ENHANCING WORKING WITH RATIONAL EXPRESSIONS AND SOLVING RATIONAL EQUATIONS

CH 1

1. Given that $f(x)=\dfrac{x-4}{2-x}$, find $f\left(-\dfrac{2}{3}\right)$.

2. Let $g(x)=\dfrac{\dfrac{5}{x-2}+\dfrac{3}{x}}{\dfrac{1}{x}-\dfrac{4}{x-2}}$, find $g(a+1)$.

3. Find k such that the slope of the line containing $\left(-\dfrac{3}{2},k\right)$ and $\left(\dfrac{1}{4},-\dfrac{4}{5}\right)$ is $\dfrac{8}{9}$.

QUIZ CH 2

Let $f(x)=\dfrac{3-x}{x+1}$ and $g(x)=\dfrac{4}{5x-3}$.

Find the following:

a. $f(x)-g(x)$ and its domain

b. $f(g(x))$ and its domain

c. $g(f(2))$

ENHANCING WORKING WITH RATIONAL EXPRESSIONS AND SOLVING RATIONAL EQUATIONS

CH 3

1. Simplify.

a. $\frac{x}{x+1}-\frac{x-4}{x-1}$

2. Solve.

a. $\frac{1}{r}+\frac{2}{1-r}=\frac{4}{r^2}$

b. $\left(\frac{x}{x+2}\right)^2=\frac{4x}{x+2}-4$

c. $x^{-4}-13x^{-2}+36=0$

ENHANCING WORKING WITH RATIONAL EXPRESSIONS AND SOLVING RATIONAL EQUATIONS

CH 4

1. Let $f(x) = \dfrac{2x+1}{x-3}$. Find the following:

a. domain

b. $f(0)$

c. zeros of f(x)

d. x such that $f(x) = -\dfrac{2}{5}$

e. x such that $f(x) < 2$

2. Solve.

$$\frac{4-x}{x-1} < x$$

3. Find the domain of

$$f(x) = \sqrt[4]{\frac{1}{x} - \frac{1}{2x-1}}$$

ENHANCING WORKING WITH RATIONAL EXPRESSIONS AND SOLVING RATIONAL EQUATIONS

CH 5

1. Simplify.

$$\log\left(1-\frac{1}{x}\right)-\log\left(1+\frac{1}{x}\right)$$

2. Solve.

$$5^{\left(\frac{1}{x}-\frac{2}{x-2}\right)}=125$$

3. Find the domain of

$$f(x)=\log_3\left(\frac{x+1}{x-3}+2\right)$$

ENHANCING WORKING WITH RATIONAL EXPRESSIONS AND SOLVING RATIONAL EQUATIONS

CH 6

1. Solve by substitution or elimination or a combination of both.

a. $$\frac{2x-1}{3}+\frac{y+2}{4}=4$$
$$\frac{x+3}{2}-\frac{x-y}{3}=3$$

b. $$\frac{2}{x}+\frac{3}{y}-\frac{2}{z}=-1$$
$$\frac{8}{x}-\frac{12}{y}+\frac{5}{z}=5$$
$$\frac{6}{x}+\frac{3}{y}-\frac{1}{z}=1$$

College Algebra/ Precalculus Exponential Functions

1. **True or False? Justify your answer.**

a. If y = 4 is a horizontal asymptote of an exponential function $f(x) = a^x$, then $as\ x \to \infty,\ f(x) \to 4$ or $as\ x \to -\infty,\ f(x) \to 4.$

b. Let $f(x) = a^x$. If $f(x) \to 0\ \ as\ x \to -\infty$, then $a > 0.$

c. Let $y = -3$ be a horizontal asymptote of an exponential function $f(x) = a^x$, $a > 1$. Then the end behavior of the exponential function is described by $as\ x \to \infty,\ f(x) \to -3.$

2. **Complete the statements.**

a. If $f(x) = a^x, a > 1$, then $\lim_{x\to\infty} a^x =$

b. If $f(x) = a^x, a = 1$, then $\lim_{x\to\infty} a^x =$

c. If $f(x) = a^x, 0 < a < 1$, then $\lim_{x\to\infty} a^x =$

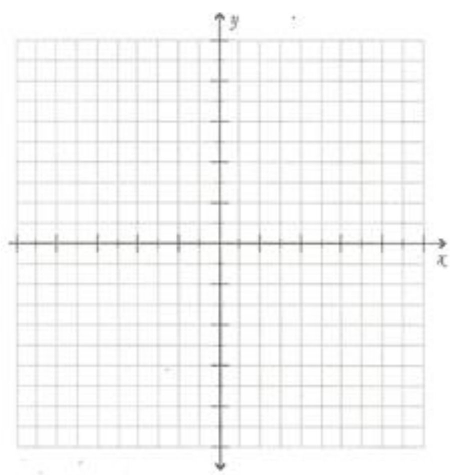
3. Sketch the graph of of $f(x) = \begin{cases} -e^{-x} & x < 0 \\ e^x & x \geq 0 \end{cases}$. Complete the statements below.

$as\ x \to \infty,\ f(x) \to$

$as\ x \to -\infty,\ f(x) \to$

$as\ x \to 0^-,\ f(x) \to$

$as\ x \to 0^+,\ f(x) \to$



Is the statement $as\ x \to 0,\ f(x) \to 1$ true?

