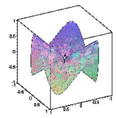


MTH 210: Circle Project : Part 1 on section D
Names:

<u>Area of a Circle</u>

What is the area of a circle? About 250 B.C., Archimedes derived the area of a circle via geometric methods. We will explore a similar method.

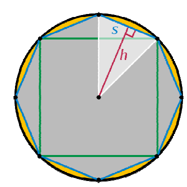
Today we will find the area A_1 of an inscribed n-gon to the circle and the area A_2 of a circumscribed n-gon about the circle.

Let n be a positive integer and cut the circle into n equal sectors.
1. Give the inside angle for each sector in terms of n.

A: <u>Inscribed an n-gon:</u>

For each sector, inscribe a triangle formed by the center of the circle and the points where the straight edges of the sector cut the circumference.

2. Find the area of each inscribed triangle in terms of sine, n and r.

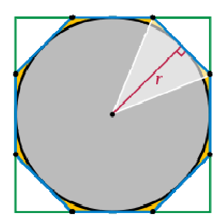


3. The area $A_1(n)$ of the inscribed n-gon equals the sum of the areas of the n inscribed triangles. Show that

$$A_1(n) = \pi r^2 \frac{\sin(\frac{2\pi}{n})}{\frac{2\pi}{n}}.$$

B: <u>Circumscribe an n-gon:</u>

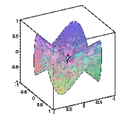
For each sector, form a triangle on the outside of the sector so that the base is tangent to the circle, as follows:



4. Find the area of each triangle in terms of tangent, n and r.

5. The area $A_2(n)$ of the circumscribed n-gon equals the sum of the areas of the n triangles. Show that

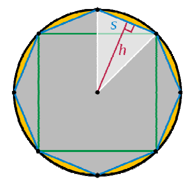
$$A_2(n) = \pi r^2 \frac{\tan(\frac{\pi}{n})}{\frac{\pi}{n}}.$$



MTH 210: Circle Project : Part 2 on section 1.6
Names:

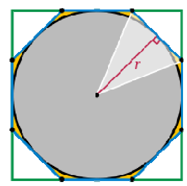
Area of a Circle

From part 1 of this project, we showed that the area of an inscribed n-gon is:



$$A_1(n) = \pi r^2 \frac{\sin(\frac{2\pi}{n})}{\frac{2\pi}{n}}$$

We also showed that the area of a circumscribed n-gon is:



$$A_2(n) = \pi r^2 \frac{\tan(\frac{\pi}{n})}{\frac{\pi}{n}}$$

C: Now let the number of triangles increase without bound, i.e. let $n \to \infty$.

1. What happens to the central angle (angle at the center of the circle) of each triangle? Let $\theta = \frac{\pi}{n}$. So what happens to θ if $n \to \infty$?

2. Write the area $A_1(n)$ and the area $A_2(n)$ as functions of θ.

3. In section 1.9 we will learn that if *f* is a continuous function, then $\lim_{x \to a} f(x) = f(a)$. In section 2.6 we will show that $\lim_{x \to 0} \frac{\sin x}{x} = 1$. Now use this information and the Squeeze Theorem to show that the area of a circle, *A*, is equal to πr^2.