



TEACHING CONCEPTUAL UNDERSTANDING THROUGH MANIPULATIVES

AMATYC

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Our goals for this workshop

You will...

- experience the power of manipulatives
- consider using manipulatives in your classes
- know where to acquire manipulatives
- be aware of online resources
 - teacher training videos
 - student worksheets
 - virtual manipulatives

First, a CAT

Developmental math students often apply memorized algorithms and procedures without understanding the concepts.

Do you agree?

If so, give an example of a topic where this occurs in your class.



Why do we use manipulatives?

- We identified a problem - students lack conceptual understanding of mathematics
 - Random application of algorithms and procedures
 - No understanding of 'why'
 - No foundation to build on
- We saw manipulatives in our kids' classrooms
 - Concrete objects used to make abstract concepts real
 - Why not use them in college?

Evolution of this project

- Have always used objects to illustrate math concepts
 - Calculus - 'toys'
 - Statistics and Probability - dice, beans, cards
- Recognized a need in developmental courses
- Researched
 - Talked with elementary and middle school teachers
 - Attended NCTM workshops

Implementation

- Got grant moneys to acquire manipulatives
- Developed curriculum to integrate into our developmental classes
 - Created student manipulative activities - student worksheet with instructor page
- Shared activities with our colleagues
- Sabbatical project 2011-12
 - Expanded the worksheet packets
 - Recorded 11 teacher-training videos
 - Identified good sources of virtual manipulatives

What is “Manipulative Mathematics”?

- Student worksheet activities
 - Students use manipulatives to develop conceptual understanding
 - Leads students to discovery of concepts
 - Results in many “Aha!” moments among our students
- Teacher support
 - Training videos
 - Instructor pages

How do we use “Manipulative Mathematics”?

- Today you’ll see how we use Manipulative Mathematics activities to teach these topics:
 - Signed Numbers
 - Fractions
 - Equations
 - Slopes

Signed Numbers

- Manipulative

- Two color counters



- Activities

- Addition of Signed Numbers
- Subtraction of Signed Numbers

Signed Numbers Packet

- Contents of Signed Numbers Packet
 - Instructor Page
 - Student Worksheet
 - Extra Practice
- **Let's do the Activities!**

Signed Numbers Teacher Video

- All Teacher Videos include:
 - Introduction to the Manipulatives
 - How do they help students learn?
 - How can you use them?
 - Suggestions for use
 - Where to find them – physical and virtual



Signed Numbers Teacher Video

Virtual Manipulatives

Addition of Signed Numbers

The interface shows the equation $6 + (-4)$ at the top. Below the equation are two buttons: a red button with a minus sign (-) and a black button with a plus sign (+). In the center, there is a large gray oval containing six black plus sign (+) chips and four red minus sign (-) chips. A trash can icon is located in the bottom right corner of the interface.

nlvm.usu.edu
Color Chips – Addition

Virtual Manipulatives

Subtraction of Signed Numbers

The interface displays the mathematical expression $5 - (-3)$ at the top. A "Continue" button is located in the top right corner. Below the expression, a gray oval contains several virtual chips. On the left side of the oval, there are five black chips, each with a white plus sign (+). On the right side, there are three pairs of chips, each pair consisting of a red chip with a white minus sign (-) and a black chip with a white plus sign (+). Above the oval, a gray box contains one such pair of chips (red minus, black plus).

