Teaching Math using Algebra in the Age of Free, Ubiquitous CASs & Other Tools

MikeLucke St Louis Community College
AMATYC November 17, 2016
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A Brief Overview

• The advance of technology in teaching math is an old story.
• What is algebra anyway?
• What can technology do to help foster student understanding of algebra/mathematics?
• How do we take advantage of the tools?
• What does it mean for our teaching & assessment?
Technology's Advance in Math Ed is NOT New!

- NAEP Results suggest that student use of G.C.s in the early 1990s hurt performance, but that this trend reversed as G.C.s became more widely used/understood by instructors. Similar results have appeared with CASs *
- Keith Devlin has argued in many formats that symbolic algebra itself is a technological innovation for problem solving that can obscure the real work of the mathematics being done.**

*Chazan, D., Levy, A. & Others (2003) in Results & Interpretations the 2003 Mathematics Assessment of the National Assessment of Educational Progress. NCTM.
**For example: http://devlinangle.blogspot.com/2016_04_01_archive.html
What IS Algebra Anyway?

In the "Old Days":

Multiplication of Large Numbers:

- Manual calculation
- Sliding rulers

Today?

WolframAlpha

<table>
<thead>
<tr>
<th>Factor x + 115</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result: x^2 + 115</td>
</tr>
</tbody>
</table>

Mathematics Changes with Technology

<table>
<thead>
<tr>
<th>Term</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equals Sign</td>
<td>1662</td>
</tr>
<tr>
<td>First &quot;Computer Proof&quot;</td>
<td>1956</td>
</tr>
<tr>
<td>Cartesian Plane</td>
<td>1637</td>
</tr>
<tr>
<td>Two Column Proof</td>
<td>1976</td>
</tr>
<tr>
<td>Computer Algebra System</td>
<td>1954</td>
</tr>
<tr>
<td>Order of Operations</td>
<td>1954</td>
</tr>
<tr>
<td>Slide Rules</td>
<td>1976</td>
</tr>
<tr>
<td>Zero</td>
<td>1994</td>
</tr>
<tr>
<td>Flux Sign</td>
<td>1814</td>
</tr>
<tr>
<td>Hand-held Calculator</td>
<td>1982</td>
</tr>
</tbody>
</table>

WATM: a set of theorems used to prove properties of numbers and operations with numbers and operations with numbers.
Multiplication of Polynomials by Monomials.

We have \( a (b + c) = ab + ac \); (§ 41)
and \( a (b - c) = ab - ac \). Hence,

101. To Multiply a Polynomial by a Monomial,

Multiply each term of the polynomial by the monomial,
and connect the partial products with their proper signs.

Find the product of \( ab + ac - be \) and \( abc \).

\[
\begin{align*}
ab + ac - be & \\
abc & \\
\hline
a^2b^2c + a^2bc^2 - ab^2c^2 \\
\end{align*}
\]

Note. We multiply \( ab \), the first term of the multiplicand, by \( abc \),
and work to the right.

Exercise 21.

Multiply:
1. \(5a + 3b\) by \(2a^2\).
2. \(ab - be\) by \(5a^2bc\).
3. \(ab - ac - be\) by \(abc\).
4. \(6a^2b - 7a^2bc\) by \(a^2bc\).
5. \(x^2 - z\) by \(-3x^2y^2\).
6. \(x^2 + 2y^2 - z\) by \(-3x^2\).
7. \(a^2 + b^2 - c^2\) by \(a^2bc^2\).
8. \(5a^2 - 3b^2 + 2c^2\) by \(4ab^2c^2\).
9. \(abc - 3a^2bc^2\) by \(-2ab^2c\).
10. \(xyz^2 + x^2y^2z\) by \(-x^2yz\).
11. \(3x - 2y - 4\) by \(5x^2\).
12. \(3x^3 - 4y^2 + 5z^2\) by \(2x^2y\).
13. \(a^2x - 5a^2x^2 + ax^3 + 2x^4\) by \(ax^2y\).
14. \(-9a^3 + 3a^2b^3 - 4a^2b^2 - b^3\) by \(-3a^4\).
15. \(3x^3 - 2x^2y - 7xy^2 + y^3\) by \(-5x^2y\).
16. \(-4xy^2 + 5x^3y + 8x^3\) by \(-3x^2y\).
17. \(-3 + 2ab + a^3y^2\) by \(-a^4\).
18. \(-z - 2xz^2 + 5x^2yz^2\) by \(-3x^2yz\).
factor $x^3 + 125$
### Mathematics Changes with Technology

<table>
<thead>
<tr>
<th>Item</th>
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<tbody>
<tr>
<td>Equals Sign</td>
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<td>First &quot;Computer Proof&quot;</td>
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</tr>
<tr>
<td></td>
<td>1890</td>
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NCTM: a set of concepts and techniques tied to the representation of quantitative relations and as a style of mathematical thinking for formalizing patterns, functions, and generalizations (PSSM 2000)

Adding it Up: representing, abstracting, and generalizing relationships among numbers and operations with numbers (2001)

What do YOU think?
What can Technology do to Develop Understanding of Algebra?

FREE Tools Available Online:

• http://www.wolframalpha.com
• http://www.symbolab.com
• http://www.desmos.com
• http://www.geogebra.org
How Can We Use These Tools?

- These tools allow us to ask new questions
- It obligates us to ask better questions
- Consider these examples:
Activities

- Function Carnival: https://student.desmos.com/?prepopulateCode=hy6dq
- Central Park: https://student.desmos.com/?prepopulateCode=7ra2
Activities

The population of the US in 2010 was 310 million people; in 2016 it's 329 million people. Find two models that fit this data. In what years will they agree?

Create a model for the thawing time of a turkey, based on wolframalpha queries.

Use function transformations to adjust the look/placement of an image.
Assessments

- Write the equation that produces the following graph:
- Seth answers the following question this way; find the mistakes
- Find the extraneous solution to the equation
- Give an example of a function that looks exponential for small x-values, but then tapers off as x gets larger:
- Which line has a steeper slope?
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Maybe Algebra isn't all there is...

- What if algebra stopped being a subject/topic and returned to being a tool?
- We could focus on asking questions and building models
- What might that look like?

https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeljimZxnhgwoVsWkTQEsfqouLWNCC6Z4/pub?output=html
Scary...

No solutions manuals

Fewer off-the-shelf resources

More challenging assessments and rubrics

Harder to articulate or quantify what students are learning

It's not necessarily what we fell in love with!
But...

It has great authenticity and integrity

Changing teaching is a marathon, not a sprint

It can have a significant impact on student understanding and engagement
Questions?
Comments?

hlueke@stlcc.edu
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