Learning by Doing: Hands-on Activities for all levels of Mathematics

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**Algebra is Artistic and more**

From Wolfram: “A figure formed by taking a series of steps of length 1, 2, …, \(n\), with an angle \(\theta\) turn after each step. The symbol for a spirolateral is \(a_1 \cdots a_k \angle \theta\), where the \(a_i\)s indicate that turns are in the \(-\theta\) direction for these steps.” For students, this means absolutely NOTHING. But what’s nice is that you can use these as a way to teach rotational symmetry, reflections, patterns and rotation. I’ve used these in liberal arts math and also math for elementary teachers in the geometry section but also in our Natures of Math course where I talk about secret codes and messages. I’ve also introduced them into College Algebra when talking about transformations of graphs.

These spirolaterals in particular have 90° rotations. A sample of what 1 – 10 turns would like is below. We’re going to tweak this to make it personal for the students.

I start by giving them the code and then the word ALGEBRA. When they master that, they work on their own names to see which pattern it follows. Note that the pattern is Right – Down - Left – Up repeating the RDLU even if the numbers aren’t done. Just like waltzing, which I do in class. They laugh. It’s all good.

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Right Down Left Up Right Down Left

and so on until you see a pattern repeat or you return to your starting position.

Your Name and Secret Code

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ALGEBRA
ALGEBRA $\Rightarrow 1375291$
Finally – fractions and creativity combined! I certainly use this in my liberal arts math classes and future teacher classes. In addition, it ties into the idea that algebra is artistic and magical because we’re going to take a simple circle and create a truncated tetrahedron and then, we’ll combine twenty of them together to make an icosahedron!

**Directions:**

Cut out the 8” circle.

**Fold any point on the circumference to the center.**

Fold again so the fold touches the corner of the first fold and a point on the circumference touches the center.

**Fold the final section** in to the center.

This is your equilateral triangle. If it has an area of 1, let’s figure out what the other shapes and areas are! (Ask the students to fold and label the different shapes) Draw each on the graph paper or hand them the blank drawings and calculate the fractional parts from there.
Questions for you to ask the class:

1. Classify all the polygons that you have made:

2. Can you find all 10 concave polygons? Draw them on the graph paper.

Introducing fractions:

Call the area of the original equilateral triangle 1.

Then the small equilateral triangles have an area of ___________.

The isosceles trapezoids have areas of _____________ and _____________.

The rhombus has an area of __________.
1. Cut out, fold and assemble the cube.
2. Find the face with a 1 on it. Write the three numbers in each of the two rows that contain the 1.
   a. Add each set of numbers.
   b. Are both sums the same?
   c. Now add the three numbers in each of the four other rows on the same face.
   d. Do you always get the same sum?
3. Find the face with the 27 on it. Add each of the six rows of three numbers on this face. What did you find?
4. There are 24 different rows of three numbers on the six faces of the cube. Can you find them all? Are their sums the same?
5. One of the numbers from 1 to 27 is missing. What number is it and where is it?
6. So there are 13 rows and diagonals in the cube that contains this middle cube numbered ____. Can you find them all? Are their sums the same?
Students in all math classes write daily. Most of the time, it’s formulas or whatever we’ve already written on the board, but very rarely do we take time for real writing. I’ve done a variety of things from one minute notecard thoughts to journals every week to research papers, all in my math classes from developmental through calculus. Every one of my classes has pre-determined notes to fill out. I’ve guided their learning with sample problems and space for more answers. I’ve interspersed that with questions about the reading or textbook or a journal prompt about how they are doing. Here are just some samplings of what I do to engage students for insights on their cognitive and affective learning.

**First Day of Class Worksheet**

Name: ___________________  Major: ___________________

Home Phone: _______________  Work/Cell Phone: ___________________

Email: _______________________  Best way to contact you: _______________________

How many hours a week do you work?  ___________  ☐ PT  ☐ FT

Last math course you took.  ___________  When was it?  ___________

Do you have a computer at home?  ☐ Yes  ☐ No  Do you have internet access?  ☐ Yes  ☐ No

If not, what is your plan to access the web for homework?

☐ Campus  ☐ friend’s  ☐ public library  ☐ Other (be specific)  _______________

In addition to this class, what are your other time commitments for the next 16 weeks (other classes, family, work, kids, trips, etc.)

What are the three most important things you want to learn from this course?

1.

2.

3.

What will be your greatest strength in mastering the skills in this math class?

What will be your biggest challenge in mastering the skills in this math class?

What is the moment in your life when you felt the most pride in achievement?

Do you have any special needs, circumstances, issues and/or considerations you want me to know about?
**Journal Prompts:** These are great with notecards or even blue books. Sometimes I make the students keep a blue book for when I’m feeling they need to think about something else. Sometimes I buy a ton of notecards and hand them out.

- My best experience with math was when .....  
- My family feels that math is ....  
- Does mathematics or a math class scare you in any way? How and/or why or why not?  
- I want to be better at math so that I .....  
- When you make mistakes, what do you do first? Do you make corrections or ask questions? Why?  
- How often do you spend doing homework or reviewing your notes? What’s holding you back from doing more? Has something changed from the beginning of the semester?  
- Design two math bumper stickers: one serious and one funny.  
- What was your grade on a quiz, exam or homework? If you weren’t happy, what can you do to improve? If you were happy, why do you think you did so well? How did you prepare for the exam? Was the room quiet enough? Were there distractions, etc.?  
- Write and solve a word problem that involves negative numbers or fractions.  
- What are the benefits of journaling in a math class? What would you do to change it and why?  
- If math could be a shape or color, it would be ............... because .................................
- The most important or useful thing I’ve learned so far is ....  
- What questions are still unanswered at the end of this week?  
- Write a possible final exam question and solve it.  
- Write a note to an incoming student. What was the best thing(s) about this class? What would you change or improve either for yourself or for me? What advice do you have for them?