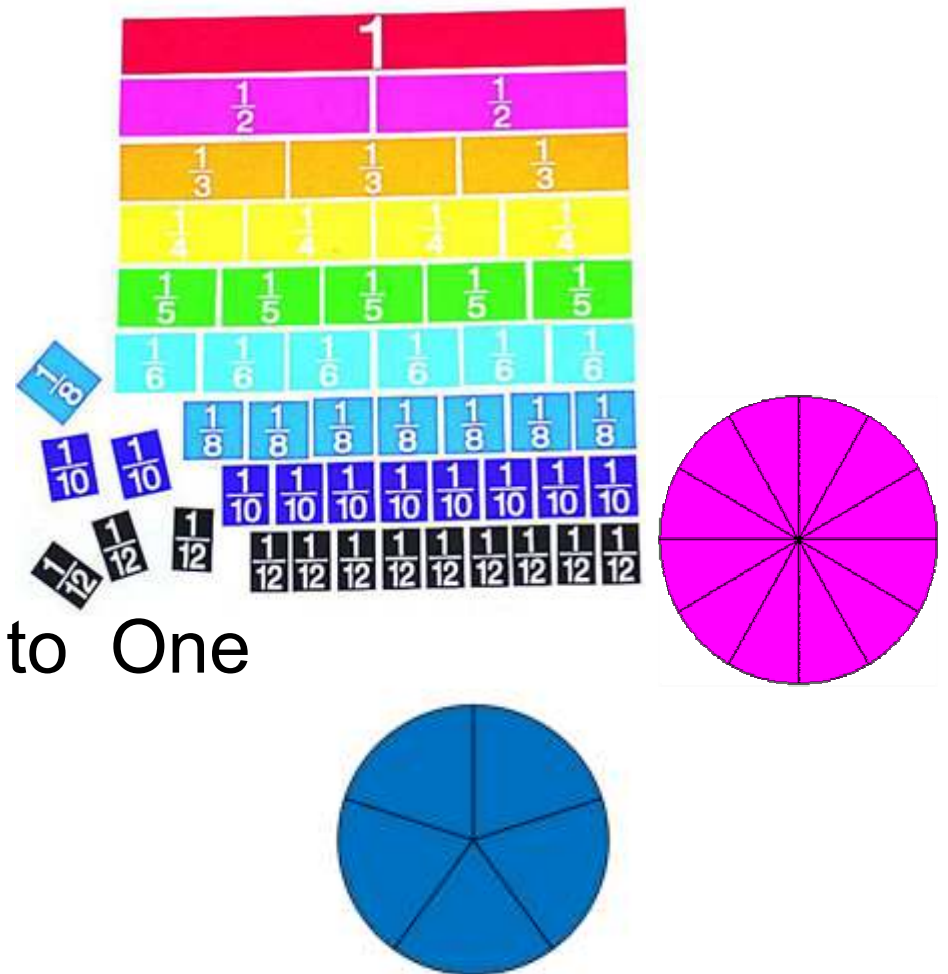
nlvm.usu.edu
Color Chips – Subtraction

Fraction Worksheets

- Conceptual understanding
 - Naming fractions
 - Equivalent fractions
 - Fractions equivalent to one
- Fraction operations
 - Multiplication, division
 - Addition, subtraction
 - Need for the LCD

Equivalent Fractions

- Manipulatives
 - Fraction Tiles
 - Fraction Circles
- Activities
 - Fractions Equivalent to One
 - Equivalent Fractions



