Authentic Problem-Solving in a Developmental Pathways Course

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Developmental Pathways ...  

- include Math Literacy for College Students, Quantway, Statway, and the New Mathways Project.

- create alternative routes to or through college-level math courses, especially non-STEM courses.

- look forward to college needs instead of backward to high school deficiencies.

- emphasize critical thinking and problem solving.

- use authentic problems and contextualized learning.
Exercise

Solve: $2x + 7 = 10$

Problem

Given only $20, how many buffalo wings can you buy if they are 25 cents each? Assume you will buy a pitcher of soda for $4 and veggies and dips for $2.50. 7.25% tax should be included along with a 20% tip.
Exercise

Simplify: \((-3x^2 y^3)^5\)

Problem

You are making brownies and debating between an 8” round pan and an 8” square pan. If you want thick brownies, which should you use and why?

Show all calculations with units included.
Exercise

Simplify: \( 2(x + 3) - 4x - 1 \)

Problem

Write an expression for the numbers of chairs around \( n \) tables arranged in this configuration:

One student arrives at this expression: \( 4n - 2(n - 1) \)

Explain this expression in terms of the tables and chairs. Verify that your expression is equivalent to it.
Exercise

Find the slope of the line passing through (-2, 5) and (0, -7).

Problem

One of your friends is taking 5 credit hours at a local community college and paid $995 in tuition and fees. Another friend is taking 13 credit hours at the same college and paid $2427. What is the tuition rate per credit hour and are there any flat fees? If so, what are they?
**Exercise**

Solve the system:

\[ 3x - 2y = -6 \]

\[ x = -4y + 8 \]

**Problem**

For which salaries is option A better than option B?

A: 5% raise

B: 3% raise plus $1000
Problems vs. Exercises

Problems require original approaches. A method is not necessarily clear at first glance.

Exercises require choosing and applying an algorithm.

Problems involve the non-routine application and connection of skills.

Exercises develop students’ ability to perform skills.
Baseball’s Magic Number

In baseball, a leading team’s magic number is an indication of how close the team is to clinching the division title.

The magic number is some combination of the number of additional wins for the leading team and additional losses for the second-place team that are required before the leading team is guaranteed to win its division.
Baseball’s Magic Number: Method 1

One way of calculating a team’s magic number is to subtract the number of wins for the leading team and the number of losses for the second-place team from 163.

Expressions:

\[ MN = 163 - (W_1 + L_2) \]  or  
\[ MN = 163 - W_1 - L_2 \]

**MN = Magic number**

**W_1 = Wins for the first-place team**

**L_2 = Losses for the second-place team**
Baseball’s Magic Number: Method 2

Find the number of games remaining for the leading team (there are 162 games in a regular season) and add one to it. Then subtract from this sum the difference between the number of losses for the second-place team and the number of losses for the leading team.

Expression:

\[ MN = [162 - (W_1 + L_1)] + 1 - (L_2 - L_1) \]

\[ L_1 = \text{Losses for the first-place team} \]
Baseball’s Magic Number: Method 3

Add one to the number of remaining games for the second-place team and then subtract the difference between the leading team’s wins and the second-place team’s wins.

Expression:

\[ MN = [162 - (W_2 + L_2)] + 1 - (W_1 - W_2) \]

W_2 = Wins for the second-place team
Baseball’s Magic Number: Skills

- Write an expression
- Define variables
- Apply the distributive property
- Combine like terms
- Perform integer calculations
- Apply the order of operations
- Interpret signed numbers
Medical Dosage

A doctor orders 200 mg of an antibiotic for an infant who weighs 15.4 lb. It should be taken every 8 hours. The medication label shows that 75–150 mg/kg per day is the appropriate dosage range.

Determine if the dosage ordered is within the desired range.
Medical Dosage: Mindset of a problem solver

<table>
<thead>
<tr>
<th>Approach</th>
<th>Work Shown</th>
</tr>
</thead>
<tbody>
<tr>
<td>What do I know?</td>
<td>Dose is 200 mg every 8 hrs for 15.4 lb infant.</td>
</tr>
<tr>
<td>The appropriate dosage range is given in terms of mg per kg per day. We</td>
<td>Range is 75–150 mg/kg per day.</td>
</tr>
<tr>
<td>have mg for 8 hours but not per day. So how much would the baby get in</td>
<td>1 dose in 8 hrs → 3 doses in 24 hours (1 day) 200 mg per dose × 3 doses = 600 mg in a day</td>
</tr>
<tr>
<td>a day?</td>
<td></td>
</tr>
<tr>
<td>The units in the range are mg per kg and the baby’s weight is in pounds.</td>
<td>2.2 lb ~ 1 kg</td>
</tr>
<tr>
<td>How many kg is 15.4 lb?</td>
<td></td>
</tr>
<tr>
<td>For every 2.2 pounds, there is 1 kg. I need to know how many 2.2’s are in</td>
<td>15.4/2.2 = 7.</td>
</tr>
<tr>
<td>15.4. So I need to divide.</td>
<td>The baby’s weight is 7 kg.</td>
</tr>
<tr>
<td>Let’s put together what we know and use units to help. The range units</td>
<td></td>
</tr>
<tr>
<td>are mg per kg per day. My dose is in terms of one day. So I need to find</td>
<td></td>
</tr>
<tr>
<td>the mg per kg that I have.</td>
<td>600 mg 85.7 mg 85.7 mg 7 kg 1 kg kg</td>
</tr>
<tr>
<td>Check the original question. Is the dose in the correct range?</td>
<td>85.7 mg/kg per day is between 75 mg/kg per day and 150 mg/kg per day, so the dosage is within the desired range. The doctor prescribed an acceptable dosage for the infant.</td>
</tr>
</tbody>
</table>
Authentic problem solving creates challenges in the classroom. What are some of them and how can we solve them?
Tips

- Use realistic and relevant contexts
- Be open to figuring out problems with the class
- Provide challenging yet accessible problems
- Allow for productive struggle
- Value multiple and varied approaches
- Don’t shield students from life’s complexities
Questions
Conclusions

1. Problems and exercises are different.
2. Skills can be taught through problem solving, but it is difficult to teach problem solving through skills.
3. A problem-solving mindset can be taught, encouraging the development of positive attitudes and behaviors.
4. Classroom challenges are worth overcoming!
For more information

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