Strategies for Solving Application Problems
(a.k.a., Word problems!)

Session:  S149
November 15, 2014 (Saturday)
1:05 – 1:55
Room:  Tennessee A

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Developed with Professor Regina Souhrada, Reading
Anne Arundel Community College
Arnold, MD
Why this topic?

Because teachers say...

- The students know how to do the math, they just don’t understand what the word problem is asking.
- The word problems are written too hard for students to get what needs to be done.
- The reading level is too hard for students.
- I need to simplify, to reword the word problems for my students, and then they can do it.
Word Problem Dilemmas

• There are essentially only **two** steps:
  1. Translate words & numbers into equations.
  2. Solve it!

• *Sorry!* There are **no secrets** for solving word problems, they must be read carefully.

• Students are often led astray by content area vocabulary and unreliable shortcuts (e.g., key/clue/cue words).

• In other content areas, mathematics is virtually always in word problems - *Ugh!*
The Beginning: George Polya (1887-1985)

- Published *How to Solve It* in 1945 (1st ed.).
- He identified four principles which form the basis for any serious attempt at problem solving.
  - Principle 1: Understand the problem
  - Principle 2: Devise a plan
  - Principle 3: Carry out the plan
  - Principle 4: Look back
- The four principles form the basis for most problems solving strategies.
I HATE WORD PROBLEMS.

ARE YOU KIDDING? WORD PROBLEMS ARE GREAT!

WITHOUT WORD PROBLEMS, MATH WOULD BE JUST SOME ABSTRACT BUNCH OF FORMULAS THAT LIVE ONLY WITHIN THE CONFINES OF A CLASSROOM OR A TEXTBOOK.

BUT IN REALITY, MATH IS EVERYWHERE YOU LOOK! IT PERMEATES EVERYTHING! YOU CAN'T ESCAPE IT! AND THAT'S WHAT WORD PROBLEMS LET US IN ON.

AND THAT'S NOT A REASON TO HATE THEM.

AND THE MORE MATH YOU LEARN, THE MORE MATH YOU SEE.

UHHH... HOUSTON, WE HAVE A WORD PROBLEM.

IF A TRAIN LEAVES THE STATION AT 9:00AM HEADING NORTH AT 55 MPH, AND ANOTHER TRAIN LEAVES THE STATION AT 10:00AM SOUTH AT 72 MPH, BOTH TRAVELING SIMULTANEOUSLY, HOW FAR APART WILL THEY BE AT 3:00PM?
Now, for the Strategies😊

The goal is to offer students a “toe-hold” for approaching and solving word problems.
Definition: Content Area Reading Strategies

- Reading strategies are not intended for students to *learn-to-read* the textbook but rather to *read-to-learn from* the textbook (online/paper).
- Reading Strategies are *actually* Learning Strategies
  - Students can use strategies to help them organize and comprehend what is read.
  - Faculty can use strategies to check on student comprehension of what is read.
Our Criteria for Choosing Various Content Area Reading Strategies

• Easily incorporated
• Little if any time needed in class after initial introduction
• Small learning curve for faculty member
• Research-based benefit for students
We suggest that you...

• not be overwhelmed with the wealth of strategies you will see.
• pick only one or two strategies to try.
• choose the one(s) that fit your teaching style, personality, and course content.
• keep in mind, all strategies can be blended, modified, extended, etc.
Strategies

• Think Aloud
• KWS Table
• Marking Word Problems
• Highlighting or Underlining
• Word Problem Map
• Problem-Solving Guide
• Sequence Map
• Word Problem Organizer
• Guided Reading for Word Problems
• Three-level Guide
• Process Notes
• Acronyms
Think Aloud

• Also called “modeling.”

• Read aloud the word problem and talk aloud your thought process as you do so. Continue to do the same as you work through the solution.

• Can easily point out vocabulary.

• Students can “see” your thought process.