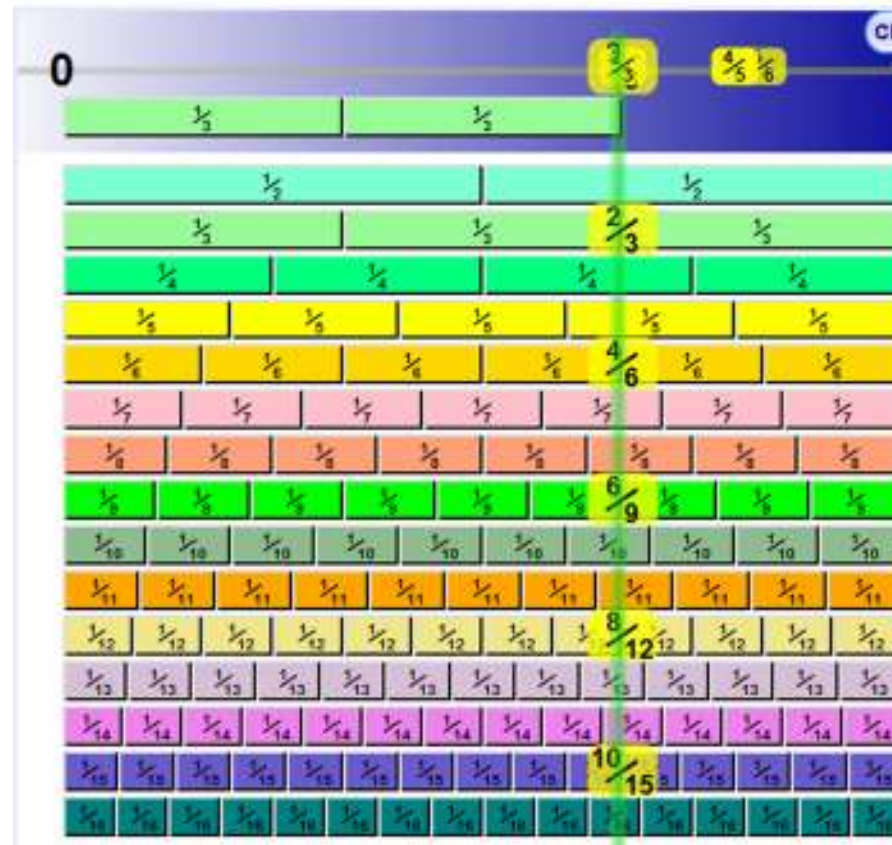
Equivalent Fractions Activities

- Gives students a visual image of fractions equivalent to one and to each other
- Students see how concrete (tiles) lead to abstract concepts
 - e.g., why you must multiply numerator and denominator by a common factor to make an equivalent fraction

Virtual Manipulative Fraction Number Line

Fraction Number Line

See [Equivalent Fractions](#) and where they fit on the [Number Line](#)

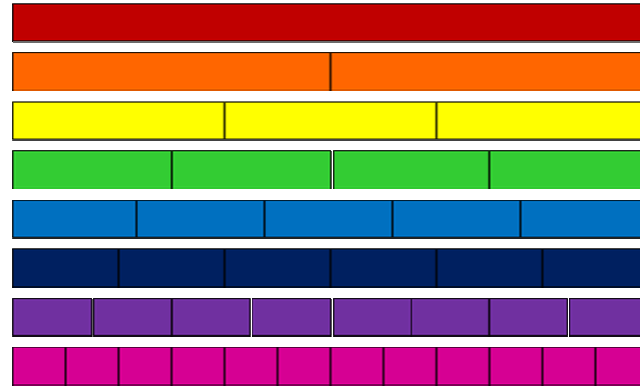


<http://www.mathsisfun.com/numbers/fraction-number-line.html>

Fraction Operations

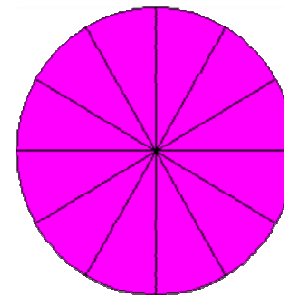
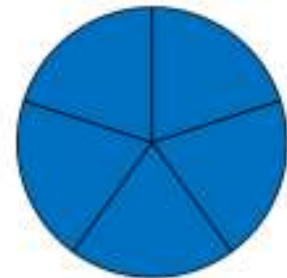
- Manipulatives

- Fraction Tiles
- Fraction Circles



- Activities

- Adding and Subtracting Fractions
- Find a Common Denominator



Fraction Operations

- Students see why like denominators do not change for addition and subtraction
- Students understand the meaning of the common denominator



