0(log n)

Speed up your career at GoPerceptive.com

perceptive software
Spent Fuel and Circuit Gain: What’s in a Log?

Steven J. Wilson, Johnson County C.C., Overland Park, KS
Stephen Hayton, Minot State U., Minot, ND

AMATYC, Jacksonville, FL
November, 2012
Disclaimer

- This presentation is **NOT** about
  - doing logs
  - rules of logs
  - computing logs
  - calculations with logs
  - etc. and so forth...

- Rather, it is about
  - understanding logs conceptually ...
POP QUIZ !!!

• No discussions with your neighbor
• One minute time limit

• Complete the sentence:
  A logarithm is ________________.
Results in a Business Calculus 1 Class:
The Five Good Answers (of 36)

• The inverse of $e^x$
• The inverse of an exponential
• A function used to determine exponent
• The exponent required to produce a given #
• Inverse of Exponential [sic]
Results in a Business Calculus 1 Class: The **Six** Basic Answers (of 36)

- A function
- A function
- A type of function
- A function
- Function
- A mathmatic function  [sic]
Results in a Business Calculus 1 Class: A Selection of the 25 Wrong Answers

• An expression to find unusual exponent rates
• A function that increases at a high rate
• Something I can use but can’t define
• The derivative of an exponential
• Annoying
• The opposite of an exponent
• No idea but I think it has something to do with the number 10
• Base function (depending on specific base)
• One of the words for math. (I don’t know)
Part 2 of the POP QUIZ!

Select ALL that apply:

A logarithm is:

a) A set of rules
b) An exponent
c) A number
d) An order of magnitude
e) A function
f) A transformation
g) An inverse
**Some Part 2 Results**

<table>
<thead>
<tr>
<th>Answer</th>
<th>From 36 students in Business Calculus I</th>
<th>From 64 students in Calc 3 or Circuits (both with a Calc 2 prerequisite)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function</td>
<td>69%</td>
<td>67%</td>
</tr>
<tr>
<td>Inverse</td>
<td>64%</td>
<td>65%</td>
</tr>
<tr>
<td>Number</td>
<td>58%</td>
<td>69%</td>
</tr>
<tr>
<td>Exponent</td>
<td>44%</td>
<td>71%</td>
</tr>
<tr>
<td>Set of Rules</td>
<td>50%</td>
<td>42%</td>
</tr>
<tr>
<td>Order of magnitude</td>
<td>42%</td>
<td>48%</td>
</tr>
<tr>
<td>Transformation</td>
<td>39%</td>
<td>41%</td>
</tr>
</tbody>
</table>

What are your thoughts about these numbers?
More Thoughts

• What issues do you find in teaching logs?
  o “Logs are just treated as another calculator exercise”
  o “Logs are just another section in a student’s journey in math”

• What’s in a log?
Understanding?

• Are we asking students to understand applications based on their [lack of] understanding of logs?

• Or are we asking students to understand logs based on their [lack of] understanding of applications?

• What applications can we expect students to understand?
  – Distance
  – Time
  – Money
  – Temperature
  – pH Levels
  – Earthquake Intensity
Distance
(how well do they – and we - really have a handle on it?)

• Collect the following information from the internet. Use the same unit of length for each. Do NOT use scientific notation.
  – The diameter of a hydrogen atom
  – Thickness of a human hair
  – The height of Lebron James (plays basketball for the Miami Heat)
  – The distance from Kansas City to Denver
  – The diameter of the earth
  – The distance from the earth to the sun
  – The diameter of the Milky Way Galaxy

• Compute the logarithm of each number.

• Explain how the logarithms are growing.

• Why use logarithms rather than the original number?
## Results

<table>
<thead>
<tr>
<th>Item</th>
<th>Values</th>
<th>Logarithms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen atom</td>
<td>0.00000000000106 m</td>
<td>- 9.80</td>
</tr>
<tr>
<td>Human hair</td>
<td>0.0001 m</td>
<td>- 4.00</td>
</tr>
<tr>
<td>Lebron James</td>
<td>2.01 m</td>
<td>0.30</td>
</tr>
<tr>
<td>Kansas City to Denver</td>
<td>970900 m</td>
<td>5.99</td>
</tr>
<tr>
<td>Diameter of the earth</td>
<td>12756000 m</td>
<td>7.11</td>
</tr>
<tr>
<td>Earth to Sun</td>
<td>1500000000000 m</td>
<td>11.18</td>
</tr>
<tr>
<td>Milky Way Galaxy</td>
<td>9500000000000000000000000 m</td>
<td>20.98</td>
</tr>
</tbody>
</table>
Log Scale on a Slide Rule

• How else can we simultaneously graph the very large and the very small?
Scale of the Universe
Flash object at http://htwins.net/scale2/

Open browser to run local copy
Money!
(they – and we - deal with this every day)

<table>
<thead>
<tr>
<th>Money</th>
<th>Base 10 Logs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>0</td>
</tr>
<tr>
<td>$10</td>
<td>1</td>
</tr>
<tr>
<td>$100</td>
<td>2</td>
</tr>
<tr>
<td>$1000</td>
<td>3</td>
</tr>
<tr>
<td>$10000</td>
<td>4</td>
</tr>
</tbody>
</table>

Images from Wikipedia
Growth of Money

At 5% interest compounded continuously, how long will it take for $5000 to grow to $B$?

\[ A = Pe^{rt} \]
\[ B = 5000e^{0.05t} \]
\[ t = 20 \ln \frac{B}{5000} \]

<table>
<thead>
<tr>
<th>B</th>
<th>t (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5,000$</td>
<td>0</td>
</tr>
<tr>
<td>$10,000$</td>
<td>13.8</td>
</tr>
<tr>
<td>$15,000$</td>
<td>21.9</td>
</tr>
<tr>
<td>$20,000$</td>
<td>27.7</td>
</tr>
<tr>
<td>$40,000$</td>
<td>41.0</td>
</tr>
</tbody>
</table>

Which scale is the log scale?
- Doubling Time (13.8 years) comes from a logarithmic expression.
- Time is a logarithmic function of the account balance.
- Time can describe the order of magnitude of the account balance.
One College Algebra Book’s Presentation of Sound Intensity

• A definition:  \( B = 10 \log \frac{I}{I_0} \) measures psychological sensation of loudness.

