Singapore Math Workshop
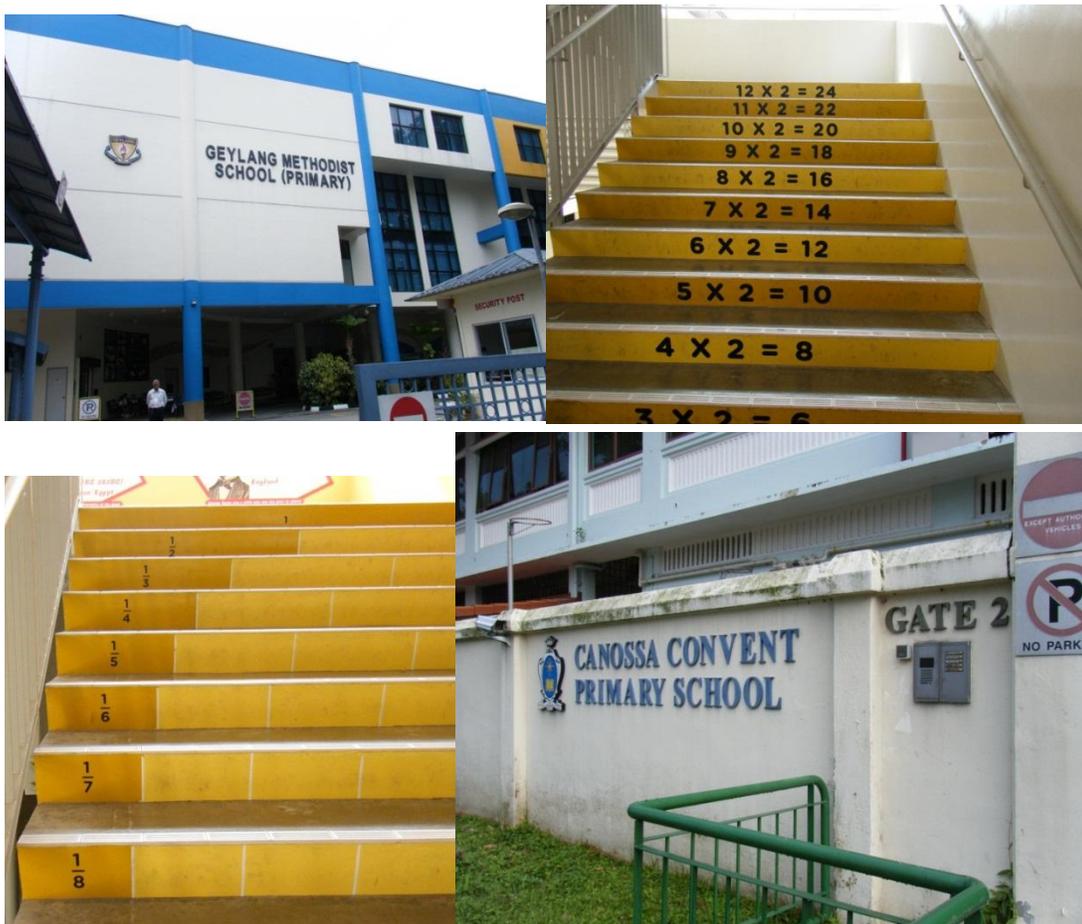
Presented by Joyce Lindstrom

AMATYC 2012

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With your table colleagues, you will investigate:

- Deck of Cards for Ordering Puzzle

- Singapore Math Text Books

- Manipulatives for Place Value

- Bar Models for Word Problems

- Divisibility Rules

Comparison Model Problems

1. Ali has $8 more than Sid. Trina has $6 less than Ali. The three of them have $76 in all. Find the amount of money each of them has.

   Yeap Ban Har, *Bar Modeling, A Problem-Solving Tool*, p. 37

2. Chris started saving some money on Monday. Each day she saved $2 more than the day before. By Friday of the same week Chris had saved $35. Find the amount Chris saved on Wednesday.

   Yeap Ban Har, *Bar Modeling, A Problem-Solving Tool*, p. 40

3. There were 3 times as many girls as boys on a bus. There were twice as many children as adults. There were 36 persons altogether on the bus. How many girls were on the bus?

   Yeap Ban Har, *Bar Modeling, A Problem-Solving Tool*, p. 56

Sample Fraction Problems

4. Kumar took 5 days to read a book. He read 1/9 of the book on Saturday, 1/4 of the remainder on Sunday, and the remaining 48 pages during the last three days. How many pages were there in the book?