• You can also ask questions of yourself or the students as you go along.
## KWS Table

<table>
<thead>
<tr>
<th>What do I Know?</th>
<th>What do I Want or need to know?</th>
<th>What Strategies might I use?</th>
</tr>
</thead>
</table>

- Helps students better understand the context (aka, story) of the problem.
- Helps with identifying important quantities (not all quantities are always named).
- Helps students reason about operations needed.
Two track stars and friends are competing, Speedy and Sporty. A sports analyst recently developed a formula to describe the distance Speedy had to run in \( t \) seconds after beginning the race. He also developed a similar formula for Sporty. How far ahead will the winner be in a 400-meter race?

- **Speedy**: \( m(t) = 0.1t^2 + 3t \)
- **Sporty**: \( n(t) = 0.095t^2 + 2.92t \)
## KWS Table

<table>
<thead>
<tr>
<th>What do I Know?</th>
<th>What do I Want or need to know?</th>
<th>What Strategies might I use?</th>
</tr>
</thead>
<tbody>
<tr>
<td>-The formulas are a function of time. -They are running a 400m race.</td>
<td>-How far ahead is the winner at the end of the race? -Who wins the race? -How long does it take the winner to cross the finish line?</td>
<td>-Make a table showing the distance run by each racer after different amounts of elapsed time. -Sketch graphs of distance as a function of time. -Create the function m(t) – n(t). -Solve m(t)=400</td>
</tr>
</tbody>
</table>
Marking Word Problems

1. **Read the problem!**
   During the summer before starting high school, Hayden earned money mowing lawns. He deposited $800 in a savings account for college. His bank offered Hayden a 6% simple interest annual rate. If Hayden doesn’t withdraw any money from the account and the interest rate stays the same, how much will he have at the end of five years?

2. **Reread the problem!**
   Then underline all numbers and math-related terms.
   During the summer before starting high school, Hayden earned money mowing lawns. He deposited $800 in a savings account for college. His bank offered Hayden a 6% simple interest annual rate. If Hayden doesn’t withdraw any money from the account and the interest rate stays the same, how much will he have at the end of five years?

3. **Find the “who” and the “what.”** Mark them with a W.
   During the summer before starting high school, Hayden earned money mowing lawns. He deposited $800 in a savings account for college. His bank offered Hayden a 6% simple interest annual rate. If Hayden doesn’t withdraw any money from the account and the interest rate stays the same, how much will he have at the end of five years?

4. **Circle the situation, issue, or circumstance.** Look for action verbs.
   During the summer before starting high school, Hayden earned money mowing lawns. He deposited $800 in a savings account for college. His bank offered Hayden a 6% simple interest annual rate. If Hayden doesn’t withdraw any money from the account and the interest rate stays the same, how much will he have at the end of five years?

5. **Box the question that needs to be answered.**
   During the summer before starting high school, Hayden earned money mowing lawns. He deposited $800 in a savings account for college. His bank offered Hayden a 6% simple interest annual rate. If Hayden doesn’t withdraw any money from the account and the interest rate stays the same, how much will he have at the end of five years?

- Students who struggle with reading need extra time and instruction on reading word problems.
- Focuses on vocabulary.
- This strategy pushes students to recognize and think about the parts of a word problem.
Highlighting or Underlining

- This is another strategy which *marks* word problems.
- It helps students to focus on vocabulary AND the numbers.
- Students must analyze and justify what they highlighted or underlined.
Let's try one 😊

A biologist studying Chase Lake springtails for a period of 10 years determined that the birthrate was 120 per year, mortality was 90 per year, immigration was 35 per year, and emigration was 50 per year.

a.) Was the population increasing, decreasing, or constant?
b.) If the population was 150 at the beginning of the 10-year period, what is it now?
c.) What is the average increase over 10 years?
Let's try one Solution 😊

A biologist studying Chase Lake springtails for a period of 10 years determined that the birthrate was 120 per year, mortality was 90 per year, immigration was 35 per year, and emigration was 50 per year.

a.) Was the population increasing, decreasing, or constant?

b.) If the population was 150 at the beginning of the 10-year period, what is it now?

c.) What is the average increase over 10 years?
Three-level Guide

- Time-consuming for faculty to prepare.
- Helps students to focus on the important facts in the problem.
- Allows students to check the usefulness of several approaches and computations.
Guided Reading for Word Problems

