Dividing Fractions

This set of activity activities are designed to help students to structure their thinking so that they are afforded the opportunity to make sense of the idea of division of fractions. The goal of this collection of activities is to allow the division algorithm to emerge from quality thinking and reasoning about the mathematics needed to divide fractions. While students will engage in many (all?) of the Standards for Mathematical Practices, the primary practice that is the focus of these activities is Practice #8: Look for and express regularity in repeated reasoning. As a result of developing this and the other mathematical practices, division of fractions will become part of a well-connected network of understanding.

The activities in this series of lessons have just a few questions in each. However, each question will likely take much time and were designed to elicit much conversation, discussion, debate, explanation, etc. That is, students should be thinking deeply about what they are doing and why they are doing it. Teachers should be challenging students to explain what “it” means as students describe the methods they used to answer each question.

This is also an excellent example of why the worksheets alone may not accomplish the intended goal if the user does not understand the purpose. By combining the workshop experience with the solutions, my hope is that the worksheets will accomplish the intended goal: for students to engage in repeated reasoning for the purpose of creating an algorithm for dividing fractions.
Part 1

1. I love Cowboy Cookies. The recipe calls for 1 cup of flour. At my house, measuring cups are hard to find and on the day I wanted to bake Cowboy Cookies, I could only find a \( \frac{1}{3} \) cup measuring cup (see picture of my kitchen drawer). Describe how I can, as accurately as possible, measure 1 cup of flour using only the \( \frac{1}{3} \) cup measuring cup. Show and/or describe all reasoning needed to resolve the situation. What mathematical operation is at play in this situation?
2. Now suppose that the only measuring cup I could find is the 2/3 cup measuring cup. Remember that the recipe calls for 1 cup of flour. Describe how I can, as accurately as possible, measure 1 cup of flour using only the 2/3 cup measuring cup.

Show and/or describe all reasoning needed to resolve the situation. What mathematical operation is at play in this situation?

3. Describe the relationship between the following two operations as they relate to the cookie situations presented in #1 and #2.

\[ 1 ÷ \frac{1}{3} \text{ and } 1 ÷ \frac{2}{3} \]
4. Now suppose that I could only find a $\frac{1}{4}$ cup measuring cup. Describe how I can, as accurately as possible, measure 1 cup of flour using only the $\frac{1}{4}$ cup measuring cup. Show and/or describe all reasoning needed to resolve the situation. What mathematical operation is at play in this situation?

5. Now suppose that the only measuring cup I could find is the $\frac{3}{4}$ cup measuring cup. Remember that the recipe calls for 1 cup of flour. Describe how I can, as accurately as possible, measure 1 cup of flour using only the $\frac{3}{4}$ cup measuring cup. Show and/or describe all reasoning needed to resolve the situation. What mathematical operation is at play in this situation?
6. Describe the relationship between the following two operations as they relate to the cookie situations presented in #1 and #2.

\[ 1 \div \frac{1}{4} \text{ and } 1 \div \frac{3}{4} \]

7. In general, suppose I have a measuring cup that can hold (hypothetically) \( \frac{1}{b} \) cups of flour and that the recipe still calls for 1 cup of flour. How many filled \( \frac{1}{b} \) cups of flour will be needed to meet the recipe requirements? Express this situation in mathematical symbols.
8. Now suppose I have a measuring cup that contains (hypothetically) $\frac{a}{b}$ cups of flour and that the recipe still calls for 1 cup of flour. How many filled $\frac{a}{b}$ cups of flour will be needed to meet the recipe requirements?

Express this situation in mathematical symbols. Hint: use your reasoning in #1-3 to guide you.
Part 2

Use the **reasoning** developed so far to think through and to respond to each of the following situations. That is, refrain from resorting to the use of a procedure or algorithm. Rather, think about the relationships between these new situations and the situations you have thought about previously in this series of tasks.

1. A recipe calls for 2 cups of flour. You only have a 1/3 cup measuring cup. How many filled 1/3 cup measuring cups are needed to meet the requirements of the recipe?

2. A recipe calls for 3 cups of flour. You only have a 1/3 cup measuring cup. How many filled 1/3 cup measuring cups are needed to meet the requirements of the recipe?