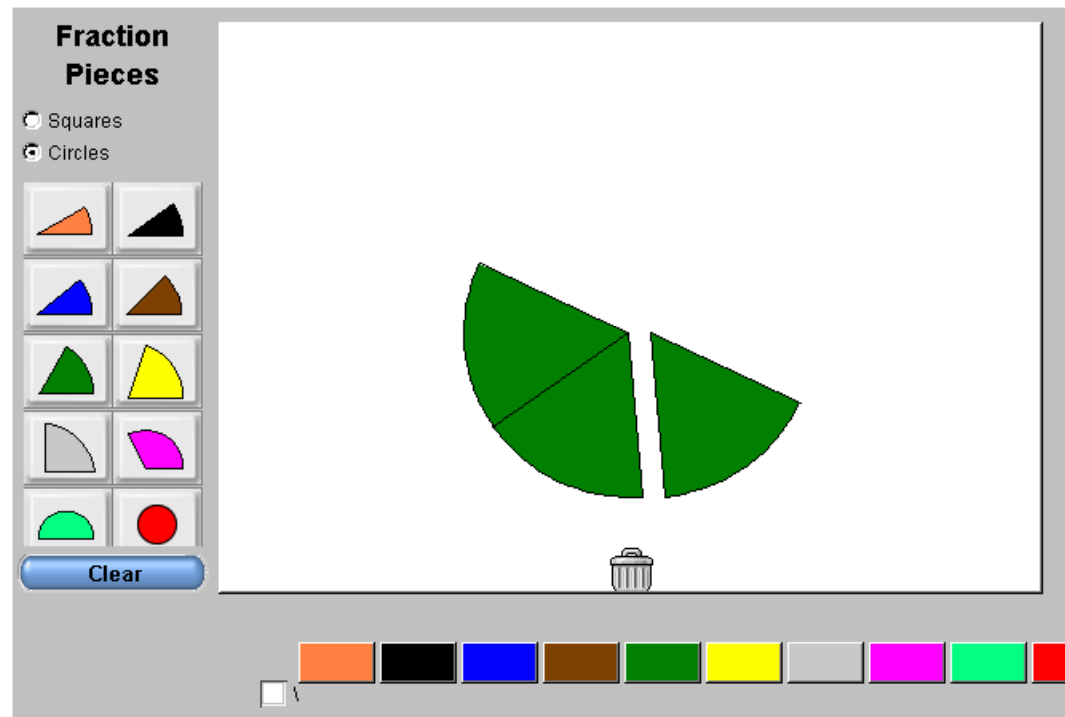
Fraction Operations Teacher Video

Fraction Operations Worksheets

- Model Fraction Addition
- Model Fraction Subtraction
- Model Finding the Least Common Denominator