- Time-consuming for faculty to prepare.
- Can *briefly* replace faculty face-to-face guidance *to a certain extent*.
- Can also include a focus on vocabulary.
- Students approach one row at a time.
How I modified the Guided Reading

**INVERSE Variation Problem Solving**

<table>
<thead>
<tr>
<th>Steps</th>
<th>Explanation</th>
<th>Problem</th>
</tr>
</thead>
</table>
| Step 1 | Read the problem and notice if any of the phrases are used:  
- Inverse variation  
- Inversely proportional  
- Varies inversely  | Ex. 4: Boyle’s law says that if the temperature stays the same, the pressure \( P \) of a gas is **inversely proportional** to the volume \( V \). If a cylinder in a steam engine has a pressure of 960 kilopascals when the volume is 1.4 cubic meters, find the pressure when the volume increases to 2.5 cubic meters. |
| Step 2 | If any of those phrases are used then use this original formula: \( y = \frac{k}{x} \). From the problem, use the values given for \( x \) and \( y \) to determine the constant of variation, \( k \). | If a cylinder in a steam engine has a pressure of 960 kilopascals when the volume is 1.4 cubic meters, \( y = \frac{k}{x} \), where \( y = 960 \) and \( x = 1.4 \). So... |
|       |             | \( 960 = \frac{k}{1.4} \)  
|       |             | \( (960)(1.4) = k \)  
|       |             | \( k = 1344 \) |
| Step 3 | Rewrite the original formula with the value for \( k \) from Step 2.  
**Note:** Some problems stop at this point, such as Ex. 3 and problems #13-20 on page 542. |  
|       |             | \( y = \frac{1344}{x} \) |
| Step 4 | Reread the problem to recall what you are asked to find.  
- You will be given \( x \) and asked to find \( y \).  
- OR you will be given \( y \) and asked to find \( x \).  
Use the rewritten formula from Step 3 to determine the value of what you are asked to find. See Ex. 4. | ...find the pressure when the volume increases to 2.5 cubic meters.  
\( y = \frac{1344}{2.5} \), where \( x = 2.5 \)  
\( y = 537.6 \) |
| Step 5 | Write the final answer in a sentence form.  
Review Ex. 4. | When the volume is 2.5 cubic meters, the pressure is 537.6 kilopascals. |

- I created one completed problem for each type of variation problem for “lecture” and students’ notes.
- After a problem type was “lectured,” students completed a problem in class using a blank form.
Process Notes

- Can think of this as a stripped-down Three-level Guide or Guided Reading.
- Helps students “see” the process needed to solve word problems.
- Has a place for vocabulary.
Let's try another😊

Find the selling price per pound of a coffee mixture made with 8 pounds of coffee A that costs $9.20 per pound and with 12 pounds of coffee B that costs $5.50 per pound.
### Let's Try It Solution

<table>
<thead>
<tr>
<th>Write the question.</th>
<th>Find the selling price per lb of a coffee mixture made with 8 lbs of coffee A that costs $9.20 per lb and with 12 lbs of coffee B that costs $5.50 per lb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>List clue words and facts.</td>
<td>Notice the units:</td>
</tr>
<tr>
<td>Identify the variable(s).</td>
<td>The unknown is the cost per lb of the coffee mixture.</td>
</tr>
<tr>
<td>Make a drawing.</td>
<td>Coffee A</td>
</tr>
<tr>
<td></td>
<td>8 lbs</td>
</tr>
<tr>
<td></td>
<td>$9.20/\text{lb}$</td>
</tr>
<tr>
<td>Coffee B</td>
<td>12 lbs</td>
</tr>
<tr>
<td></td>
<td>$5.50/\text{lb}$</td>
</tr>
<tr>
<td>Choose a strategy.</td>
<td>Make a table since this is a mixture problem.</td>
</tr>
<tr>
<td>Solve the problem.</td>
<td></td>
</tr>
<tr>
<td>In a formula:</td>
<td>Coffee $\frac{\text{lbs}}{8}$ $\frac{\text{$}}{9.20}$ $\frac{\text{$}}{12}$ Total ($$)$</td>
</tr>
<tr>
<td>A $\frac{189.60}{20\text{lbs}} = \frac{$6.98}{\text{lb}}$</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>5.50</td>
</tr>
<tr>
<td>Mix</td>
<td>20</td>
</tr>
<tr>
<td>?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{139.60}{20\text{lbs}}$</td>
</tr>
<tr>
<td>Write your answer in a complete sentence that answer the question.</td>
<td>A mixture of 8 pounds of coffee A and 12 pounds of coffee B will cost $$6.98/\text{lb}$.</td>
</tr>
<tr>
<td>Checks:</td>
<td><strong>Credibility (Does your answer make sense?)</strong>: Yes, because the mixture cost should be between the cost of coffee A and the cost of coffee B.</td>
</tr>
<tr>
<td>Mathematical</td>
<td><strong>Double-checked calculations!</strong></td>
</tr>
</tbody>
</table>
Word Problem Map