<div align="right">Cynthia Seto, <i>Teaching Fraction, Ratio and Percentage Effectively</i>, p. 49</div>

5. The ratio of the number of boys to the number of girls in a school hall was 5:7. After ½ of the boys and 63 girls left the hall, there was an equal number of boys and girls remaining in the hall. How many girls were there in the hall at first?

<div align="right">Cynthia Seto, <i>Teaching Fraction, Ratio and Percentage Effectively,</i> p. 80</div>

6. There were 126 children at a concert. The number of boys was ¾ as many as the number of girls. When an equal number of boys and girls left the concert, the number of boys and girls remaining at the concert was 5:7. How many boys left the concert?

<div align="right">Cynthia Seto, <i>Teaching Fraction, Ratio and Percentage Effectively</i>, p. 79</div>

Sample Questions from the Primary School Leaving Exam

7. Keane bought some marbles and gave half of them to Leon. Leon bought some stamps and gave half of them to Keane. Keane used 5 stamps and Leon gave away 11 marbles. The ratio of the number of stamps to the number of marbles Keane had left then became 1:7 and the ratio of the number of stamps to the number of marbles Leon had left became 1: 5. How many stamps did Leon buy?

<div align="right">[ACS 2010 SA1 Q18]</div>

8. Kentucky Town and Merrily Town are 234 km apart. Dawn left Kentucky Town for Merrily Town at 8.42 a.m. travelling at an average speed of 85 km/h. At the same time, Jennifer left Merrily Town for Kentucky Town. They met each other at 10.30 a.m.

   (a) What was Jennifer's average speed when she met Dawn?

   (b) If Jennifer were to increase her speed by 26 km/h before meeting Dawn, how much less time would she take before meeting her?

9. The number of balls in Box A is ½ of the number of balls in Box B. 10% of the balls in Box A and 10% of the balls in Box B was moved to Box C. As a result, the number of balls in Box C increased by 20%. There are 72 balls in Box C now. How many balls were there in Box B at first?

   [RGPS 2010 SA1 Q16]

10. Dolly had 80 more stickers than Jenny. Dolly gave 25% of her stickers to Jenny. Jenny in return gave 60% of her stickers to Dolly. In the end, Dolly had 100 stickers more than Jenny. How many stickers did Dolly have at first?   [NHPS 2010 SA1 Q18]

11. The tickets for a show were priced at $10 and $5. The number of ten-dollar tickets available is 1½ times the number of five-dollar tickets. 5 out of 6 ten-dollar tickets and all the five-dollar tickets were sold. The amount of money collected from the sale on the tickets was $5600. How much more would have been collected if all the tickets were sold?

   [Ai Tong 2010 CA1 Q10]

Now Try These with Singapore Math Bar Modeling

12. How many gallons of 50% antifreeze must be mixed with 80 gallons of 20% antifreeze to obtain a mixture that is 40% antifreeze?

<div align="right">Lial, Hornsby, McGinnis Beginning and Intermediate Algebra, 4th Edition, p. 158</div>

13. Airplanes usually fly faster from west to east than from east to west because the prevailing winds go from west to east. The air distance between Chicago and London is about 4000 miles, while the air distance between New York and London is about 3500 miles. If a jet can fly eastbound from Chicago to London in the same time it can fly westbound from London to New York in a 35-mph wind, what is the speed of the plane in still air?

<div align="right">Encyclopaedia Britannica, quoted in Lial, Hornsby, McGinnis Beginning and Intermediate Algebra, 4th Edition, p. 451</div>

14. Ms. Teng, a high school mathematics teacher, gave a test on perimeter, area, and volume to her geometry class. Working alone, it would take her 4 hours to grade the tests. Her student teacher, Jonah Schmidt, would take 6 hours to grade the same tests. How long would it take them to grade these tests if they work together?

<div align="right">Lial, Hornsby, McGinnis Beginning and Intermediate Algebra, 4th Edition, p. 452</div>

Investigation of Divisibility by 9

Let $n = a_t \times 10^t + ... + a_2 \times 10^2 + a_1 \times 10^1 + a_0 \times 10^0$

Complete this table:

t	10^t	$10^t - 1$	Divisible by 9?	Divisible by 11?	$10^t + 1$	Divisible by 11?
0						
1						
2						
3						
4						
5						
6						

Example: 53,812 can be written as:

$5(9999 + 1) + 3(999 + 1) + 8(99 + 1) + 1(9 + 1) + 2(1)$

Which can also be written as, where we know that each of the first four terms and, hence, their sum, is divisible by 9:

$(5 \times 9999) + (3 \times 999) + (8 \times 99) + (1 \times 9) + (5 \times 1) + (3 \times 1) + (8 \times 1) + (1 \times 1)(2 \times 1)$

Divisible by 9   Divisible by 9?

