*S067: Ready for a New Life in Developmental Math?*

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# Section 1.5 Percentages and Percent Change

**Goals:**

- To understand the meaning and use of percent in everyday situations
- To investigate percent increase and decrease
- To explore how to compare quantities using percentages
- To examine the difference between absolute change and percent change
- Introduce how the Consumer Price Index is used to measure inflation

**Math Talk**

What is a percent? You have probably seen the symbol % and encountered the term in daily life. For example,

- A students grade on the test was 87%
- 90% of the class had a passing grade on the test
- The number of students taking college math has increased by 10%.
- The bookstore is selling calculators at 15% off the list price.
- A student received a scholarship covering 50% of her tuition.

A percent is just a fraction with a denominator of 100.

$$n\ \% = \frac{n}{100}, \quad \text{where } n \text{ is a number.}$$

Percent can be interpreted as *per 100* or *out of 100* or *divided by 100* and is often denoted with the symbol %. So, 75% means 75 out of 100 and is equal to $75/100$, which can be reduced to the fraction $3/4$ or expressed as the decimal 0.75. These values are just different forms of the same number and which form you choose depends on the situation. For example, suppose a student attends 75% of his math classes. Each phrase given below has the same meaning but the first and last phrase are more likely to be used to explain the student's attendance record.

- a student attends 75% of his math classes.
- a student attends $75/100$ of his math classes
- a student attends 0.75 of his math classes
- a student attends $3/4$ of his math classes

Even though $75\% = \frac{75}{100}$, this does not necessarily mean that the student attended 75 out of 100 classes. There might have been 40 classes during the semester and the student made it to only 30 of them. The 75% indicates that if there were 100 classes, the student would have attended 75 of them. In a similar fashion, $3/4$ represents that the student attended about 3 out of every 4 classes.

Because of his poor attendance, the above student had a 60% average going into the final exam. What does this mean, aside from the fact that he needs to ace the final? Again, percent means per 100 or divided by 100. So, 60% can be expressed in any of the following forms.

$$60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5} = 0.6 = 0.60$$

Most students simply recognize 60% as a poor average, but one could interpret it to mean receiving 60 out of a possible 100 points. In reality, the student might have received 240 points out of a possible 400 points, i.e.,

$$\frac{240}{400} = 0.6$$

Although 0.6 is the same as 60%, the latter form is what we are accustomed to receiving as a grade. Therefore, a percent is that part of a whole, which has been divided into 100 equal parts.

The words *percent* and *percentage* are often used to mean the same thing. But some people like to make the following distinction. When you take a percent of something, that thing can be called the **base** and the resulting solution is the **percentage**. For example, in stating "75% of 40 equal 30", the percent is 75, the base is 40, and the percentage is 30. In this text we will not make a distinction between the words percent and percentage.

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The first Discovery Adventure will explore the business concept of marking up and marking down an item. As you work through the problem remember when calculating a percent *of* a number, the word "of" implies multiplication.

**Discovery Adventure 1: A Changing Base**

The weather is hot and there is heavy demand for air conditioners. A local retailer decides to *markup* a $200 air conditioner by 25%. Once the heat wave ends the retailer will *markdown* the price by 25%. After the markdown, will the air conditioner return to its original price? Go to part *a* to start your investigation.

***a.*** A **markup** is a percent of the base price that is added to the base price. Find the markup on the original base price of the air conditioner.

***b.*** What is the price of the air conditioner after the markup?

***c.*** A **markdown** is a percent of the base price that is subtracted from the base price. Find the markdown on the price from part *b* that occurs after the heat wave is over.

***d.*** What is the price of the air conditioner after the markdown?

***e.*** How is it that a 25% markup followed by a 25% markdown does not give us the original price of $200?

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***f.*** Suppose you buy the air conditioner after the heat wave in a state with a 5% sales tax. What is the final price you will pay for the air conditioner?

**Feedback**

In Discovery Adventure 1 the changing base meant that you were taking 25% of a different number in calculating the markup and markdown. The markup was on a base price of $200, which adjusted the price as follows.

$$\begin{aligned} \text{Markup} &= 25\% \text{ of } \$200 \\ &= 0.25 \cdot 200 \\ &= \$50 \end{aligned} \qquad \begin{aligned} \text{Ajusted price} &= \text{Original price} + \text{Markup} \\ &= \$200 + \$50 \\ &= \$250 \end{aligned}$$

So the markup, $50, is the dollar amount you add to the original price. The markdown is on the adjusted base price of $250. Applying the markdown means to readjust the price as follows.

$$\begin{aligned} \text{Markdown} &= 25\% \text{ of } \$250 \\ &= 0.25 \cdot 250 \\ &= \$62.50 \end{aligned} \qquad \begin{aligned} \text{Readjusted price} &= \text{Adjusted price} - \text{Markdown} \\ &= \$250 - \$62.50 \\ &= \$187.50 \end{aligned}$$

Therefore, the markdown, \$62.50, is the dollar amount you subtract from the adjusted price. Since the markup of \$50 is less than the markdown of \$62.50, the readjusted price of the air conditioner is less than the original price.

