1. The listing method of the set \( \{ x | (x+1)(x-2)(x^2+1)=0, x \in \mathbb{R} \} \) can be expressed as ________.

2. The range of the equation \( f(x) = 2x+3 \) is \( \{-1, 2, 5, 8\} \), the domain is \( \{ \text{____} \} \).

3. If \( f(2x) = 3x^2+1 \), then the equation of \( f(x) \) is ________.

4. If set \( A \) satisfy \( \{1\} \cup A = \{1, 3, 5\} \), then set \( A = \text{_______} \).

5. The graph of \( f(x) = ax^{-1}-3 \) must pass the fixed point ________.

6. \( f(x) = \begin{cases} 
2, & x \leq 0 \\
3x^2-4, & x > 0 
\end{cases} \) then \( f(f(-2)) = \text{_______} \).

7. If \( A = [1, 4) \), \( B = (-\infty, a) \), if \( A \subseteq B \), then the domain of real number \( a \) is ________.

8. \( a = 0.3^2 \), \( b = 2^{0.3} \), \( c = \log_{0.3} 2 \), list \( a, b, c \) from the smallest to the largest ________.

9. \( U = \{1, 2, 3, 4, 5\} \), \( A \cap (C \cup B) = \{1\} \), \( B \cap (C \cup A) = \{5\} \), \( (C \cup A) \cap (C \cup B) = \{2\} \), then set \( A = \text{_______} \).

10. Function \( y = -x^2+4ax \) is absolutely increasing on the interval \([1, 3]\), then the domain of real number \( a \) is ________.

11. \( f(x) \) is an odd function, then \( f(\sqrt{3}+2)+f(\sqrt{3}-2)=\text{_______} \).

12. The range of \( f(x) = \frac{x^2}{x^2+10} \) \( \text{_______} \), \( x \in \mathbb{R} \) is ________.
13. Suppose positive integer m satisfies $10^{m-1} < 2^{512} < 10^m$, then $m = \underline{11}$ ($\lg 2 \approx 0.3010$).

14. $y = x$ is the line of symmetry of $f(x) = (x^2)^x$ and $g(x)$, let $h(x) = g(1 - |x|)$, then $h(x)$ has the following characteristics:
   (1) The symmetry for $h(x)$ is the origin
   (2) $h(x)$ is even
   (3) The minimum value for $h(x)$ is 0
   (4) $h(x)$ is decreasing on the interval $(0, 1)$

   The correct statement is \underline{1} (fill in all the correct #s).

15. Set $A = \{1, a^2 + 1, a^2 - 3\}$, $B = \{-4, a-1, a+1\}$, and $A \cap B = \{-2\}$ find the value of $a$.

16. Compute:
   (1) $(2^{-\frac{3}{5}})^0 + 2^{-2} \cdot (2^{\frac{1}{4}})^{-\frac{1}{2}} - (0.01)^{0.5}$
   (2) $2^{3\log_2 4} + 3^{\log_2 1} - \log_{10} 3 \cdot \log_3{2} - \log{5}$

17. $\alpha, \beta$ are two zeros of $y = x^2 - 2kx + 6$.
   (1) Find the equation of $f(k) = (\alpha - 1)^2 + (\beta - 1)^2$ and find the domain.
   (2) Find the minimum value of $f(k)$ and find $k$'s value when $f(k)$ has a minimum value.

18. $f(x) = ax^2 + bx + cx$, inequality the solution of the inequality $f(x) > -2x$ is $(1, 3)$
   (I) If $f(x) + 6a = 0$ has 2 equal real roots, find the equation for
       $f(x)$.
   (II) If the maximum value of $f(x)$ is a positive number, find
        the range of $a$. 
19. The value \( y \) (dollars) of a diamond and the square of its weight \( x \) (in carats) are positively correlated. A 3 carats of the diamond is $54,000.

(I) Write a function of \( y \) into term of \( x \).

(II) If cut the diamond into 2 piece according to 1:3, find the percentage of loss.

(III) If cut the diamond into 2 pieces, and the weight of the 2 pieces are \( m \) carats and \( n \) carats, prove: when \( m=n \), the percentage of loss is the greatest.

(Note: Percentage of loss = \( \frac{\text{Origin Value} - \text{Present Value}}{\text{Origin Value}} \times 100\% \))

20. If function \( f(x) \) satisfies: for any \( x \in D \), there exists a constant \( M > 0 \), and if \( |f(x)| \leq M \) exists, then \( f(x) \) is a limit function of \( D \), then \( M \) is the upper limit of \( f(x) \).

Knowing that \( f(x) = 1 + \alpha \left( \frac{1}{2} \right)^x + \left( \frac{1}{4} \right)^x \), \( g(x) = \frac{1 - m \cdot 2^x}{1 + m \cdot 2^x} \)

(1) When \( \alpha = 1 \), find the range of \( f(x) \) in the interval of \((-\infty, 0)\), and tell if \( f(x) \) in the interval \((-\infty, 0)\) is an upper limit function and explain the reason.