Virtual Manipulative

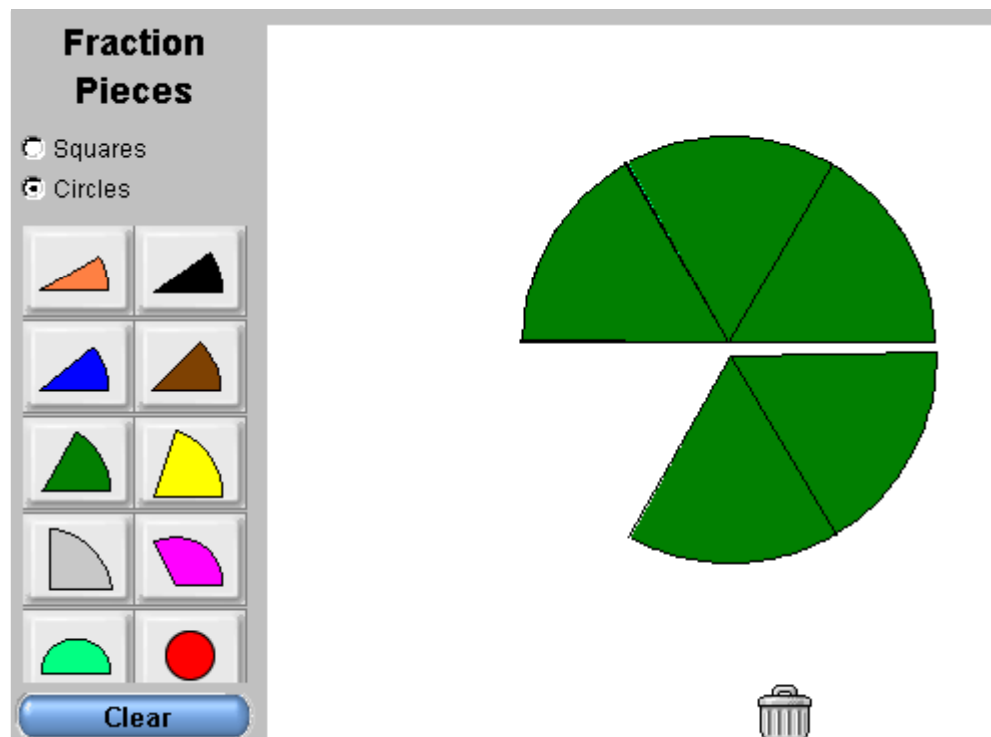
Adding and Subtracting Fractions



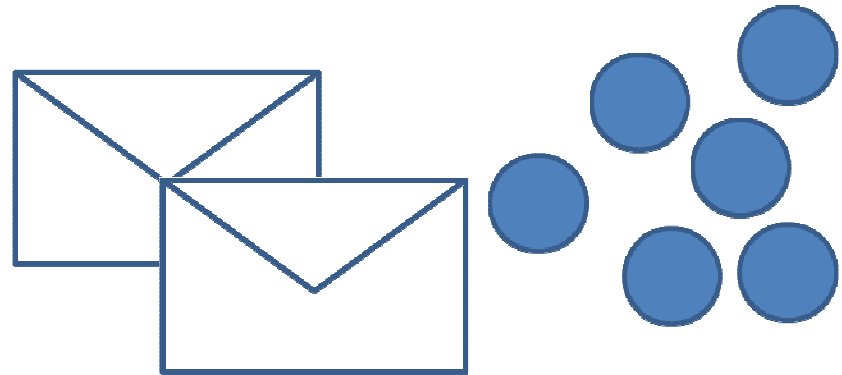
http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html

Virtual Manipulative

Finding a Common Denominator



Solving Equations



- Manipulative
 - Envelopes and Counters (or paperclips)
- Activities
 - Model the Subtraction Property of Equality
 - Model the Division Property of Equality



Solving Equations Teacher Video

Solving Equations Worksheet

- To solve an equation is to discover the value of the unknown
- Students see that they must do the same operation to both sides of the equation

Virtual Manipulatives

Equations

Explorelearning 'gizmos'

- 5 minutes free use
- Two-color counters model for 1-step equations
- Cups and counters model of 2-step equations
- www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=226

Explorelearning Gizmos

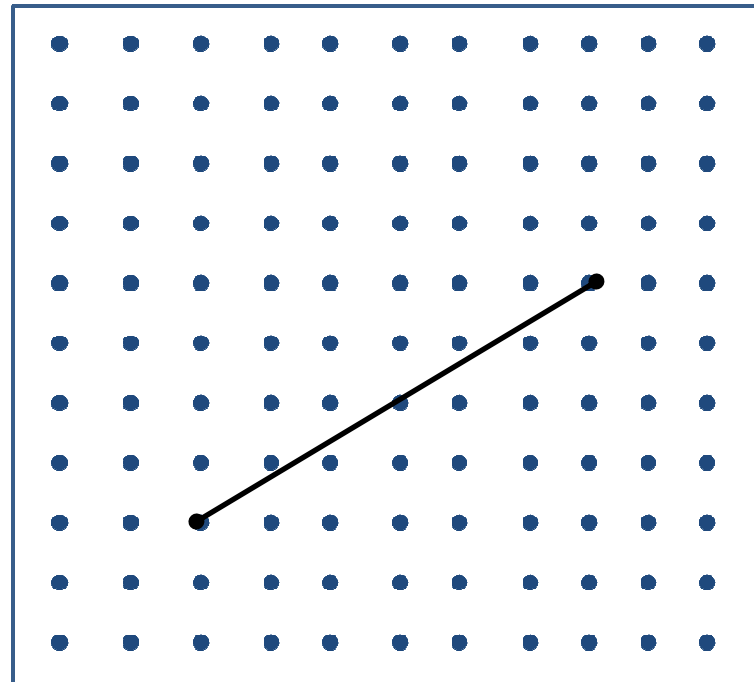
Equation: $7 = 2x + 1$ for x

The diagram shows a balance scale with two pans. The left pan contains 7 red beads and is labeled with the number 7. The right pan contains 2 blue circular weights and 1 red bead, and is labeled with the expression $2x + 1$. An equals sign is positioned between the two pans, indicating that the total weight on both sides is equal.