Hence, a percent increase on a quantity followed by the same percent decrease on the adjusted quantity will not return the original quantity.

If you are not concerned with knowing the markup value, but want to directly calculate the adjusted price you can perform the single calculation given below.

$$\begin{aligned} \text{Adjusted price} &= 125\% \text{ of Original price} \\ &= 1.25 \cdot \ \$200 \\ &= \$250 \end{aligned}$$

Do you understand why this one calculation gives the same answer as the previous two calculations? See problem 1 from the section exercises.

Also, the markdown is essentially taking 25% off the adjusted price. But taking 25% off is the same as calculating 75% of the adjusted price, i.e.,

$$\underbrace{\$250 - 0.25(\$250)}_{25\% \text{ off}} = \$187.50 \quad \text{and} \quad \underbrace{.75 \ \$250}_{75\% \text{ of}} = \$187.50$$

Markups and markdowns are examples of increasing or decreasing a base by a percent of the base. In Discovery Adventure 2, you will explore the idea of rate as **percent change**.

**Discovery Adventure 2: Comparing Quantities**
According the to U.S. Census Bureau, Las Vegas, Nevada and Phoenix, Arizona were two of the fastest-growing cities in the United States during the last decade of the twentieth century. Phoenix's population grew from 983,403 in 1990 to 1,321,045 in 2000, while Las Vegas went from 258,295 in 1990 to 478,434 in 2000.

***a.*** What operations can you perform on these population figures to compare how the size of each city has changed? Explain the importance of each operation.

***b.*** What is the difference in the size of Las Vegas's population between 1990 and 2000? This difference is the **absolute increase** in the population.

***c.*** What is the absolute increase in the population of Phoenix over this decade?

***d.*** What do the answers to parts *b* and *c* indicate about population change in these two cities? Explain.

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***e.*** For the city of Las Vegas, find the ratio of the 2000 population to the 1990 population. This is a comparison using division. Round to the hundredths place.

***f.*** For the city of Phoenix, find the ratio of the 2000 population to the 1990 population. Round to the hundredths place.

***g.*** What do the answers to parts *e* and *f* indicate about population change in these two cities? Explain.

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***h.*** To find the **relative increase** in the population of Las Vegas, divide the absolute increase from part *b* by the previous size of the population in 1990. Express this relative increase in percent form. Note that when this relative increase is expressed as a percent it is often called the *percent change* in the population.

***i.*** Now find the percent change in Phoenix's population over this decade.

***j.*** What do the answers to parts *h* and *i* indicate about population change in these two cities? Explain.

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***k.*** In the U.S. Census Bureau rankings of the ten fastest-growing cities between 1990 and 2000, Las Vegas is ranked first and Phoenix is ranked fifth. How is this possible when Phoenix's population added 117,503 more people than Las Vegas?

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**Feedback**

Although Phoenix did add more people to their population than Las Vegas, the absolute increase does not tell the whole story. When using subtraction you should have found that Phoenix increased their population by 337,642, while Las Vegas increased by 220,139. However, when using division to compare each city's population change, a very different picture appears. The ratio of the 2000 population to the 1990 population of each city follows.

$$\text{Las Vegas ratio: } \frac{478,434}{258,295} \approx 1.85 \qquad \text{Phoenix ratio: } \frac{1,321,045}{983,403} \approx 1.34$$

This means that Las Vegas is 1.85 times larger in 2000 than it was in 1990, while Phoenix is 1.34 times as large.

If we divide the absolute increase by the previous population (in 1990), then a relative increase is obtained. This *percent change* is calculated as follows.

$$\text{LasVegas \% Change} = \frac{220,139}{258,295} \approx 0.85 = 85\% \qquad \text{Phoenix \% Change} = \frac{337,642}{983,403} \approx 0.34 = 34\%$$

So Las Vegas's population increased by 85%, while Phoenix's population increased by 34%. How much faster is the population of Las Vegas changing compared with Phoenix?