College Algebra/ Precalculus Logarithmic Functions

1. Suppose that $G(x) = \log_3(2x+1) - 2$.

a. What is the domain of G?

b. What does G(x) approach as $as\ x \rightarrow -\frac{1}{2}^{+}$?

c. What is the equation of the vertical asymptote of the function?

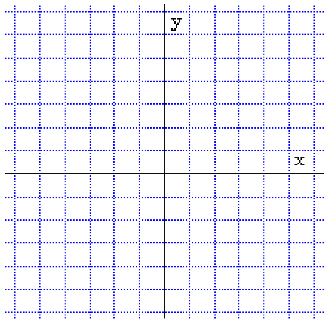
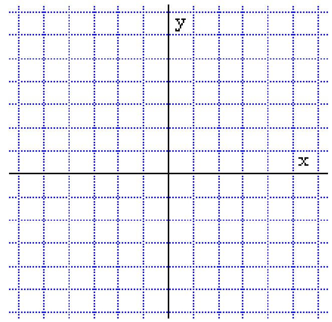
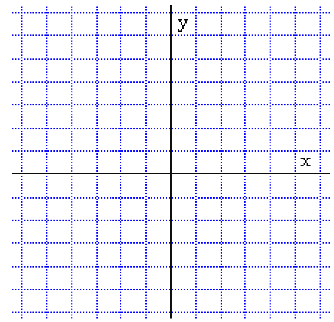
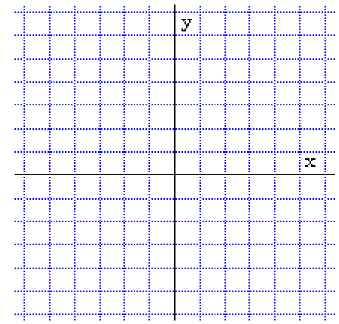
2. Use the properties of logarithms and transformations to describe how the graph of the given function compares to the graph of $f(x) = lnx$.

a. $g(x) = \ln(-x)$

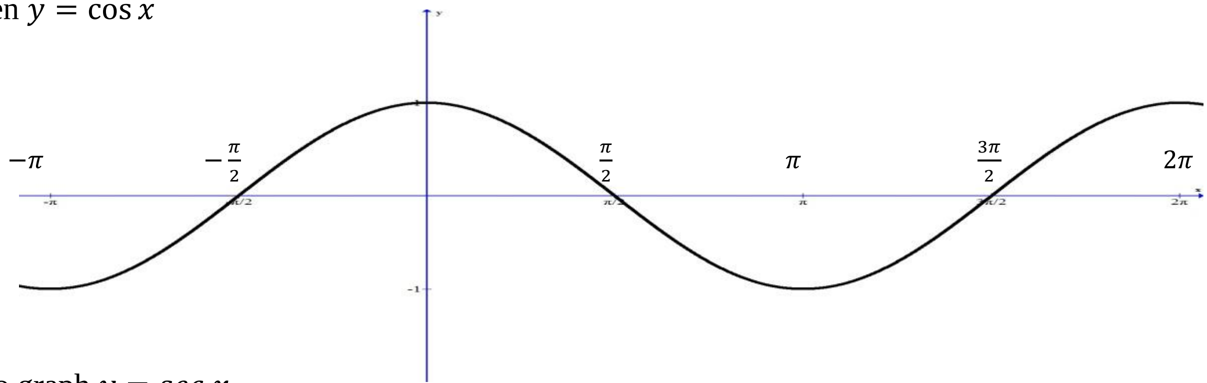
b. $h(x) = \ln\left(\frac{x}{e^2}\right)$

How do the given transformations impact the end behavior of the parent function $f(x) = \ln(x)$?
Do either of these transformations affect the equation of the vertical asymptote?

College Algebra **Domain/Range**

Function	$y=3$	$f(x)=(x-2)^2+1$	$f(x)=1-\|x\|$	$y=\sqrt{x+1}$
Graph				
Domain				
Range				
Complete the statements	$y\to$ $\quad as\ x\to\infty$ $y\to$ $\quad as\ x\to-\infty$	$f(x)\to$ $\quad as\ x\to\infty$ $f(x)\to$ $\quad as\ x\to-\infty$ $f(x)\to$ $\quad as\ x\to 2$	$f(x)\to$ $\quad as\ x\to\infty$ $f(x)\to$ $\quad as\ x\to-\infty$ $f(x)\to 1$ $\quad as\ x\to$	$y\to$ $\quad as\ x\to\infty$ $y\to$ $\quad as\ x\to 3$ $y\to$ $\quad as\ x\to-1^+$

1. Given $y = \cos x$



Use it to graph $y = sec\ x$

Step 1: Note that $\sec x = \frac{1}{\cos x}$ so start by graphing $y = \cos x$ as before but dotted or lightly.

Step 2: Where does $\cos x = 0$? What does this tell us about the domain of $y = \sec x$?
Wherever the cosine graph crosses the x-axis you get $\cos x = 0$, so $\sec x$ will be undefined there. Put in vertical asymptotes in those places.