(2) If 3 is the upper limit of \( f(x) \) in the interval \([0, +\infty)\), find the domain of the real number \( \alpha \).

(3) If \( m > 0 \), the upper limit of \( g(x) \) in the interval \([0, 1]\) is \( T(m) \), find \( T(m) \) the domain of \( T(m) \).
Third Year Math Exercises (High School)

1. Given the top of the parabola \( C \) is the origin, its focus is on the x-axis, line \( y = x \) and the parabola \( C \) intercept at two points \( A \) and \( B \). If \( P(2,2) \) is the midpoint of \( AB \), then the equation of the parabola \( C \) is ________.

2. In \( \triangle ABC \), \( \overrightarrow{AB} \cdot \overrightarrow{BC} = 3 \), \( \triangle ABC \)'s surface area \( S_{\triangle ABC} \in [\frac{\sqrt{3}}{2}, \frac{3}{2}] \), then the angle between \( \overrightarrow{AB} \) and \( \overrightarrow{BC} \) is ________.

3. Suppose the focus of the parabola \( y^2 = 2px \) \( (p > 0) \) is \( F \), Point \( A(0,2) \) If line segment's midpoint \( B \) is on the parabola, then the distance from \( B \) to the directrix of the parabola is ________.

4. Given that the distance between the moving point \( P \) and fixed point \( (2,0) \) and the distance between \( P \) and straight line \( L: x = -2 \) are equal, then point \( P \)'s equation is ________.

5. In rectangle system \( xOy \), Suppose \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) \( (a > b > 0) \) the focus is \( 2c \). From \( O \) as the center make a circle with radius \( a \). If the tangent lines passing \( P(\frac{a^2}{c}, 0) \) and the circle \( M \) are perpendicular, then the eccentricity of the \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) is ________.

6. Make a straight line passing through \( (0,1) \). The line share only one common point with parabola \( y^2 = 4x \). How many such lines exist ?

7. Line \( y = b \) and \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) intersected at two points \( A \) and \( B \), \( \angle AOB = 90^\circ \) \( (O \) is the origin). The slope of the asymptote is ________.
8. \( P \) is a point on the left curve of \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) \((a>0, b>0)\). \( F_1, F_2 \) are the left and right foci. \( \overrightarrow{PF_1} \cdot \overrightarrow{PF_2} = 0 \), \( \tan \angle PF_2F_1 = \frac{2}{3} \), the eccentricity of \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) is ________.

9. Given piecewise defined \( \begin{cases} x \geq 0, y \geq 0 \\ y \leq -2x + 2\sqrt{2} \\ y \leq kx + \sqrt{2} \end{cases} \), the range \( D \) is completely covered by a circle with radius 1, then the range of the real number \( k \) is ________.

10. From the moving point \( P \) toward \( \frac{x^2}{4} + y^2 = 1 \) make 2 tangent lines \( PA, PB \). The points of tangency are \( A, B \), \( \angle APB = 90^\circ \), the equation of the moving point \( P \) is ________.

11. The line passes through the focus of \( y^2 = 4x \) intersects the parabola in 2 points \( A, B \). passing \( B \) make a perpendicular line to the directrix \( L \), the point of intersection is \( C \). Given \( A(4,4) \), then line \( AC \)'s equation is ________.

12. \( O \) is the origin for \( y^2 = 4x \), \( A, B \) are two moving points on the parabola, \( OA \perp OB \), when line \( AB \)'s angle is \( 45^\circ \), the surface area of \( \triangle AOB \) is ________.

13. Given \( x^2 = 4y \), point \( F \) is the focus of the parabola, the point of intersection of the directrix and \( y \)-axis is \( M \), \( N \) is a point of on the parabola, \( |NF| = \frac{\sqrt{3}}{2} |MN| \), \( \angle NMF \) ________.
14. Compare "The sum and difference of the law of Sine and cosine", for the given functions

\[ S(x) = \frac{e^x - e^{-x}}{2} \quad \text{and} \quad C(x) = \frac{e^x + e^{-x}}{2} \]

Write a correct computational equation———

15. On There are 2 points A, B on \( y^2 = 4x \), Point F is the focus of the parabola, O is the origin, if \( \overrightarrow{FO} + 2\overrightarrow{FA} + 3\overrightarrow{FB} = \overrightarrow{0} \), then the point of intersection of line AB and X-axis is———

16. F is the left focus of \( \frac{x^2}{16} - \frac{y^2}{9} = 1 \). On the right side of F of the X-axis, there is a point A, The points of intersection of the circle with radius FA and the 2 curves (above X-axis) are M, N, The value of \( \frac{|FN| - |FM|}{|FA|} \) is———