> In general, **absolute change** and **percent (or relative) change** can be calculated using the following formulas.
>
> $$\text{Absolute change} = \text{New value} - \text{Old value}$$
>
> $$\text{Percent (or relative) change} = \frac{\text{Absolute change}}{\text{Old value}}$$

The ratio comparing a new to an old value is different from the percent change but contains similar information. For example, let's look at the ratio of new population to old population and the percent change in population for Las Vegas.

$$\frac{\text{New population}}{\text{Old population}} = \frac{478,434}{258,295} \approx 1.85 \qquad \frac{\text{New population} - \text{Old population}}{\text{Old population}} = \frac{478,434 - 258,295}{258,295} \approx 0.85$$

If we convert these two values to percent form, then the ratio of New to Old is 185%, while the percent change is 85%. As we stated previously the 85% represents the increase in population. To understand the meaning of 185%, you must first understand that taking 100% of something, like a population, is the same as multiplying by 1. In effect, taking 100% of a population simply gives you the whole population back. This makes sense if we remember that percent means divided by 100, so $100\% = 100/100 = 1$. Therefore, taking 185% of a population means getting a new population that is100% of the previous population plus an 85% increase.

In the next Discovery Adventure we take a look at why percents don't always add up.

**Discovery Adventure 3: Bargaining a New Contract**

The union is bargaining with management on a new three-year contract. The union wants increases of 3% the first year, 3% the second year, and 3% the third year. Management has proposed 9% the first year, 0% the second year, and 0% the third year. As a member of the union, with a yearly salary of $30,000, you are wondering which proposal is best.

***a.*** Show how to calculate your yearly salary for each year of the new contract using management's proposal.

***b.*** Based on management's proposal, find the total amount of money you received in salary over all three years of the contract.

***c.*** Show how to calculate your yearly salary for each year of the new contract using the union's proposal.

***d.*** Based on the union's proposal, find the total amount of money you received in salary over all three years of the contract.

***e.*** Which proposal will give you the highest yearly salary at the end of the 3-year contract? Explain why the two proposals produce different ending salaries.

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***f.*** Which proposal totals to the most money earned over all 3 years of the contract? Explain why the two proposals produce different totals.

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***g.*** Which proposal seems best for a union worker who would makes $30,000 per year. Justify your answer.

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**Feedback**

Three consecutive 3% increases is not the same a one 9% increase. Using management's proposal, during the first year you receive a 9% increase on your $30,000 salary. This increase, 9% of $30,000, is calculated as follows.

$$\text{Increase} = 0.09 \cdot 30,000 = \$2700$$

So during the first year of the contract your new salary is obtained by adding the increase to the old salary. That is

$$\text{New salary} = 30,000 + 2700 = \$32,700$$

Since the second and third years of management's proposal have a 0% increase, you would be making $32,700 per year during each year of the contract. Therefore, the total salary received over 3 years is calculated as follows.

$$\text{Total} = 3 \cdot \$32,700 = \$98,100$$

The union's proposal begins with a 3% increase the first year so

$$\begin{aligned}\text{First year salary} &= \underbrace{0.03 \cdot 30,000}_{900} + 30,000 \\ &= \$30,900\end{aligned}$$

Now during the second year you receive a 3% increase on the first year salary. Therefore, you are receiving an increase on the original salary of $30,000 and the $900 raise you obtained during the first year of the contract. That is,

$$\begin{aligned}\text{Second year salary} &= \underbrace{0.03 \cdot 30,900}_{927} + 30,900 \\ &= \$31,827\end{aligned}$$

In the third year you continue to receive a 3% increase on the previous years salary ($31,827). This is a 3% increase on the original $30,000, the $900 first year raise, and the $927 second year raise.  Thus,

$$\begin{aligned}\text{Third year salary} &= \underbrace{0.03 \cdot 31,827}_{954.81} + 31,827 \\ &= \$32,781.81\end{aligned}$$

The total salary for all 3 years of the proposed union contract is calculated below.

$$\text{Total} = 30,900 + 31,827 + 32,781.81 = \$95,508.81$$

In summary, by the end of the contract, the 3%–3%–3% union proposal gives you a higher yearly salary than the 9%-0%–0% management proposal.  However, when examining the earnings over the entire 3-year contract, we see that the union proposal produced a total of $95,508.81 while the management proposal totaled $98,100.  So which proposal would you vote for?

In the fourth and final Discovery Adventure you will explore how the cost of living increases each year due to a continued rise in prices of most goods and services.

**Discovery Adventure 4: Keeping Up With Inflation**

When prices of goods and services consistently rise over time, the value of money to purchase those goods and services will fall.  This long-term trend is known as **inflation**.  The standard measure of inflation is a *cost-of-living index* called *The Consumer Price Index* (CPI).  Note: There are actually two CPI's: CPI-U for all urban consumers and CPI-W for urban wage earners and clerical workers. The table 1.1 gives the annual average CPI-U for the years 1981–2010.