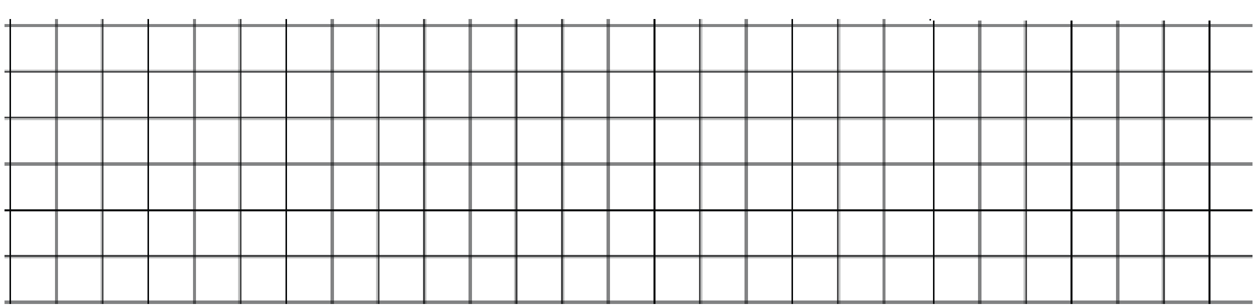
Step 3: Plot some points for $y = sec\ x$. Start with the values of x that give the maximum and minimum y values for $y = \sin x$. Then plot some additional points.

x	$-\pi$	0	π	$-\frac{3\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$
$y = \cos x$							
$y = \sec x$							

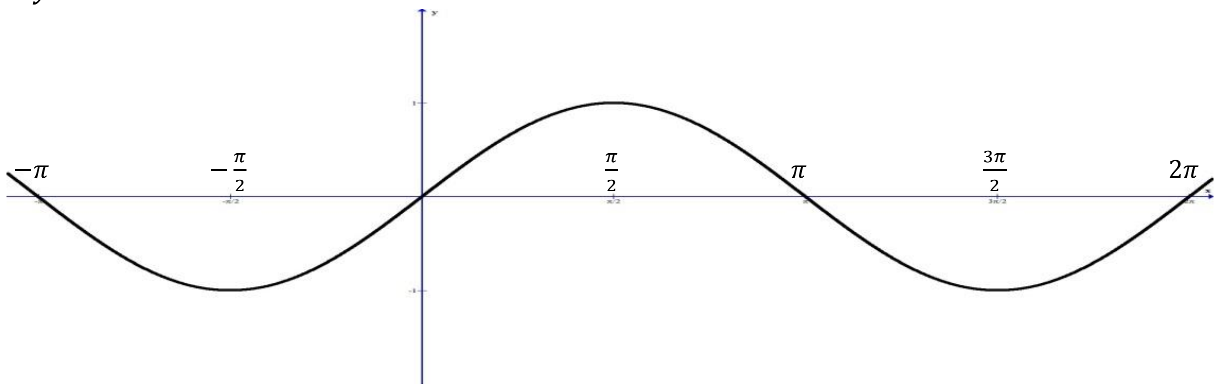
Step 4: The two graphs share the high and low points, but what happens at the places where $\sin x = 0$?

Start with $x = \frac{\pi}{2}$. What happens to $\frac{1}{\sin x}$ as you get closer and closer from the right side, in other words, as $x \to \frac{\pi}{2}^-$, $y \to$_______. And $x \to \frac{\pi}{2}^+$, $y \to$_______.

Use this behavior near the asymptotes to complete the remainder of the graph.



2. Given $y = \sin x$



Use it to graph $y = \csc x$

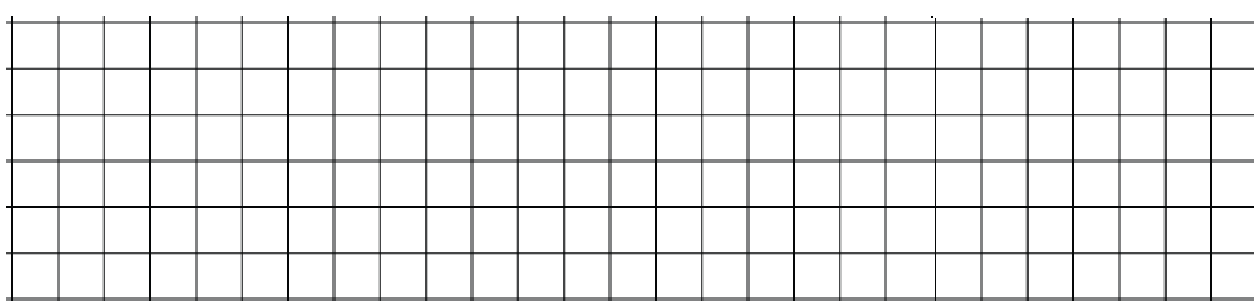
Step 1: Note that $\csc x = \frac{1}{\sin x}$ so start by graphing $y = \sin x$ as before but dotted or lightly.

Step 2: Where does $\sin x = 0$? What does this tell us about the domain of $y = \csc x$?
Wherever the sine graph crosses the x-axis you get $\sin x = 0$, so $\csc x$ will be undefined there. Put in vertical asymptotes in those places.

Step 3: Plot some points for $y = \csc x$. Start with the values of x that give the maximum and minimum y values for $y = \sin x$. Then plot some additional points.

x	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$-\frac{3\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$
$y = \sin x$							
$y = \csc x$							

Step 4: The two graphs share the high and low points, but what happens at the places where $\sin x = 0$?
Start with $x = 0$. What happens to $\frac{1}{\sin x}$ as you get closer and closer from the right side, in other words, as $x \to 0^-, y \to$_______. And $x \to 0^+, y \to$_______.
Use this behavior near the asymptotes to complete the remainder of the graph.



