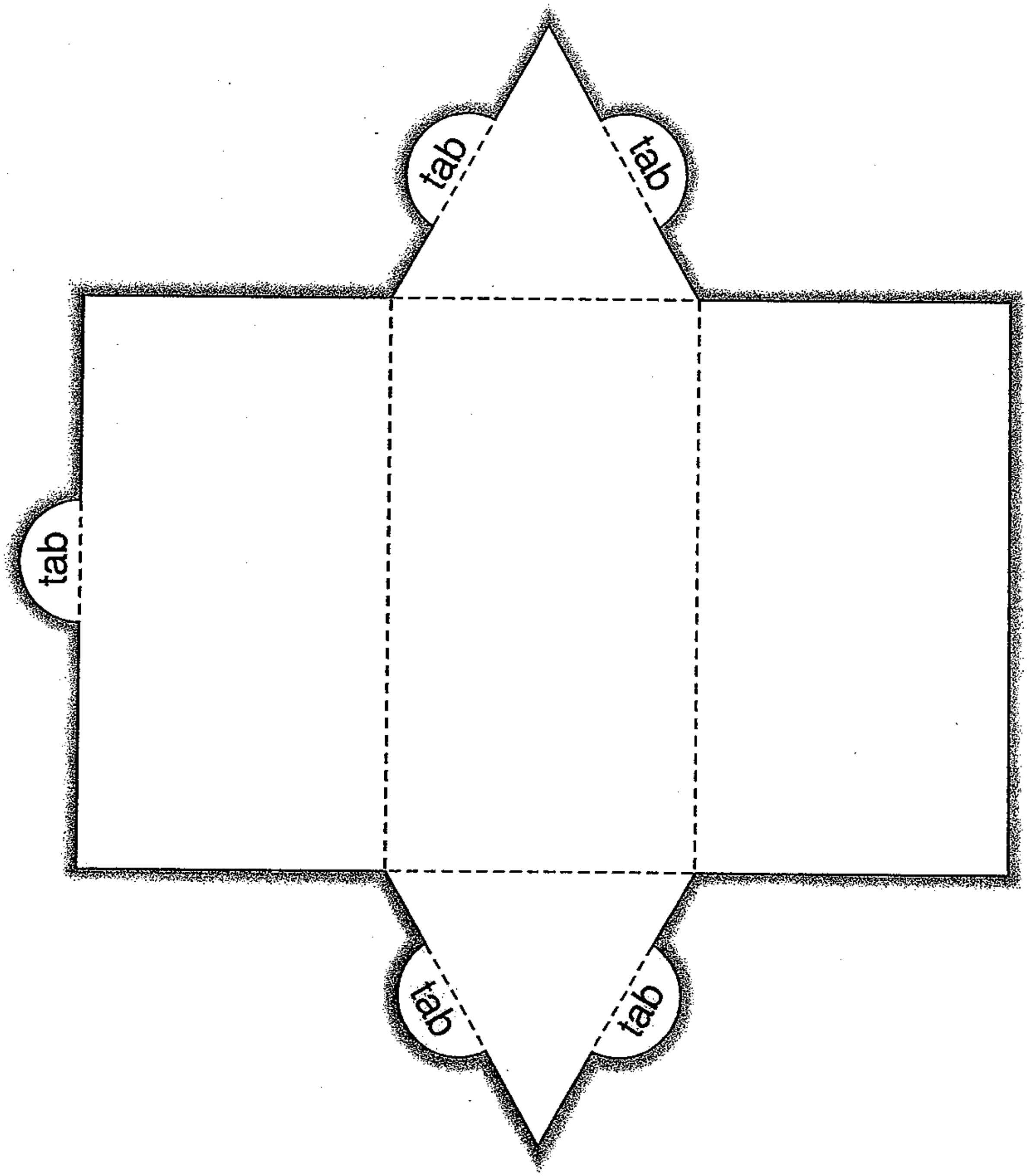
As the water is filling the trough, what measurements of the space the water is occupying are changing?

As the water is filling the trough, what measurements of the space the water is occupying are staying the same?

What are the overall dimensions of the trough?

How is this helpful in answering the question asked?

Triangular Prism Model



Solid of Revolution Project

On your card you will find a function a region to graph and an axis of revolution. Your task is to first *carefully* graph the region indicating the axis of revolution and then make a 3-D model of the solid of revolution created when the region is spun around the indicated axis. Your model needs to be accurate and to scale using the scale you choose and document on your graph.

Solid of Revolution Model Project Grading Sheet

_____ Graph of the region with axis of spin labeled (10 points)

_____ Accuracy of solid (20 points)

_____ Correct Scaling (15 points)

_____ Creative use of material (5 points)

Region: Bounded by $f(x) = x^2 + 2, x = 3,\ x - axis$

Axis of revolution: $y - axis$

Graphing Activity

Materials needed for each student:

Transparency with graph grid

Marker for writing on the transparency

Divide the class into groups of three students for each group. Give the group a function that has critical integer critical values and integer values for which the second derivative is zero or undefined. Have one of the students graph the function on their transparency, another student graph the first derivative of the function and the third student graph the second derivative. Once the graphs are completed, have them stack the transparencies so the origins coincide. Have the students compare the three graphs and answer the questions:

1) Over the interval that the function is decreasing, what do you notice about the graph of the first derivative?
2) Over the interval that the function is increasing, what do you notice about the graph of the first derivative?
3) Over the interval that the first derivative's graph is increasing, what do you notice about the graph of the second derivative? What do you notice about the graph of the function?
4) Over the interval that the first derivative's graph is increasing, what do you notice about the graph of the second derivative? What do you notice about the graph of the function?