Thus, to determine if 53, 812 (or any number) is divisible by 9, it remains to be determined if the sum of the digits is divisible by 9.

Conclusion: A number is divisible by 9 if and only if the sum of the digits is a multiple of 9.

Investigation of Divisibility by 11

By an argument similar to that of divisibility by 9,

Let $n = a_t \times 10^t + \ldots + a_2 \times 10^2 + a_1 \times 10^1 + a_0 \times 10^0$

Complete this table:

t	10^t	$10^t - 1$	Divisible by 9?	Divisible by 11?	$10^t + 1$	Divisible by 11?
0						
1						
2						
3						
4						
5						
6						

Example: 53,812 can be written as:

$5(9999 + 1) + 3(1001 - 1) + 8(99 + 1) - 1(11\text{-}1) + 1(1)$

Which can also be written as, where we know that each of the first four terms and, hence, their sum, is divisible by 11:

$5(9999) + 3(1001) + 8(99) + 1(11) + 5(1) - 3(1) + 8(1) - 1(1) + 2(1)$

          Divisible by 11   Divisible by 11?

Thus, to determine if 53, 812 (or any number) is divisible by 11, it remains to be determined if (sum of odd-position digits) – (sum of even-position digits) is a divisible by 11.
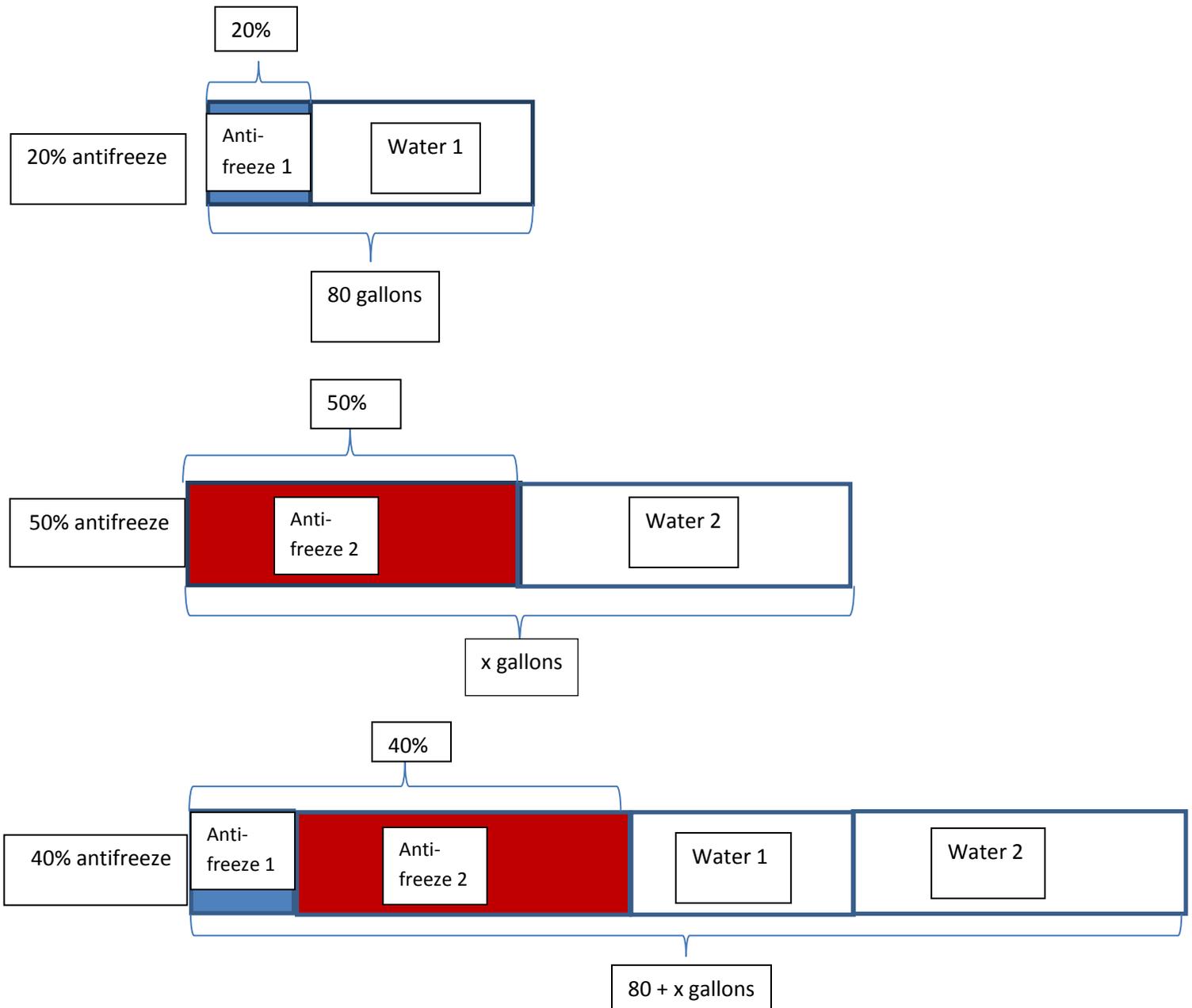
Conclusion: A number is divisible by 11 if and only if the number formed by (sum of odd-position digits) – (sum of even-position digits) is a multiple of 11.