3. Given the graphs of $y = \sin x$ and $y = \cos x$, graph $y = \tan x$

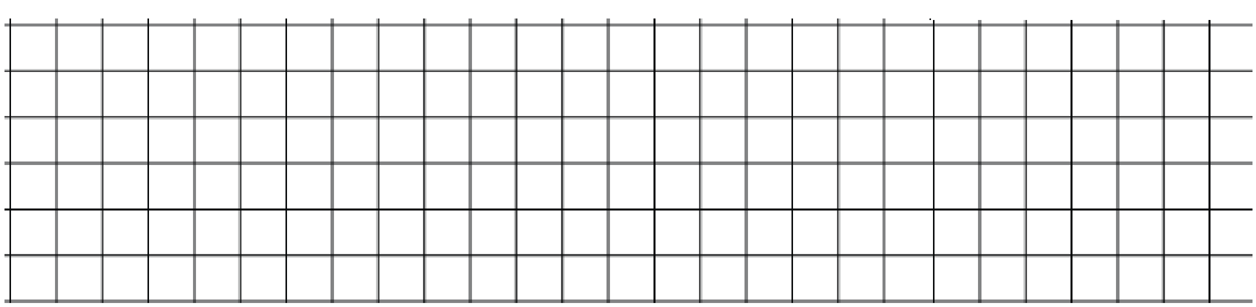
Step 1: Note that $\tan x = \frac{\sin x}{\cos x}$ so start by graphing $y = \cos x$ as before but dotted or lightly.

Step 2: Where does $\cos x = 0$? What does this tell us about the domain of $y = \tan x$?
Wherever the cosine graph crosses the x-axis you get $\cos x = 0$, so $\tan x$ will be undefined there. Put in vertical asymptotes in those places.

Step 3: Plot some points for $y = \tan x$. Start with the values of x that give the similar y values for $y = \sin x$ and $y = \cos x$. Then plot some additional points.

x	$-\frac{3\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$-\pi$	0	π
$y = \sin x$							
$y = \cos x$							
$y = \tan x$							

Step 4: Now consider what happens at the places where $\cos x = 0$?
Start with $x = \frac{\pi}{2}$. What happens to $\frac{\sin x}{\cos x}$ as you get closer and closer from the right side, in other words, as $x \to \frac{\pi}{2}^{-}$, $y \to$_______. And $x \to \frac{\pi}{2}^{+}$, $y \to$_______.
Use this behavior near the asymptotes to complete the remainder of the graph.



4. Given the graphs of $y = \sin x$ and $y = \cos x$, graph $y = \cot x$

Step 1: Note that $\cot x = \frac{\cos x}{\sin x}$ so start by graphing $y = \sin x$ as before but dotted or lightly.

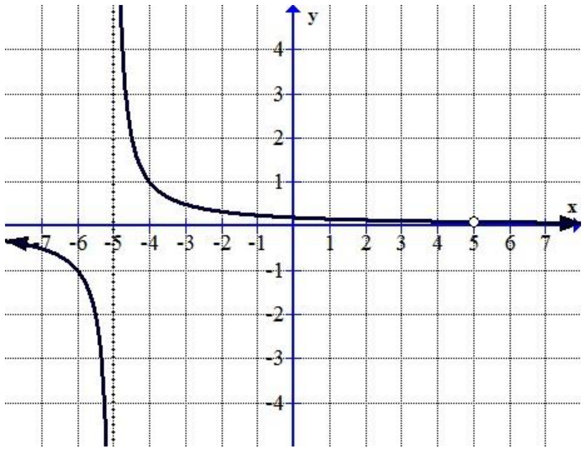
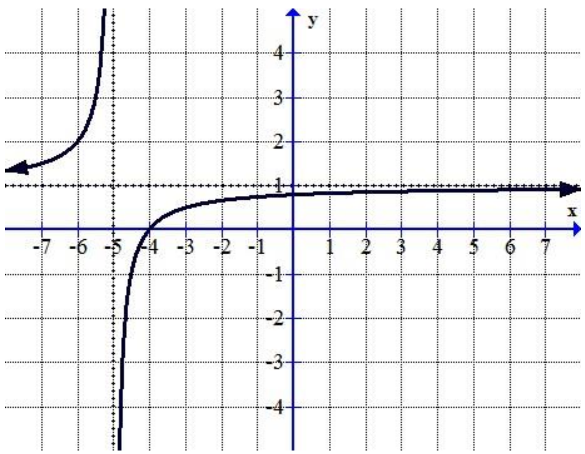
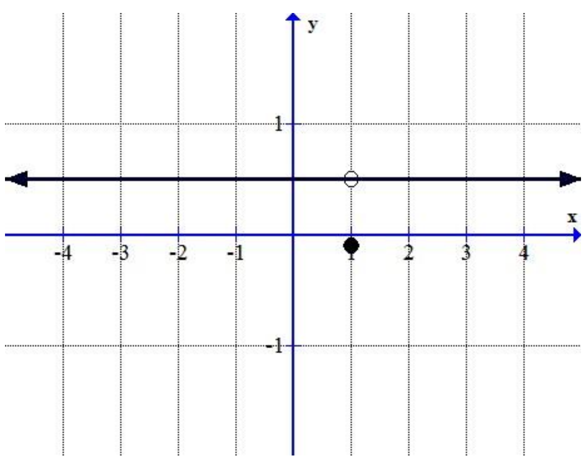
Step 2: Where does $\sin x = 0$? What does this tell us about the domain of $y = \cot x$?
Wherever the sine graph crosses the x-axis you get $\sin x = 0$, so $\cot x$ will be undefined there. Put in vertical asymptotes in those places.