3. A recipe calls for 2 cups of flour. You only have a 1/4 cup measuring cup. How many filled 1/4 cup measuring cups are needed to meet the requirements of the recipe?
4. A recipe calls for 3 cups of flour. You only have a 1/4 cup measuring cup. How many filled 1/4 cup measuring cups are needed to meet the requirements of the recipe?

5. A recipe calls for \(c\) cups of flour. You only have a 1/\(b\) cup measuring cup. How many filled 1/\(b\) cup measuring cups are needed to meet the requirements of the recipe?
6. A recipe calls for 2 cups of flour. You only have a 2/3 cup measuring cup. How many filled 2/3 cup measuring cups are needed to meet the requirements of the recipe?

7. A recipe calls for 3 cups of flour. You only have a 2/3 cup measuring cup. How many filled 2/3 cup measuring cups are needed to meet the requirements of the recipe?

8. A recipe calls for 2 cups of flour. You only have a 3/4 cup measuring cup. How many filled 3/4 cup measuring cups are needed to meet the requirements of the recipe?
9. A recipe calls for 3 cups of flour. You only have a 3/4 cup measuring cup. How many filled 3/4 cup measuring cups are needed to meet the requirements of the recipe?

10. A recipe calls for \( c \) cups of flour. You only have an \( \frac{a}{b} \) cup measuring cup. How many filled \( \frac{a}{b} \) cup measuring cups are needed to meet the requirements of the recipe?

11. Write a concluding statement that describes the results of the reasoning and work you have completed in Part 2 of this activity.
The following set of activities is designed to develop the idea of division of fractions from a proportional reasoning perspective.
Every morning when I wake up, I feed my two dogs, Tobie and Gracie. It seems like I am buying large bags of dog food so often that I wondered one morning...how many scoops of dog food do I deposit into their doggie dishes before running out of dog food?

I use a clear plastic cup (see picture) to scoop the dog food. I determined that it takes approximately 93 scoops (rounded to the nearest whole scoop) to exhaust the supply of dog food. I then need to go to the dog food store to buy a new 28 pound bag of food for Tobie and Gracie.

1. Draw a pair of line segments to represent the situation involving the number of scoops and the total amount of dog food in the bag. Label as much information on the segments as you can. You will use these segments to help you answer subsequent questions.

2. If I cut the “scoops” line segment into ____________ pieces, each piece represents ________________ copies of the total number of scoops or _____________ scoops.

3. To keep the situation in proportion (you may want to discuss what this means), cut the “pounds” line segment into ____________ pieces where each piece represents ________________ copies of the total number of pounds of dog food or _____________ pounds.

4. Represent, using fractions, the number of pounds of dog food that fit into one scoop. Explain how you know.
Activity 2 – The Dog Food Saga Continues

In Activity 1, the number of scoops of dog food was rounded to the nearest whole number of scoops. In reality, it takes \(92 \frac{3}{4}\) scoops before I run out of dog food and have to buy another 28 pound bag of dog food. As you respond to the items on this page, use fractions to represent all quantities.

1. Draw a pair of line segments to represent the situation involving the number of scoops and the total amount of dog food in the bag. Label as much information on the segments as you can. You will use these segments to help you answer subsequent questions.

2. If I cut the “scoops” line segment into _____________ pieces, each piece represents ______________ copies of the total number of scoops or _____________ scoops.

3. To keep the situation in proportion (you may want to discuss what this means), cut the “pounds” line segment into ______________ pieces where each piece represents ______________ copies of the total number of pounds of dog food or _____________ pounds.

4. Represent, using fractions, the number of pounds of dog food that fit into one scoop. Explain how you know.
Activity 3 – Baking Cookies

I love Cowboy Cookies. The recipe shown calls for \( \frac{2}{3} \) cup of flour. At my house, measuring cups are hard to find and on the day I wanted to bake Cowboy Cookies, I could only find a \( \frac{1}{2} \) cup measuring cup. Describe how I can, as accurately as possible, measure \( \frac{2}{3} \) cup of flour using only the \( \frac{1}{2} \) cup measuring cup.
Activity 4 – Coffee Blends

Some people like to create their own coffee blends by mixing together different kinds of freshly roasted and ground coffee beans. One website, www.thecaptainscoffee.com, recommends different blends for people to try. One such blend is shown.

Suppose a person has $5 \frac{7}{8}$ pounds of Ethiopian Harrar coffee that they wish to blend with Sumatran Mandeling (of which they have abundant supply). How many pounds of this blend can be made?