- Gives a “pictorial map” for solving word problems.
- Helps students organize their thoughts as they tackle the problem.
- Provides a focus on vocabulary.
Word Problem Solving Guide

- **Gives students a “toehold” for approaching a word problem.**

- **Also helps students organize their thoughts as they tackle the problem.**

- **Offers students the feeling of progress due to the flow.**
Example of Word Problem Solving Guide

Write problem here: Section 2.3, #3, p. 171

A pilot flew a plane from an altitude of 20,000 feet to an altitude of 15,000 feet. What was the change in altitude? The plane was flying over the Mojave desert at 500 miles per hour.

What are you trying to find out (what is the unknown, the variable)?
- the change in altitude

What useful information is given?
- started at 20,000 ft
- ended at 15,000 feet

Ideas of what I can do to solve this:
- subtract
  (use change in value formula p. 167)

Solve problem:

\[
\text{change} = \text{end value} - \text{beginning value} \\
= 15,000 - 20,000 \\
= -18,500 \text{ ft}.
\]

Verify answer:

\[
\frac{18,500}{20,000} + \frac{1500}{20,000} = \frac{20,000}{20,000} = 1.
\]

Use for scratch work, drawing, or sketching:

Write answer in sentence form:
The pilot had -18,500 foot change in altitude.

Explain why or why not your answer is reasonable:
The change had to be between 20,000 ft and 15,000 ft, and it is 18,500 ft. It also had to be negative since the pilot lost altitude.
Problem-Solving Organizers

**Problem-Solving Organizer**

What is the question? What are you asked to do?

What is your mathematical plan?

1. 
2. 

First, I... 
Then, I... 

Write a conclusion statement. Look back at the question. Be sure your conclusion answers the original question.

**Advanced Problem-Solving Organizer**

THINK - What is the question? What are you being asked to do? Rewrite the question.

SOLVE - What is your mathematical plan? Show your work.

Step 1 
Step 2 
Step 3 

EXPLAIN - What was your thought process? How did you solve the problem?

First, I... 
Then, I... 
And finally, I... 

WRITE a conclusion. Make sure your conclusion answers the original question.
Example of the Advanced Problem-Solving Organizers