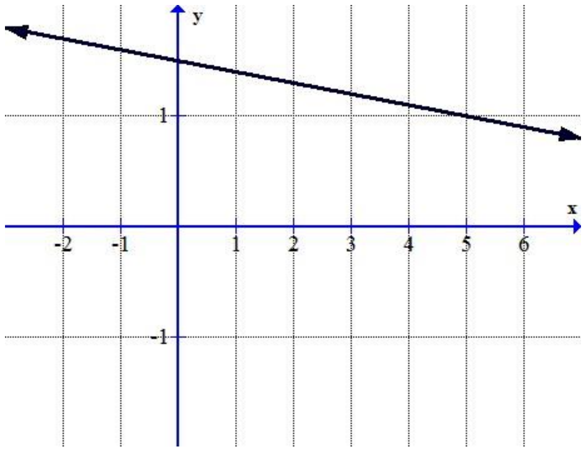
Step 3: Plot some points for $y = \cot x$. Start with the values of x that give the similar y values for $y = \sin x$ and $y = \cos x$. Then plot some additional points.

x	$-\frac{3\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$
$y = \sin x$							
$y = \cos x$							
$y = \cot x$							

Step 4: Now consider what happens at the places where $\sin x = 0$?
Start with $x = 0$. What happens to $\frac{\cos x}{\sin x}$ as you get closer and closer from the right side, in other words, as $x \to 0^-, y \to$_______. And $x \to 0^+, y \to$_______.
Use this behavior near the asymptotes to complete the remainder of the graph.

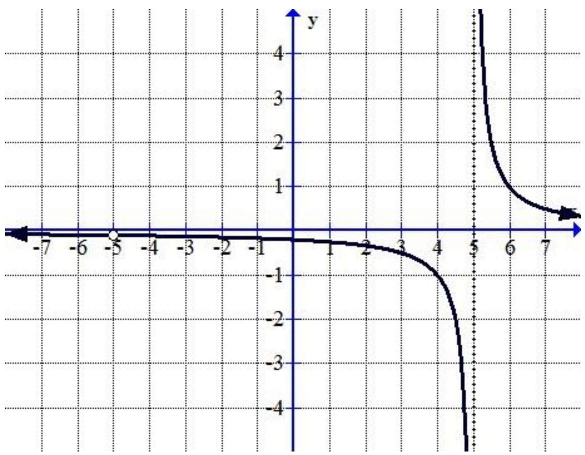
no x-intercept				
no y-intercept				
no intercepts				
odd				
average rate of change on $[-1,1]$ is constant				
even				

	$f(x) = \dfrac{x-5}{x^2-25}$	$as\ x \to 5, f(x) \to \dfrac{1}{10}$
	$f(x) = -\dfrac{1}{x+5} + 1$	$as\ x \to \infty, f(x) \to 1$ $as\ x \to -5^+, f(x) \to -\infty$
	$f(x) = \begin{cases} \dfrac{x-1}{2x-2}, x \neq 1 \\ -\dfrac{1}{10}, x = 1 \end{cases}$	$as\ x \to 5, f(x) \to \dfrac{1}{2}$



$$f(x) = -\frac{1}{10}x + \frac{3}{2}$$

$$as\ x \to 5, f(x) \to 1$$
$$as\ x \to \infty, f(x) \to -\infty$$



$$f(x) = \frac{x+5}{x^2-25}$$

$$as\ x \to 5^-, f(x) \to -\infty$$
$$as\ x \to -5, f(x) \to -\frac{1}{10}$$

LIMIT MATCHING

Designed by

Karen Summerson

Ivana Seligova

INSTRUCTIONS

Print 1 set for each group

Cut cards apart

Have students match

USES

Reviewing rational functions and limit concepts

College Algebra – Calc I

$f(0)=1$				
$f(-1)=0$				
$f(-1)=2$				
$f(1)=1$				
$f(2)=-1$				
$f(-1)=f(1)$				

College Algebra/Precalculus Piecewise Functions 1

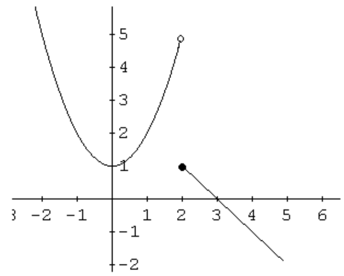
1. Use the graph of the function f(x) given below to find:

a. as $x \to \infty, f(x) \to$

b. as $x \to -\infty, f(x) \to$

c. as $x \to 2^-, f(x) \to$

d. as $x \to 2^+, f(x) \to$



2. Sketch a graph of a piecewise function f(x) given the properties.

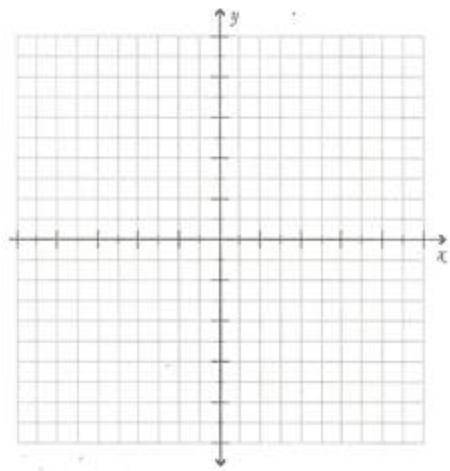
$as\ x \to -1^-, f(x) \to 1$

$as\ x \to -1^+, f(x) \to 2$

$as\ x \to 2^-, f(x) \to -1$

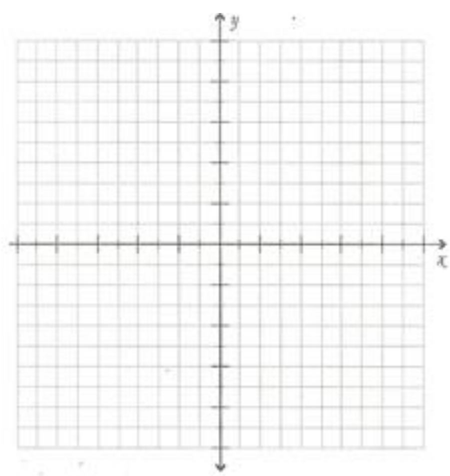
$as\ x \to 2^+, f(x) \to -2$

$f(-1) = 1 \quad and \quad f(2) = -2$



3. Graph the piecewise function. Then answer the limit questions.

$$f(x) = \begin{cases} x+1 & x \neq 3 \\ 2 & x = 3 \end{cases}$$



a. $as\ x \to 3^-, f(x) \to$

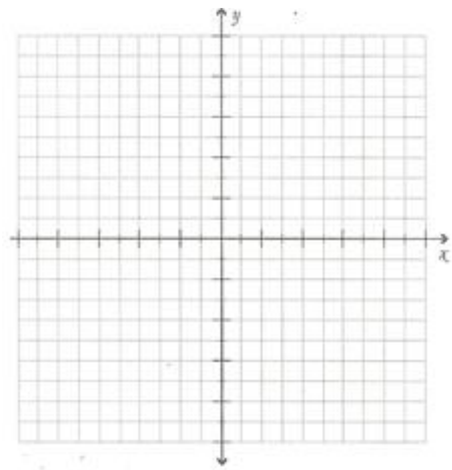
b. $as\ x \to 3^+, f(x) \to$

c. $f(3) =$

Precalculus **Piecewise Functions 2**

1. Graph the piecewise function then answer the limit questions.

$$f(x) = \begin{cases} \sin x & x \geq 0 \\ \cos x & x < 0 \end{cases}$$



a. $as\ x \to 0^-,\ f(x) \to$

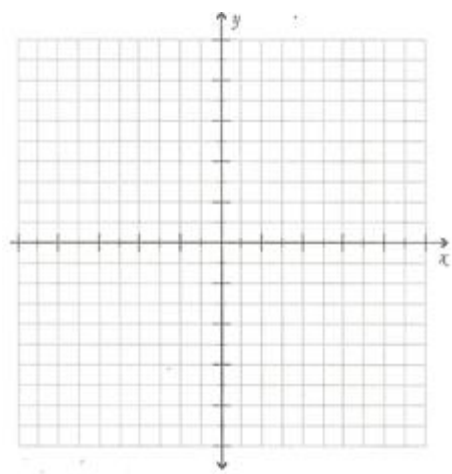
b. $as\ x \to 0^+,\ f(x) \to$

c. $as\ x \to 0,\ f(x) \to$

d. $f(0) =$

2. Graph the piecewise function then answer the limit questions.

$$f(x) = \begin{cases} \tan \frac{\pi x}{4} & -1 < x < 1 \\ x & x \leq -1 \quad or \quad x \geq 1 \end{cases}$$



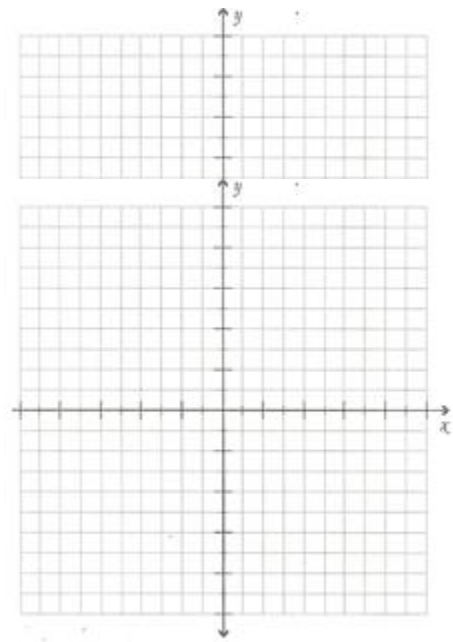
a. $as\ x \to 1^-,\ f(x) \to$

b. $as\ x \to 1^+,\ f(x) \to$

c. $as\ x \to -1^-,\ f(x) \to$

d. $as\ x \to -1^+,\ f(x) \to$

3. Rewrite the function $f(x)=\dfrac{|x-3|}{x-3}$ **so that it is defined piecewise.**



a. Sketch the graph of *f*(*x*).

b. Complete the statements:

$as\ x \to 3^{+},\ f(x) \to$

$as\ x \to 3^{-},\ f(x) \to$

$as\ x \to 3\ ,\ f(x) \to$

4. Let f(x) be a piecewise function and *b* a positive real number. Graph the piecewise function then answer the limit questions.