Test these numbers for divisibility by 11: 2838; 34, 521; 26,341; 10,234,341

Possible Solutions to Problems 12 – 14

12. How many gallons of 50% antifreeze must be mixed with 80 gallons of 20% antifreeze to obtain a mixture that is 40% antifreeze?

20%

| 20% antifreeze | Anti-freeze 1 | Water 1 |

80 gallons

50%

| 50% antifreeze | Anti-freeze 2 | Water 2 |

x gallons

40%

| 40% antifreeze | Anti-freeze 1 | Anti-freeze 2 | Water 1 | Water 2 |

80 + x gallons

20% of 80 gallons + 50% of x gallons = 40% of (80 + x) gallons

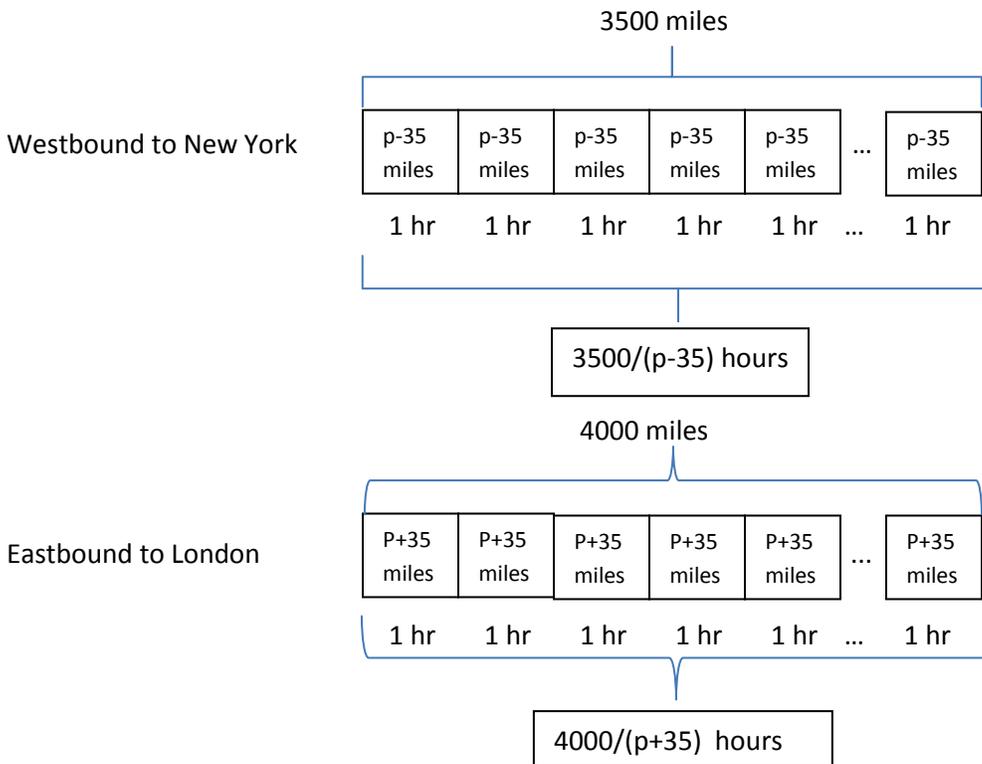$$.20(80) + .50(x) = .4(80 + x)$$

$$16 + .50(x) = 32 + .40(x)$$

$$.10(x) = 16$$

$$x = 160 \text{ gallons}$$

13. Airplanes usually fly faster from west to east than from east to west because the prevailing winds go from west to east. The air distance between Chicago and London is about 4000 miles, while the air distance between New York and London is about 3500 miles. If a jet can fly eastbound from Chicago to London in the same time it can fly westbound from London to New York in a 35-mph wind, what is the speed of the plane in still air?