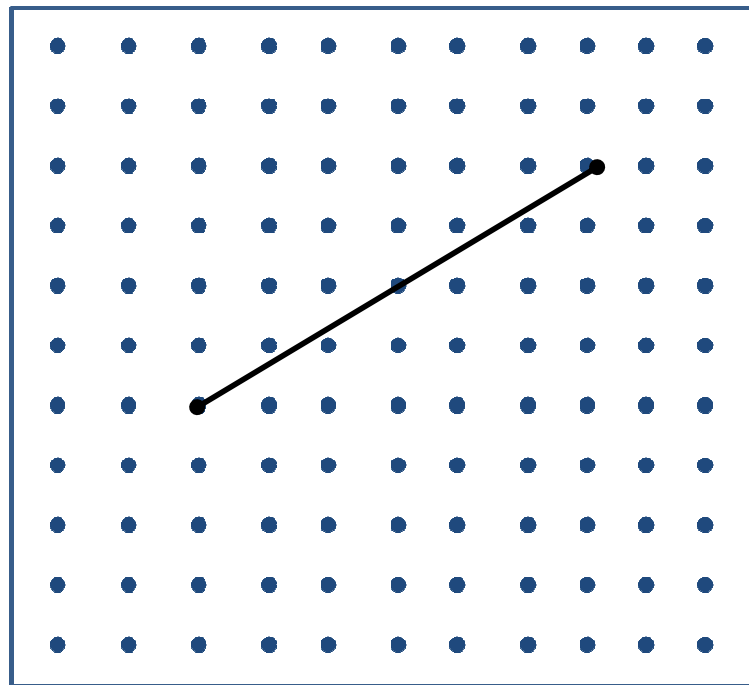
Exploring Slopes

- Manipulative
 - Geoboards, rubber bands
- Activity
 - Exploring Slopes



Exploring Slopes Worksheet

Students count out the rise and the run





Exploring Slopes Teacher Video

Manipulatives at Santa Ana College

- Basic Math, Prealgebra, Elementary Algebra
- Manipulatives in the classroom
 - Class sets of manipulatives
 - Full sets in classroom where most sections meet
 - One section meeting in computer classroom
- Activities included in *Foundations of Algebra* manuscript
- Professional development workshops

What do our students say?

- My favorite was the two color counters because I have always had a hard time with negative numbers and this made me understand.
- The fraction pieces helped a lot and I was able to understand completely
- Fractions were a weakness of mine and the manipulative activity helped me improve.

What do our students say?

- Envelopes and Paper Clips - this activity made it easy and fun to understand how to solve these equations because the activity showed how many coins you had outside and from that you would get an idea as to how many were inside the envelope.
- The envelopes and paper clips helped me understand the concept of that chapter way better
- Geoboard because I never understood slope till having it visual in my hands.

Virtual vs physical manipulatives

- Student preferences
 - Virtual 6 Physical 15 Both 2
- Why virtual manipulatives?
 - It's easier and faster.
 - You can always go back if you still don't get it.