**Table 1.1**   *CPI-U 1981-2010 (Source: US Bureau of Labor Statistics)*

| Year | CPI |
|---|---|
| 1981 | 90.9 |
| 1982 | 96.5 |
| 1983 | 99.6 |
| 1984 | 103.9 |
| 1985 | 107.6 |
| 1986 | 109.6 |
| 1987 | 113.6 |
| 1988 | 118.3 |
| 1989 | 124.0 |
| 1990 | 130.7 |

| Year | CPI |
|---|---|
| 1991 | 136.2 |
| 1992 | 140.3 |
| 1993 | 144.5 |
| 1994 | 148.2 |
| 1995 | 152.4 |
| 1996 | 156.9 |
| 1997 | 160.5 |
| 1998 | 163.0 |
| 1999 | 166.6 |
| 2000 | 172.2 |

| Year | CPI |
|---|---|
| 2001 | 177.1 |
| 2002 | 179.9 |
| 2003 | 184.0 |
| 2004 | 188.9 |
| 2005 | 195.3 |
| 2006 | 201.6 |
| 2007 | 207.3 |
| 2008 | 215.3 |
| 2009 | 214.5 |
| 2010 | 218.1 |

***a.*** The CPI measures changes in the average price of goods and services from the reference period, 1982-84, which equals 100. So a 2005 index of 195.3 means prices increased 95.3% from the period 1982-84. If you bought $200 worth of common groceries during 1982-84, how much would they cost in the year 2005?

***b.*** Divide the CPI for 2005 by reference period index of 100. What does the result tell you about the cost of living in 1982-84 compared with 2005?

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***c.*** Compare the CPIs for the years 2000 and 2005 by finding the ratio of the CPI in 2005 to the CPI in 2000. Round to the hundredths place.

***d.*** What does the ratio in part *c* indicate about inflation over this time period?

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***e.*** Imagine your annual salary for 2000 was $40,000. How much would you need to earn in 2005 to be able to purchase the same amount of goods and services as in 2000? In other words, what salary do you need to earn in 2005 to keep the same standard of living as 2000?

***f.*** Suppose the average price of a cup of coffee at your favorite coffee shop went from $0.95 in year 2000 to $1.20 in year 2005. Use ratios, like part *c*, to see how the average price of a cup of coffee in 2005 compared with the average price in 2000. Round to the hundredths place.

***g.*** Based on your answers to parts *c* and *f*, how did the price increase in your cup of coffee compare with the overall rise in prices as measured by the CPI?

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***h.*** Find the absolute increase in the CPI between 2000 and 2005?

***i.*** Find the percent increase in the CPI between 2000 and 2005? Round to the nearest tenth of a percent.

***j.*** What does the percent increase found in part *i* indicate about prices?

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***k.*** Think about the meaning of your answers to parts *c* and *i*. Explain the similarities and differences between the two values when rounded to the same place value.

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***l.*** Suppose your annual salary went from $40,000 in 2000 to $44,200 in 2005. Find the percent increase in your salary between 2000 and 2005.

***m.*** Based on the information in parts *i* and *l*, has your salary kept up with the cost of living? Explain.

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***n.*** During which two years did the CPI have the largest absolute increase? What was the percent increase between those two years?

***o.*** What happens to a person's buying power during the two years in part *n*? Explain.

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**Feedback**

Inflation occurs when there is a sustained increase in the prices of basic consumer goods and services over an extended period of time. Because of inflation the value of money needed to purchase those goods and services will decrease over time. We can see how the buying power of a dollar decreases over time by examining the CPI ratios, which compare CPIs for different years using division. For example, the cases given below find the two ratios that can compare the CPIs for the years 2000 and 2005.

$$\text{Case 1: } \frac{\text{CPI 2005}}{\text{CPI 2000}} = \frac{195.3}{172.2} \approx 1.13 \qquad \text{Case 2: } \frac{\text{CPI 2000}}{\text{CPI 2005}} = \frac{172.2}{195.3} \approx 0.88$$

The Case 1 ratio indicates that what you could purchase in 2000 for a dollar would require about $1.13 to buy in 2005. In general, you will pay 1.13 times as much for items in year 2005 as in 2000. This can also be interpreted as the overall cost of goods and services in 2005 is 113% as much as 2000.

The Case 2 ratio indicates that the buying power of a 2000-dollar had decreased to about 88 cents in 2005. In general, the number of dollars spent on goods and services in the year 2000 will be able to purchase about 88% of the same amount of items in the year 2005.
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When a problem requires you to find a percent, an important question to ask is "Percent of what?" For example, suppose the tuition at Inflation College, currently $100 per credit, increases $20 per credit. Ghazi, who has a salary of $20,000, says this increase is only 0.001 or 0.1%, while Tammy claims an increase of 0.2 or 20%. Who is right?

Ghazi is talking about the increase as a percent of his salary, while Tammy's value is a percent increase on the current tuition amount. If Tammy has an annual salary of $2000, what percent of her salary is the tuition increase? Which of these values, percent of salary or percent of current tuition, seems to be the most relevant?