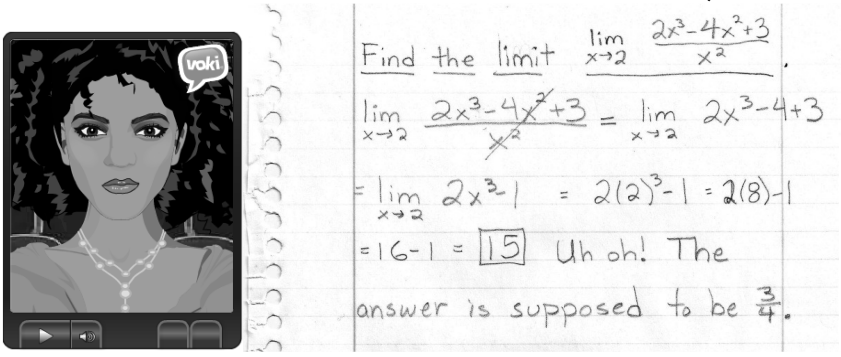
Students saw and listened to this avatar named Sharon, saw her work, and were asked these questions:



Briefly explain Sharon's steps. Why do you think this solution is incorrect?

What would you say to Sharon to help with the problem?

From your understanding of what a limit is, does Sharon's answer seem reasonable or unreasonable? Explain.

What specific steps or strategies could Sharon use to avoid this type of error?

After submitting their responses, students were shown 3 more avatars that gave various explanations. The students were then given the opportunity to provide additional comments.

Here are some typical student responses:

"Sharon begins her attempt at finding the limit by trying to cancel out the denominator. She incorrectly cancels x^2 from the denominator with 4x^2 in the numerator, while 4x^2 is still part of the equation in the numerator. Instead of automatically attempting to cancel out x in the denominator, Sharon should plug 2 in for x to see if the limit can be found with substitution method before attempting to factor and cancel out. So by substituting 2 for x we get: [2(2)^3 - 4(2)^2 + 3] / (2)^2 = [2(8) - 4(4) + 3]/ 4 = (16-16+3)/4 =3/4 Sharon's answer does not seem reasonable because her answer 15 is entirely too high. By using the substitution method before trying to factor and cancel out, Sharon can avoid similar mistakes in the future."

"Incorrect canceling is a common mistake when solving even simple algebraic equations. Although Sharon's mistake seems not unreasonable, failure to recognize these mistakes may lead to major frustration when solving more advanced problems. Being careful and deliberate in your calculations is important!"

"Sharon crossed out the x^2 on the top/bottom leaving 2x^3-4+3. She then simplified this by substituting 2 for x. Then she solved getting 2(2)^3-1=2(8)-1. She then solved and got 15. I would tell sharon to look at the problem and look underneath lim. This will give you the number the limit is approaching and which you should substitute into the problem. The new problem should look like this; [2(2)^3-4(2)^2+3]/(2)^2 When simplified the answer will turn out to be 3/4, which is the desired answer for this problem. Sharon seems to have forgotten a rule in math that is made all the time which is crossing out like terms when they are a part of an equation. When the variable (x) is being multiplied by something not added/subtracted, it can be crossed out with the bottom. That is not the case with this problem. You would substitute with what the limit is approaching (2) and solve. She should look at the problem and decide whether or not she should substitute or factor. She should substitute when there is no chance of the bottom=zero. If there is a chance of the bottom=zero than Sharon should factor. "

"To avoid the problems Sharon had she just have used direct-substitution to begin with. Sharon should also look at the problem and apply the basic rules of math to it. "

"X squared cannot be factored out of the problem, because factoring only works with multiplication and division problems (not a subtraction problem as shown). A better way to do this problem would be substitute x with 2 and do the problem as is. The answer seems sort of reasonable if the rules are bent, but this is MATH !!!"

"Review Direct subsitution rules."

