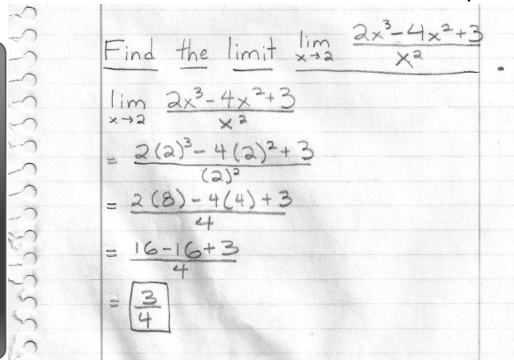
Students saw and listened to this avatar named Allison, saw her work, and were asked these questions:



Briefly describe the steps Allison took. Do you think this solution is correct?

From your understanding of what a limit is, does this method make sense? Explain.

Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Can you share another way of finding this limit?

After submitting their responses, students were shown 3 more avatars that gave various explanations. The students were then given the opportunity to provide additional comments.

Here are some typical student responses:

"Allison substituted the 2 in for all of the X's in the problem. She then used algebra to come to her solution. I think that Allison's solution is correct. It does make since because you want your x value to approach the value it is intending. In this case the x value was two. This is the only way I know of to find this limit. "
"I forgot that you can actually graph this to find your answer, so there is more than one way to do this. I also forgot to add that this way will not work every time. In cases where the denominator is 0, you have to use factoring as your first step!"

"Allison plugged in the limit and took the proper algebraic steps to reach the correct the solution. Yes, I believe her solution is correct. From my understanding of limits, yes it makes sense you can do this algebraically since there are no holes, jumps, bumps, or asymptotes present. As long as there is no "funny business", this method should work. If the limit is not continuous, I think this method would result in failure to find the correct solution. If you were given the needed information, you could find the limit via graphical means as well. You could also use the power rule to arrive at the correct answer without having to do as much algebraic work. "
"Not in particular, I think I covered everything I wanted to discuss."

"Allison simply took the first step that we were taught to try and that is the direct substitution. In the direct substitution step, one will take the value that the variable approaches and substitute that value for the variable like in Allison's case the variable (x) was approaching 2 so she substituted x for 2 wherever x was present in the equation. With my understanding of limits, this methods makes complete since because there was no way for the denominator in the equation to be equaled to zero unless the limit was to approach zero, but it was approaching 2. This method does not work every time. It is the method that should be tried first because it is the simplest method, and it does not require any factoring. A special circumstance that could prevent a problem like this is if someone direct substitutes a number for the variable in the denominator of the equation and the denominator will equal zero. The only other way I could think of to find the limit would be to factor x^2 out of the numerator so to cancel out the x^2 in the denominator."
"I also worked the problem out and got 3/4 as well. I think that checking your answer with a graph would be a good idea as well."

"Allison plugged the 2 in the place of the x and then she solved through the algebra. She finally came to find the solution of the limit which was 3/4. I do believe this solution is true. From my understanding this method does make sense to find a limit. A limit tries to identify the point at which there is a hole at a certain x-value. I believe this method will work every time. One certain circumstance where you would have to modify it is when the

solution can be factored or ""unFOILed"". Another way this limit could have been found is by breaking up the solution into three different fractions. You can do this because all three terms in the numerator have the same denominator. From this point you would plug the two in place of the x and do the algebra to find the limit."
"After hearing what the other videos from the students said, I did notice I left out some things. First I should clear up that this problem was not a discontinuous problem. This problem worked because Allison could substitute or plug in the 2 for the x. Since there was not a zero in the denominator this problem was continuous and able to be solved. If the denominator was 4-x^2 then the solution would have been continuous everywhere except for x=4. At x=4 this problem would be discontinuous because that would cause the denominator to zero. At this point there would be a hole."

"Allison used direct substitution and then simplified. Her method does make sense to me, but there are certain circumstances that would prevent that from working. Like if the denominator equals zero, in this case it would be at 0. The other way I would use to find this limit would be to factor out an x^2 on the top and then cancel out and simplify. After simplified, substitute in the number that x approaches."
"I formerly said that you could factor x^2 out of the top and then cancel it out. After this I said to simplify and plug the x value. I am not sure that this method would have worked. I guess a graphing calculator would be a good explanation!"

"Allison takes the limit given and immediately inputs the x value to try and solve the problem, allowing her to come to the conclusion of 3/4 as the limit. I do think Allison's solution is correct. Her method makes sense because we want to find the limit at the point of x=2. This method of finding a limit would not work if the point was 0, she would not be able to solve the problem in such a way, but that is the only time this would not work. Another way of finding the limit is by using graphing."

"Allison plugged in a 2 to the equation and then just worked it out. The solution is correct. The method Allison used makes sense under the circumstances. However, her method will not work every time. If there is a hole or jump in the graph then you cannot just plug it in the way she did. A hole or jump in the graph indicates the fraction is being divided by zero. Another way to finding the limit is to graph the equation and find the y value the limit is closest to as it approaches 4 on x axis."
"I have nothing to add to my explanation."

"First, Allison plugged in 2 for all x-values. Then, using the order of operations, simplified the expression. I think the solution, ¾ is correct. From my understanding of what a limit is, the method Allison used made sense to me because like the definition of a limit states, f(x) gets close to some limit as x gets close to some value. Plugging in a value for all x-values to evaluate a limit will not work every time. For example, if after plugging in a value for all x-values in the expression, one arrives at a solution with 0 as the denominator. Because one cannot divide by 0, this solution suggests the limit does not exist; however, if one were to first simplify the rational expression, he or she then could plug in a value for all x-values and arrive at the correct solution. If one were to graph the function, he or she could find the limit graphically. "

"Allison first started by plugging in the value of 2 into each x because the limit is as x is approaching 2. This is how she found the limit. I believe her solution is correct. Allison's method makes sense because she wanted to know what the value was as it got closer to the number 2. Therefore plugging in 2 would give you what the ultimate value or the "limit" would be. I think Allison's method will work every time if there is a limit and an equation given. This is a great way to find the limit and Allison did a great job!"

"Allison plugged in 2 for every x she saw. I do not think her work is correct because i think the top needed to be factored a little before plugging in for x. I don't think this method will work every time only some times."
"I would like to change my answer. After listening to what other students said I now understand that direct substitution would work because the denominator does not equal zero. However if the denominator did equal zero then the method Allison used would not work."

"Allison found the limit simply by direct substitution. She plugged 2 into the equation and readily solved the problem. After plugging the numbers in, she ended with three fourths. I believe that this answer is correct and her work was done in an orderly manner. It is clear and her method makes sense. Allison's method may not work every time, because not all limits can be evaluated by direct substitution. For example, if "x" was 0, the equation would not be able to be solved by substitution because the denominator would be 0. I do not know another way of finding this limit, but I believe that direct substitution is the easiest and most time efficient way."
"My answer was correct, but i did not mention that Allison's answer was continuous. She had no holes, jumps or asymptotes. When x=2, then y=3/4, so her method of direct substitution worked perfectly."