• The reference intensity:  \( I_0 = 10^{-12} \text{ W/m}^2 \)

• Table of common sounds and their decibel levels

• Example: Find the decibel intensity level of a jet engine during takeoff if intensity was 100 W/m².

So what has the student learned? Psychology? Isn’t this “just another calculator exercise”?
Sound Intensity: The Real Issue

• Circuit Gain
  o ... or What Are you Doing to Your Ears?

• You increase the volume from 3.1 to 3.2. What is the percent increase in intensity on your ears?
  o Hint: 3% is wrong. \[
  \frac{10^{3.2}}{10^{3.1}} = 10^{0.1} \approx 1.2589
  \]
  o About a 26% increase!

• What about an increase from 90.1 to 90.2 dB?
Noise!
(which they think they understand)

<table>
<thead>
<tr>
<th>Sound sources (noise) Examples with distance</th>
<th>Sound pressure Level $L_p$ dB SPL</th>
<th>Sound pressure $p$ N/m$^2$ = Pa</th>
<th>Sound intensity $I$ W/m$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet aircraft, 50 m away</td>
<td>140</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Threshold of pain</td>
<td>130</td>
<td>63.2</td>
<td>10</td>
</tr>
<tr>
<td>Threshold of discomfort</td>
<td>120</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>Chainsaw, 1 m distance</td>
<td>110</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Disco, 1 m from speaker</td>
<td>100</td>
<td>2</td>
<td>0.01</td>
</tr>
<tr>
<td>Diesel truck, 10 m away</td>
<td>90</td>
<td>0.63</td>
<td>0.001</td>
</tr>
<tr>
<td>Kerbside of busy road, 5 m</td>
<td>80</td>
<td>0.2</td>
<td>0.0001</td>
</tr>
<tr>
<td>Vacuum cleaner, distance 1 m</td>
<td>70</td>
<td>0.063</td>
<td>0.00001</td>
</tr>
<tr>
<td>Conversational speech, 1 m</td>
<td>60</td>
<td>0.02</td>
<td>0.000001</td>
</tr>
<tr>
<td>Average home</td>
<td>50</td>
<td>0.0063</td>
<td>0.0000001</td>
</tr>
<tr>
<td>Quiet library</td>
<td>40</td>
<td>0.002</td>
<td>0.00000001</td>
</tr>
<tr>
<td>Quiet bedroom at night</td>
<td>30</td>
<td>0.00063</td>
<td>0.000000001</td>
</tr>
<tr>
<td>Background in TV studio</td>
<td>20</td>
<td>0.0002</td>
<td>0.000000001</td>
</tr>
<tr>
<td>Rustling leaves in the distance</td>
<td>10</td>
<td>0.000063</td>
<td>0.0000000001</td>
</tr>
<tr>
<td>Hearing threshold</td>
<td>0</td>
<td>0.00002</td>
<td>0.00000000001</td>
</tr>
</tbody>
</table>

Table from: http://www.sengpielaudio.com/TableOfSoundPressureLevels.htm
Spent Fuel
(using understanding of logs for an unknown application)

• How long will spent fuel with a half-life of 87.7 years take to lose $p$ of its radioactivity?

\[
y = y_0 e^{-kt}
\]

\[
\frac{1}{2} y_0 = y_0 e^{-k(87.7)}
\]

\[
k = \frac{\ln 2}{87.7} \approx 0.0079
\]

\[
(1 - p) y_0 = y_0 e^{-0.0079t}
\]

\[
t = -127 \ln (1 - p)
\]

<table>
<thead>
<tr>
<th>$p$</th>
<th>$t$ (yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>87.7</td>
</tr>
<tr>
<td>0.9</td>
<td>291.3</td>
</tr>
<tr>
<td>0.99</td>
<td>582.7</td>
</tr>
<tr>
<td>0.999</td>
<td>874.0</td>
</tr>
<tr>
<td>0.9999</td>
<td>1165.3</td>
</tr>
<tr>
<td>0.99999</td>
<td>1456.7</td>
</tr>
</tbody>
</table>

• Where is the log connection?
  o Time describes the order of magnitude of the remaining radioactivity.
What’s in a Log?

• Functions, inverses, numbers
  o And about 2/3 of students are learning these

• Exponents
  o In spite of the emphasis from college algebra textbooks, it is taking a long time for students to learn this

• Order of Magnitude
  o Less than half of students know this, even though it describes a key feature of logarithms, and college algebra textbooks rarely mention it
Contact Information

• Steven J. Wilson, Professor of Mathematics
  • Johnson County Community College, Overland Park, KS
  • swilson@jccc.edu, 913-469-8500, www.milefoot.com

• Stephen Hayton, Asst. Professor of Math & Computer Science
  • Minot State University, Minot, ND
  • stephen.hayton@minotstateu.edu, 701-858-3075