"While this problem may seem to be correct the illegal cancel of the x squared, which eventually throws off the whole problem causing it to come out as 15 instead of 3/4. Watch for the illegal cancel as you can you see if you would've just been a little more careful the error could've been avoided the illegal cancel throws off the whole problem. To me, her answer seems reasonable as to what a limit is and that may be bias based on the fact that I'm impartial to working with fractions :) Watch for the cancel, try to plug in the number before you complete the entire problem just so you have an idea on what to expect before all the work is done."
"when doing limits remember to substitute the 2 in for x instead of doing. illegal cancels"

"When cancelling, Sharon made an illegal math move. You cannot cancel when there's addition or subtraction. Sharon did this causing her entire problem to be wrong. In order to help Sharon, I would tell her to remember to only cancel when there is no addition or subtraction. When looking at the graph of the equation, it does not make any sense to say that the limit as X approaches 2 is 15. The actual answer is 3/4, which does make sense when you look at the graph. Sharon needs to look at the graph before and after she works out the limit algebraically to make sure her answer makes sense. She also needs to make sure she remembers not to cancel in addition or subtraction operations. "

"Sharon first crossed out the denominator with the x2 from the numerator. She then simplified the solution and once the solution was as simple as it could be, she plugged in the limit 2. The 2 was substituted and the problem was able to be completely solved. This solution was incorrect because Sharon crossed out the denominator when it was not able to be crossed out yet. I would tell sharon to remember her Algebra rules. You can't cross out variables when they are in a polynomial. Her answer is reasonable for a student. When I first did it, I couldn't figure out what she did wrong. It is because I forgot my factoring rules. Once I was refreshed, I remembered. She could review and study her old notes from previous classes, since Math builds on top of each other. "
"Sharon, I was correct with what I stated earlier, but I failed to mention that you should always try to do the problem with trying direct substitution first. In this problem you are able to use direct substitution and so it would be much easier. "

"Sharon's solution is incorrect because she attempted to cancel the 4x^2 with the x^2 in the denominator which is not possible because the 4x^2 is a part of the trinomial in the numerator, and it is incorrect to cancel out add-ins. I would encourage Sharon to first try direct substitution to solve the limit before factoring anything, which may help her to avoid canceling errors. Although Sharon's algebraic processes were incorrect, her final answer and the actual way in which she solved the limit were correct. So, for the numbers that she was using she did get a reasonable answer. I would specifically tell Sharon to try direct substitution first and to make sure to always factor and then cancel. "
"As one of the students mentioned, if you were to graph the function you would find that 15 is an unreasonable answer. Sharon may have wanted to draw a quick sketch to check her answer and then she would have noticed that she had made an error. Most importantly she needs to review the rules for simplifying rational expressions. "

"Sharon tried to cancel out the x^2, when she could have just plugged in a 2 where the x's are in the equation. I would tell Sharon to plug in a 2 in the place of the x's, then solve the rest of the algebraically. Obviously the answer is unreasonable because it is not correct. However, fifteen can be the limit for a different equation. To avoid this type of error, Sharon should always try direct substitution first."
"Just as I suspected, Sharon could have just plugged and chugged the 2, since the limit was approaching two. This would have given her the correct answer the easiest way. Her way was incredibly wrong because her algebra was incorrect; she needed to divide every piece on top by the piece on the bottom."

"Sharon cancelled out the two x^2 on top and bottom of the fraction, when you can't do that because there is a 4 in front of it. I would tell Sharon to see if she can factor out anything before trying to cancel the x^2. It seems unreasonable because you're dealing with x approaching 2, which is much smaller than 15. She should try checking to make sure that all her equations are simplified before cancelling things."
"Sharon, I believe I told you wrong before. I was mistaken and thought the top part of the fraction read 2x^3 - 4x^2 + 3x. After taking another look at it I see that there is not an x behind the 3, so factoring out an x would not work. I think you should use direct substitution by plugging in 2 to the equation since it is already simplified. Like I said before, you cannot cancel the x squared that way so something you might want to go back and study to practice that skill would be to look over dividing polynomial expressions."