GROUP PROBLEM FOR YOUR TABLE

There are two parts to this problem. Each part is of equal value.

a) On each day of his life Denth always tells the truth or he always lies. Each week he has
   at least one "truth" day and at least one "lie" day. One day he made four of the five
   listed statements. Which statement didn't he make?
   1) Half my sister's age is an odd number.
   2) My sister's age is a prime number.
   3) My sister has a Florida driver's license.
   4) I sometimes tell lies.
   5) I always tell the truth.

b) In this part of the problem, you need to determine an antiderivative. This is it:

$$\int \frac{12x+17}{(6x^2+17x+10)(6x^2+17x+12)+2016}\, dx$$

Challenging and Exciting Students Through Math Competitions – Sample Problems
AMATYC Annual Conference Nov. 17, 2016
Presenters: Cathy Panik and Mary Beth Headlee

AMATYC Student Math League:

What is the value of $(\log_{624} 625)(\log_{623} 624) \dots (\log_6 7)(\log_5 6)$?
    A.   2
    B.   2.5
    C.   4
    D.   5
    E.   6

Answer: C

Math Olympics – Individual Exam (no calculator):

Find $\frac{dy}{dx}$, where $y = \int_{\pi}^{\ln x^2}(e^t - 1)dt$.

    A) $e^{x^2}\ln 2 - 1$
    B) $xe^{x^2} - \ln 2$
    C) $2x - 2x^{-1}$
    D) $2e^x \ln x - e^{\pi}$
    E) None of these

Answer: C

Math Olympics – Individual Exam (calculator required):

Calculate: $\lim\limits_{x \to 4} \dfrac{3^{\cos(x-4)+1}+7-x^2}{(\ln(x))(\tan(x-4))}$

    (Radian mode and round to three decimal places)

Answer: -5.771

Math Olympics – Team Competition (no calculator):

There are two parts to this problem, but they are not related.
Each part is of equal value.

a) The sum of two real numbers is 9 and the sum of their reciprocals is 3/5. What is the exact value of the sum of their squares?

b) Bag A contains 8 red marbles and 8 blue marbles. Bag B contains 8 red marbles, 8 blue marbles, and y yellow marbles, with $y > 0$. If y is chosen so that the probability of randomly taking two marbles of the same color without replacement from Bag A is the same as randomly taking two marbles of the same color without replacement from Bag B, then what is the value of y?

Answer: a) 51 b) 29

Math ACES – Non-Calculator Problem:

Decompose $\dfrac{7x+33}{x^3-27}$ into $\dfrac{A}{x-3} + \dfrac{Bx+C}{x^2+3x+9}$. Find A+B+C^3

(A) -125

(B) -121

(C) -5

(D) -1

(E) None of the above

Answer: A

Math ACES – Non-Calculator Problem:

Evaluate $\int (cos\theta - cos^3\theta)\sqrt{sin\theta - 2}\, d\theta$

(a) $\frac{2}{3}(sin\theta - 2)^{3/2} + \frac{2}{7}(sin\theta - 2)^{7/2} + C$

(b) $(sin\theta - 2)^{1/2} + (sin\theta - 2)^{5/2} + C$

(c) $\frac{2}{7}(sin\theta - 2)^{7/2} + \frac{8}{5}(sin\theta - 2)^{5/2} + \frac{8}{3}(sin\theta - 2)^{3/2} + C$

(d) $\frac{2}{5}(sin\theta - 2)^{5/2} + \frac{2}{3}(sin\theta - 2)^{3/2} + \frac{1}{2}(sin\theta - 2)^{1/2} + C$

(e) None of the above

Answer: C

Math ACES – Calculator Problem:

The probability that the first five cards dealt from a standard deck of cards results in a royal flush is $\frac{1}{649,740}$. If Lou uses a standard deck to deal 100 five-card hands, shuffling all 52 cards after each hand, approximately what is the probability that Lou deals at least one royal flush?

    (a) 0

    (b) 8.63579×10^{-30}

    (c) 2.78944×10^{-16}

    (d) 1.53896×10^{-4}

    (e) None of the above

Answer: D

Answers to the Group Problem for Your Table:

a) Denth did not make statement 4.

b) $\frac{1}{\sqrt{2015}} \tan^{-1} \left(\frac{6x^2 + 17x + 11}{\sqrt{2015}} \right) + C$