Encyclopaedia Britannica, quoted in Lial, Hornsby, McGinnis *Beginning and Intermediate Algebra*, 4th Edition, p. 451

p = speed of plane in still air

3500 miles

Westbound to New York

| p-35 miles | p-35 miles | p-35 miles | p-35 miles | p-35 miles | ... | p-35 miles |

| 1 hr | 1 hr | 1 hr | 1 hr | 1 hr | ... | 1 hr |

3500/(p-35) hours

4000 miles

Eastbound to London

| P+35 miles | P+35 miles | P+35 miles | P+35 miles | P+35 miles | ... | P+35 miles |

| 1 hr | 1 hr | 1 hr | 1 hr | 1 hr | ... | 1 hr |

4000/(p+35) hours

Time eastbound = time westbound

$$\frac{3500}{p-35} = \frac{4000}{p+35}$$

3500(p + 35) = 4000(P - 35)

3500p + 122500 = 4000p − 140000

262500 = 500p

525 = p

The plane travels at 525 mph in still air.

14. Ms. Teng, a high school mathematics teacher, gave a test on perimeter, area, and volume to her geometry class. Working alone, it would take her 4 hours to grade the tests. Her student teacher, Jonah Schmidt, would take 6 hours to grade the same tests. How long would it take them to grade these tests if they work together?

Lial, Hornsby, McGinnis *Beginning and Intermediate Algebra*, 4th Edition, p. 452
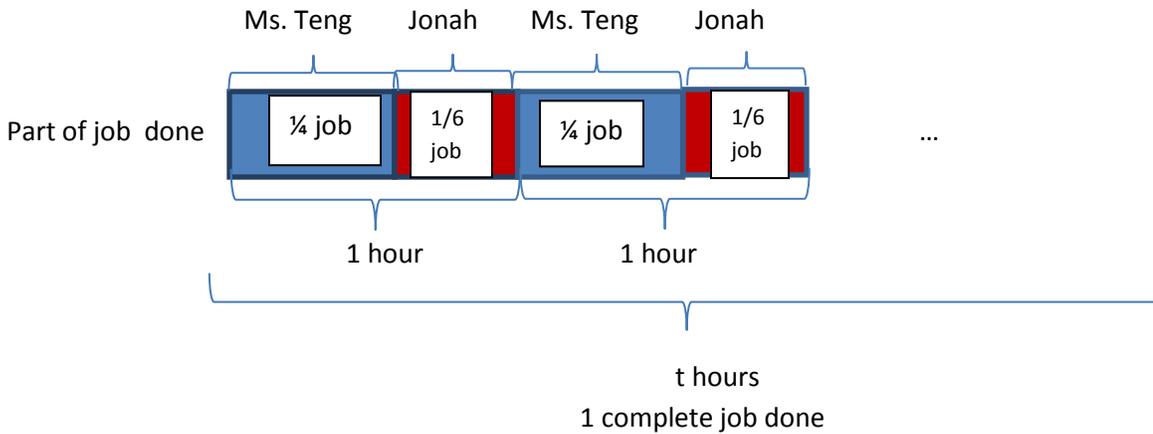
Each hour Ms. Teng and Jonah completed ¼ + 1/6 of the job. It took them t hours to get the whole job done, at which time their parts would add up to 1.

In t hours, Ms. Teng completed t(1/4) = t/4 of the job.
In t hours, Jonah completed t(1/6) = t/6 of the job.

Together, they got the job done:

$$t/4 + t/6 = 1$$

Multiply both sides by 12: $\quad 12(t/4) + 12(t/6) = 12(1)$

$$3t + 2t = 12$$
$$5t = 12$$
$$t = 12/5 \text{ hours} = 2.4 \text{ hours} = 2 \text{ hours and 24 minutes}$$

Ms. Teng and Jonah need to work together for 2 hours and 24 minutes to get all the tests graded.