$$f(x)=\begin{cases} 0 & 0 \le x < b \\ b & b < x \le 2b \end{cases}$$

a. $as\ x \to b^{-},\ f(x) \to$

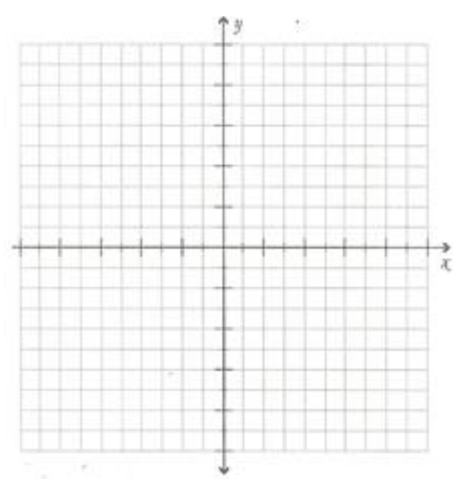
b. $as\ x \to b^{+},\ f(x) \to$

c. $f(0) =$

d. $f(b) =$

e. $f(2b) =$

5. Let $f(x) = [[x]]$ **and let b be any integer. Sketch the graph of *f*(*x*) and answer the questions below.**



a. $as\ x \to b^{-},\ f(x) \to$

b. $as\ x \to b^{+},\ f(x) \to$

6. Let $f(x)$ be defined piecewise. For what value of the constant c will $f(x)$ approach the same value as x approaches 3 from either side?

$$f(x) = \begin{cases} cx + 1 & if \quad x \le 3 \\ cx^2 - 1 & if \quad x > 3 \end{cases}$$

Piecewise Functions

1. $f(x)=\begin{cases}1-x & \text{if } x\le -1\\ \sqrt{x+1} & \text{if } x>-1\end{cases}$

2. $f(x)=\begin{cases}1+x & \text{if } x<0\\ x^2 & \text{if } x\ge 1\end{cases}$

3. $f(x)=\begin{cases}x^2+1 & \text{if } x<0\\ -x^2-1 & \text{if } 0<x\le 2\end{cases}$

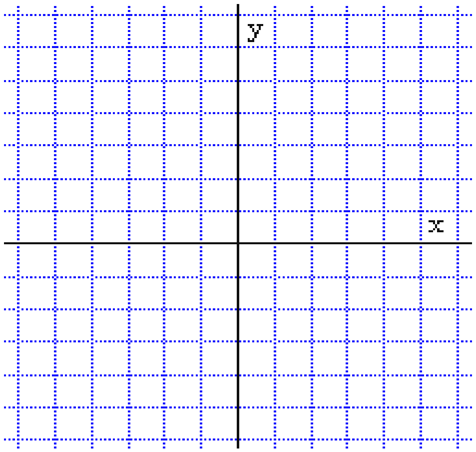
4. $f(x)=\begin{cases}-1 & \text{if } x\le -1\\ x^3 & \text{if } -1<x<1\\ 1 & \text{if } x\ge 1\end{cases}$

5. $f(x)=\begin{cases}2x & \text{if } x\ne 2\\ -1 & \text{if } x=2\end{cases}$

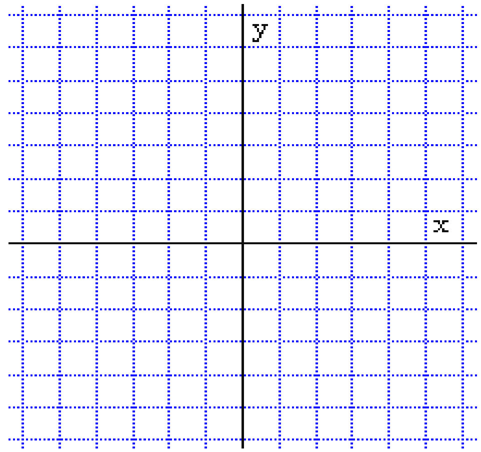
6. $f(x)=\begin{cases}x & \text{if } x<-1\\ -x^2 & \text{if } |x|\le 1\\ -x & \text{if } x>1\end{cases}$

Piecewise Functions – Sketches

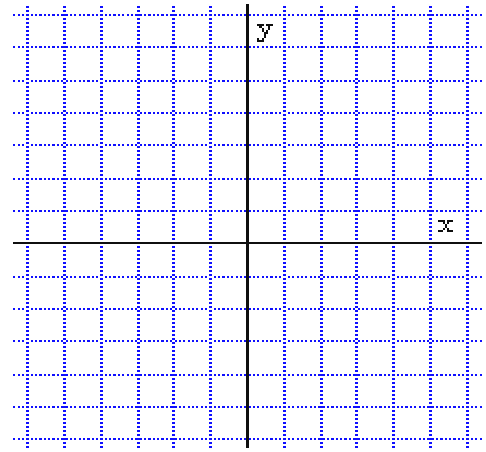
1. $$f(x)=\begin{cases}1-x & \text{if } x\le -1\\ \sqrt{x+1} & \text{if } x>-1\end{cases}$$