<table>
<thead>
<tr>
<th>THINK</th>
<th>What is the question? What are you being asked to do? Rewrite the question.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>How long to double on initial investment of $2000 to $4000?</td>
</tr>
<tr>
<td></td>
<td>Looking for time.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SOLVE</th>
<th>What is your mathematical plan? Show your work.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Use compound interest formula, $A = P(1 + \frac{r}{n})^{nt}$</td>
</tr>
<tr>
<td></td>
<td>$4000 = 2000\left(1 + \frac{0.05}{4}\right)^{4t}$</td>
</tr>
<tr>
<td></td>
<td>$2 = 1.0125^{4t}$</td>
</tr>
<tr>
<td>Step 2</td>
<td>Take log of both sides</td>
</tr>
<tr>
<td></td>
<td>$\log 2 = \log 1.0125^{4t}$</td>
</tr>
<tr>
<td></td>
<td>$\log 2 = 4t \log 1.0125$</td>
</tr>
<tr>
<td></td>
<td>$t = \frac{\log 2}{4 \log 1.0125}$</td>
</tr>
<tr>
<td></td>
<td>$t = 13.994408$</td>
</tr>
<tr>
<td></td>
<td>$t \approx 14$ yrs.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EXPLAIN</th>
<th>What was your thought process? How did you solve the problem?</th>
</tr>
</thead>
<tbody>
<tr>
<td>First, I...</td>
<td>determined the correct formula</td>
</tr>
<tr>
<td></td>
<td>inserted known values</td>
</tr>
<tr>
<td></td>
<td>simplified some</td>
</tr>
<tr>
<td>Then, I...</td>
<td>used logarithms and applied the power property</td>
</tr>
<tr>
<td>And finally, I...</td>
<td>solved for t and answered the question.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WRITE a conclusion. Make sure your conclusion answers the original question.</th>
</tr>
</thead>
<tbody>
<tr>
<td>It will take nearly 14 years for $2000 to double to $4000 at 5% interest compounded quarterly.</td>
</tr>
</tbody>
</table>
And one more😊

How long does it take an investment of $2000 to double if it is invested at 5% interest compounded quarterly?
**And one more Solution😊**

**Advanced Problem-Solving Organizer**

**THINK** - What is the question? What are you being asked to do? Rewrite the question.

How long to double an initial investment of $2000 to $4000? 
Looking for time.

**SOLVE** - What is your mathematical plan? Show your work.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use compound interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$</td>
<td>Take log of both sides: $\log A = \log P + n \log\left(1 + \frac{r}{n}\right)$</td>
<td>Solve for $t$: $t = \frac{\log A - \log P}{n \log\left(1 + \frac{r}{n}\right)}$</td>
</tr>
</tbody>
</table>

$A = 4000, P = 2000, r = 5\% = 0.05, n = 4$  
$A = 4000, P = 2000, r = 5\% = 0.05, n = 4$  
$t = 13.999408 \approx 14$ yrs.

**EXPLAIN** - What was your thought process? How did you solve the problem?

First, I...  
determined the correct formula  
inserted known values  
simplify some

Then, I...  
used logarithms  
and applied the power property

And finally, I...  
solved for $t$ and answered the question.

**WRITE a conclusion.** Make sure your conclusion answers the original question.

It will take nearly 14 years for $2000 to double to $4000 at 5\% interest compounded quarterly.
Sequence Map

- Is more simplistic.
- Provides structure for analyzing the story of the word problem.
- Incorporates highlighting/marking.
Acronyms

- CUBE
- CUBES
- KNWS
- SOLVE
- STAR
- UPS Check

More details of some are given in the Handout.
CUBE vs. CUBES

- **C**ircle the numbers.
- **U**nderline the important words.
- **B**ox the question.
- **E**liminate unnecessary information.

- **C**ircle the numbers.
- **U**nderline the question
- **B**ox any math action word.
- **E**valuate the solution steps needed and **E**liminate unnecessary information.
- **S**olve and check.
KNWS

- Helps students decode the given information, determine the question, and select an appropriate solution method.

- Faculty can evaluate student understanding and check for misconceptions before solving.

- It does not ask students to solve.
SOLVE

• Study the problem.
• Organize the facts.
• Line up a plan.
• Verify the plan with action.
• Examine the results.
STAR

• Search the word problem.
• Translate the word problem.
• Answer the word problem.
• Review the solution.
UPS Check

- **U**nderstand the problem.
- **P**lan how to solve the problem.
- **S**olve the problem.
- **C**heck your answer.
Final Thoughts

• Do not “just assign” new types of word problems; offer the students a strategy and then model it for them or work a problem together.

• Consider letting students work in groups when learning a new solving strategy.

• Consider asking students to explain or write about their thought process when first using a solving strategy.

• Different word problems may lend themselves to different solution strategies - your call!
Thank-you for attending!

If you would like more information please email me at

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Have a wonderful Thanksgiving!