Driving the design of the STEM-Prep pathway

What’s meaningful to students and essential to success

Spring 2015

The creative principle [of science] resides in mathematics.
—Albert Einstein
(On the Method of Theoretical Physics. The Herbert Spencer Lecture, delivered at Oxford, June 10, 1933)

What is the STEM-Prep pathway?

This pathway is a sequence of courses designed to prepare developmental mathematics–level students interested in STEM careers to enter—and succeed in—calculus and other technical courses that require strong algebraic skills and a mastery of functions. The new courses under development to support the STEM-Prep pathway include Reasoning With Functions I and II.

Students begin the STEM-Prep pathway after completing the New Mathways Project developmental course Foundations of Mathematical Reasoning and its co-requisite Frameworks for Mathematics and Collegiate Learning.

With three semesters of coursework, students can complete this accelerated pathway to calculus:

- Semester 1: Foundations and Frameworks (which students take concurrently when they traditionally would be entering a beginning algebra course)
- Semester 2: Reasoning With Functions I
- Semester 3: Reasoning With Functions II

What’s the STEM-Prep content design team doing?

To identify the most appropriate content for the STEM-Prep pathway, we invited prominent leaders in mathematics education research and instruction to participate in a content design team.

This team convened in spring and summer 2014 through a series of phone conferences and face-to-face meetings facilitated by Frank Savina (STEM-Prep pathway coordinator, Dana Center) and Stuart Boersma (STEM-Prep pathway lead author, Central Washington University, Ellensburg, Washington).

The content design team began this process by drawing on its members’ experience in the classroom, knowledge of mathematics education research, and understanding of how undergraduate institutions approach calculus to identify the aspects of calculus that tend to be most demanding and difficult for students.

STEM-Prep Pathway Content Design Team

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1 The content design team’s process is presented in more detail in “They Will Need It for Calculus,” available via http://www.utdanacenter.org/higher-education/new-mathways-project/new-mathways-project-curricular-materials/stem-prep-pathway-i-and-ii/reasoning-with-functions-i-ii
While it is well known that weak algebraic skills can cause students difficulty in calculus, the content design team concluded that success in calculus was even more strongly dependent on a deep conceptual understanding of function processes and covariational reasoning.

**Some Difficult Concepts in Calculus**
- *Function notation*—including distinguishing input from output and variables from parameters; using different symbols to represent equal processes
- *Exponential and logarithmic growth*—including the fact that each is a function in its own right and also an inverse of the other
- *Communicating about change and rates of change*—including understanding what it means for negative quantities to increase or decrease; interpreting graphs of \( f, f', f'' \); understanding limiting behavior; identifying rate as a quantity in and of itself; examining change over entire intervals
- *Riemann sums*—including making connections between the use of technology and summation notation; using and conceptualizing approximation rules
- *Function as process*—including thinking of a function as a complete activity; understanding functions are processes that can be reversed and composed
- *Covariation*—including understanding dynamic geometric relationships; communicating change with correct language; understanding the chain rule and how compositions of functions transmit change
- *Limits and approximations*—including understanding the definite integral as an accumulator

**What’s meaningful to students and what’s essential to success?**

That is, how do we incorporate meaningful problem solving in mathematics coursework, and what masteries and understandings are essential to student success in calculus?

The content design team used these two driving questions to focus its thinking on the most crucial content for the STEM-Prep pathway. This focus, grounded in research,\(^2\) entails streamlining the traditional precalculus syllabus. While the team recognized the many content areas in traditional precalculus courses that are clearly helpful in preparing students for calculus, team members focused on identifying those content areas that were both immediately meaningful to the students and essential to student success.

The content—and the pedagogical approach—for the STEM-Prep courses *Reasoning With Functions I* and *II* were thus designed to better prepare students to master difficult concepts in calculus by giving them targeted skills and conceptual understanding to succeed.

The final content and sequencing of this course material was informed by current research in mathematics education and recommendations from the Mathematical Association of America’s outreach work to partner disciplines.\(^3\)

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\(^2\) See the Resources section at end of this document.

\(^3\) See References and Suggested Reading in the Resources section at the end of this document.
As the team continued to examine essential content more deeply, four overarching principles emerged. To succeed in calculus and beyond, it is crucial that students first achieve . . .

- **Deep understanding of the function process:** A strong conceptual understanding of the process view (rather than the action view) of function gives students a critical mathematical foundation to support future learning in STEM fields. By stressing the process view of a function, the STEM-Prep curriculum will prepare students to analyze function outputs on entire intervals of inputs, reason about inverting functions by reversing a process, and make stronger connections between the graph of a function and the function’s relationship to generalized inputs and outputs. Students will understand that a function is independent of a formula and that to conceptualize fully the notion of function composition, a process view of functions is essential.

- **Proficiency in covariational reasoning:** The ability to analyze two quantities simultaneously, how those quantities change, and how they covary enables students to better understand the unique and dynamic problem situations addressed by calculus and related disciplines. The STEM-Prep curriculum will provide students many opportunities to explore dynamic function relationships and help students more easily conceptualize the notions of an average rate of change and the transition between an average rate of change and an instantaneous rate of change.

- **Fluency in communication with functions and function notation:** Students will communicate orally and in writing as they analyze function behavior in multiple representations. The STEM-Prep curriculum will engage students at the notational level by having them directly examine the need for function notation and by requiring them to interpret to and from function notation. Such communication skills are essential to developing students’ mathematical knowledge and logical reasoning skills.

- **Facility with meaningful approaches to algebraic reasoning:** Students will engage with the curriculum content as they develop their algebra and problem solving skills within authentic STEM contexts. Students will create, explore, and interpret mathematical models and use algebra as a way of extracting additional information from a model or mathematical problem. This approach to algebra will give students an immediate appreciation of the usefulness of algebra and algebraic reasoning.

The content design team strongly believes that an active pedagogy seriously and intentionally built on these principles will help students succeed in calculus.

**What’s next?**

Since May 2014, Savina and Boersma have been refining the work of the content design and structure design teams and collaborating with authors on the writing process. Savina and Boersma continue to present on project progress at professional meetings of the Mathematical Association of America, American Mathematical Association of Two-Year Colleges, and STEMtech.

To receive updates on events and releases of materials through the monthly Dana Center Higher Ed In Brief, email us at mathways@austin.utexas.edu.
Resources

Learning Outcomes

Reasoning With Functions I: Student Learning Outcomes (Draft for Implementation Spring 2016): Developed by the content design team and Dana Center staff, these learning outcomes are available here: http://www.utdanacenter.org/wp-content/uploads/nmp_reasoning_with_functions1_learning_outcomes_sept20141.pdf

A more detailed description of the content for this course is available in the Brief Course Outline available via the NMP website, www.utdanacenter.org/mathways.

Reasoning With Functions II: A similar outline is in preparation.

References and Suggested Reading


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About this resource

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About the development of this document
The work on STEM Prep began in earnest in January 2014. This resource is one of several that will document the development of the STEM-Prep pathway, for which the Dana Center is developing two college-level transferable math courses—Reasoning With Functions I and Reasoning With Functions II. This pathway will move students from developmental math through college-level precalculus with learning outcomes specific to STEM preparation.

As the STEM-Prep development process continues, we plan to issue revisions and additional documents to reflect on—and clarify—the approaches and strategies the NMP is employing to support students in learning rigorous mathematics content and progressing to and through a program of study that leads to a certificate or degree.

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About the Dana Center
The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations.

For more information:
• about the New Mathways Project, see www.utdanacenter.org/mathways.
• about the Texas Association of Community Colleges, see www.tacc.org.