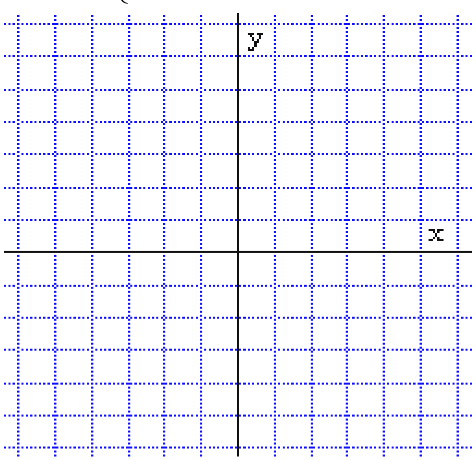
2. $$f(x)=\begin{cases}1+x & \text{if } x<0\\ x^2 & \text{if } x\ge 1\end{cases}$$



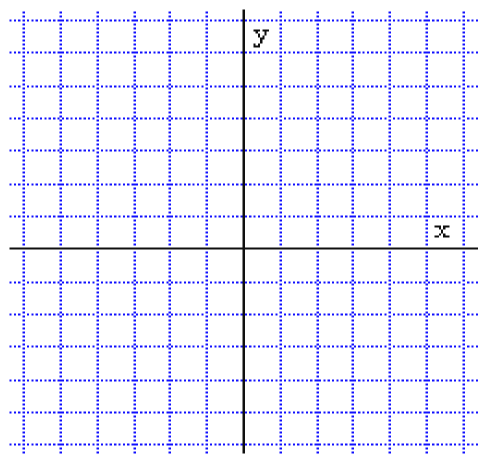
3. $$f(x)=\begin{cases}x^2+1 & \text{if } x<0\\ -x^2-1 & \text{if } 0<x\le 2\end{cases}$$



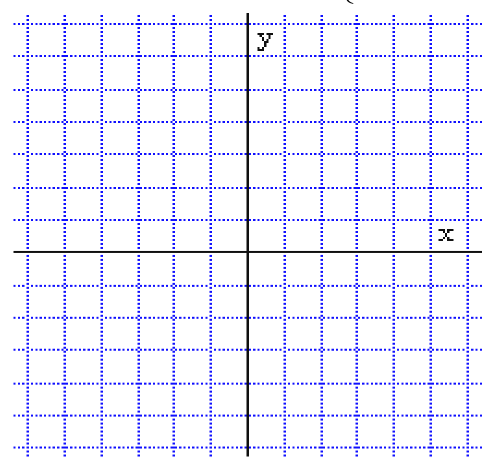
4. $$f(x)=\begin{cases}-1 & \text{if } x\le -1\\ x^3 & \text{if } -1<x<1\\ 1 & \text{if } x\ge 1\end{cases}$$



5. $$f(x)=\begin{cases}2x & \text{if } x\ne 2\\ -1 & \text{if } x=2\end{cases}$$



6. $$f(x)=\begin{cases}x & \text{if } x<-1\\ -x^2 & \text{if } |x|\le 1\\ -x & \text{if } x>1\end{cases}$$



Follow up Question:

1.

increasing on $(-1,\infty)$	$y \to 1 \;\; as\; x \to 0^-$			
$y \to 1 \;\; as\; x \to 0^-$				
$y \to -1 \;\; as\; x \to 0^+$				
increasing on $(-1,1)$				
$y \to 4 \;\; as\; x \to 2$				
decreasing on $(0,\infty)$				

Let $f(x)$ be a polynomial function. Complete the table below.

DEGREE EVEN/ODD	LEADING COEFFICIENT +/-	$as\ x \to \infty$	$as\ x \to -\infty$
EVEN		$f(x) \to \infty$	$f(x) \to$
		$f(x) \to -\infty$	$f(x) \to \infty$
ODD	**-**	$f(x) \to$	$f(x) \to$
EVEN	**+**	$f(x) \to$	$f(x) \to$
ODD		$f(x) \to \infty$	$f(x) \to$
	-	$f(x) \to -\infty$	$f(x) \to -\infty$

Chlorine Levels
Chlorine is often added to swimming pools to control microorganisms. The ideal level will be between 1 and 3 ppm. If the level of chlorine rises above 3 ppm (parts per million), swimmers will experience burning eyes. If the level drops below 1 ppm, there is a possibility that algae will form. Chlorine must be added to pool water at regular intervals. In a simplified description, if no chlorine is added to a pool during a 24-hour period, approximately 20% of the chlorine will dissipate into the atmosphere and 80% will remain in the water.

a. Determine a recursive sequence a_n, that expresses the amount of chlorine present after n days if the pool has a_0 ppm of chlorine initially and no chlorine is added.

b. If a pool has 7 ppm of chlorine initially, construct a table to determine the first day on which the chlorine level will drop below 3 ppm.

c. What happens after 7 days, as the chlorine level continues to drop?

d. What do you predict will happen to the chlorine level after several weeks if no new chlorine is added?

Adapted from Swokowski/Cole Pre-Calculus 12th ed., www.reference.com, www.troublefreepool.com, https://stab-iitb.org/newton-mirror/askasci, https://www.physicsforums.com/threads/swimming-pool-chlorine-questions.70792/

D: $(-\infty,\infty)$ R: $(0,\infty)$				
D: $(-\infty,0)\cup[1,\infty)$ R: $(-\infty,\infty)$				
D: $(-\infty,0)\cup(0,2]$ R: $[-5,-1)\cup(1,\infty)$				
D: $(-\infty,\infty)$ R: $[-1,1]$				
D: $(-\infty,\infty)$ R: $(-\infty,4)\cup(4,\infty)$				
D: $(-\infty,\infty)$ R: $(-\infty,0]$				