Virtual vs physical manipulatives

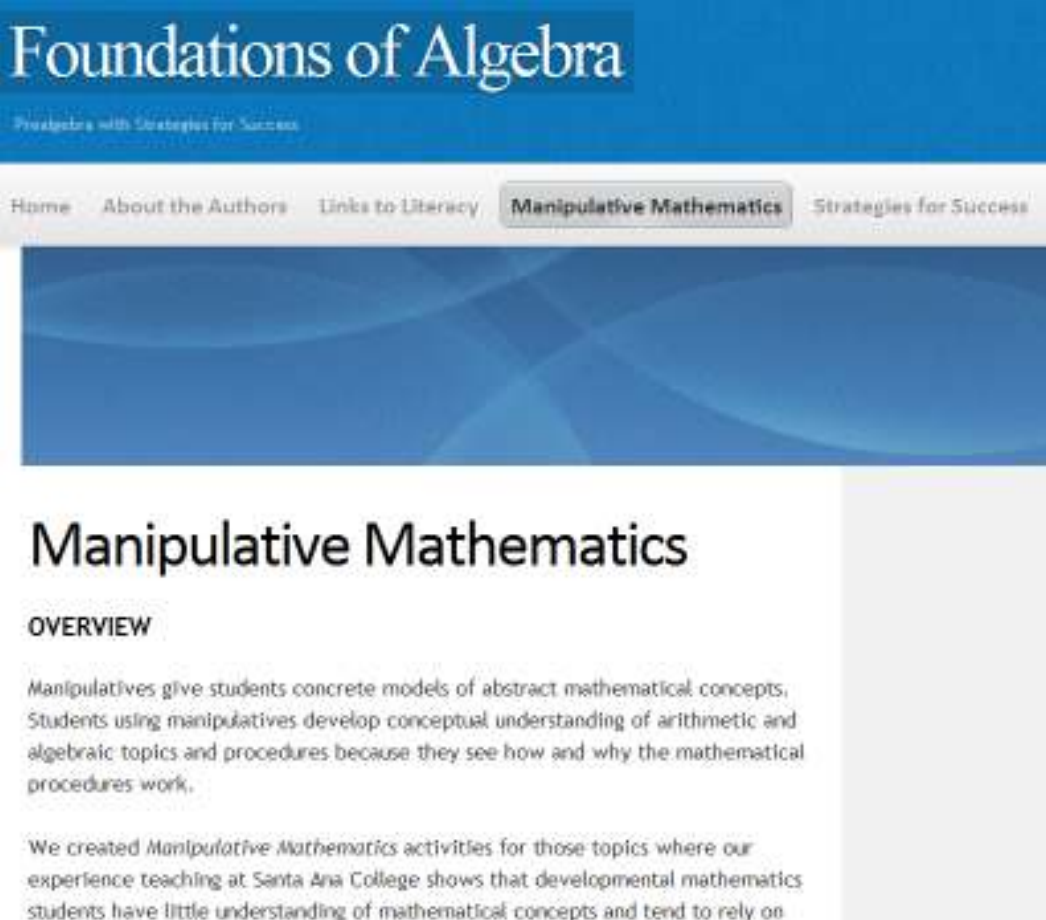
- Why physical manipulatives?
 - It's more interaction.
 - I remember it better if I have something solid in front of me.
 - It makes me actually think more about the problem.
 - Virtual manipulatives may cause distractions because it is on a computer and I have other interests to do on a computer rather than math.

Where can you find manipulatives?

- Teacher supply stores
- Catalogs and websites
 - National Council of Teachers of Mathematics
 - Cuisinaire
 - Learning Resources
 - ETA
 - Lakeshore Learning
- Math for Elementary School Teachers textbooks

Resources available online

- <http://www.foundationsofalgebra.com/manipulative-mathematics/>



The screenshot shows the website for 'Foundations of Algebra'. The header is blue with the title 'Foundations of Algebra' and the subtitle 'Prealgebra with Strategies for Success'. A navigation menu includes 'Home', 'About the Authors', 'Links to Literacy', 'Manipulative Mathematics' (which is highlighted), and 'Strategies for Success'. Below the navigation is a large blue graphic with abstract shapes. The main content area has the heading 'Manipulative Mathematics' and a sub-heading 'OVERVIEW'. The text under 'OVERVIEW' explains that manipulatives provide concrete models for abstract concepts and that the site was created based on the authors' experience at Santa Ana College.

Foundations of Algebra

Prealgebra with Strategies for Success

Home About the Authors Links to Literacy **Manipulative Mathematics** Strategies for Success

Manipulative Mathematics

OVERVIEW

Manipulatives give students concrete models of abstract mathematical concepts. Students using manipulatives develop conceptual understanding of arithmetic and algebraic topics and procedures because they see how and why the mathematical procedures work.

We created *Manipulative Mathematics* activities for those topics where our experience teaching at Santa Ana College shows that developmental mathematics students have little understanding of mathematical concepts and tend to rely on



Manipulative Mathematics website

- Teacher training videos
- Links to virtual manipulative websites
- Student activity worksheets – password protected (email us!)



Questions???



We're here to help you!

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