Camera Ready:
How to guide the problem-solving habits of students
by
Dr. James Vicich and Dr. Phil Clark

Session Number S052

“Expert problem solvers have access to rich, well-connected knowledge of mathematical concepts and possess confidence following a long history of successful problem solving. They also have an ability to imagine and conjecture possible solution paths, to monitor their progress and dynamically revise or abandon solution paths, and to verify that a solution is reasonable and makes sense. In contrast, developmental mathematics college students rarely plan a solution in advance, may demonstrate an inability to consistently monitor their progress, and have varying degrees of success recognizing that a solution attempt is not progressing toward the desired goal.\(^\text{19}\) When their initial strategy is not productive, these students have difficulty switching to an alternative strategy. Faculty and students need to take these characteristics into consideration and employ and engage in classroom activities that focus on boosting students’ confidence and building a reservoir of problem-solving strategies. When students are given opportunities to use multiple approaches to solve problems, they come to recognize that mathematics is more than computation or getting the single right answer—it is a balance of process and product—a combination of good thinking and meaningful answers.”

Quote from *Beyond Crossroads*, pp 22 – 23, (Blair, 2006)

This work was supported in part by MSP grant #1103080 through the National Science Foundation. Opinions expressed are those of the authors and not necessarily those of the NSF.

Oct 29, 2016
©Dr. James Vicich james.vicich@gmail.com,
These materials are selected from a paper under review for publication and may not be duplicated by any method.
This paper is based on a synthesis of findings from a series of design experiments (as described by Schoenfeld (2014a) and Cobb, et al. (2003)) for the purpose of having research inform teacher training as well as students’ mathematical performance.

The idea of camera ready solutions can have a significant impact on the social constructs of a math classroom and promote active engagement for all students in the classroom. Schoenfeld (2014b) claims that there’s consistent evidence that classrooms that produce powerful mathematical thinkers have these five properties:

- *High quality content and practices.*
- *Meaningful, carefully structured challenge.* Solving complex problems takes perseverance; students should neither be spoon-fed nor lost.
- *Equitable opportunity.*
- *Students as sense makers.*
- *A focus on building and refining student thinking*

What do you mean when you say you want student work to be camera ready? What elements are crucial/necessary/needed?

- Neatly organized (large enough to read easily with appropriate spacing).
- Thorough (All steps are shown to convince everyone in the room).
- All figures labeled and referenced properly using correct notation.
- Coherent solutions.
- Oral presentation skills are also required of the speaker (See Vicich (2007)) which include: Be poised, maintain a body position so audience can see your work, items 1 & 2 from list above, use appropriate voice (clear & loud enough for the room), maintain eye contact with audience.

*Note: All elements are crucial/necessary.*

How do teachers help their students understand what is meant by camera ready?

On the first day you may pair students to reduce anxiety and fear of speaking. Assign an open-ended question (see p.9 of this packet for criteria of task selection). Create a safe-environment for critiquing students’ work: “We respect the student as a person and critique only the mathematical ideas presented.” Prior to any student presentations, require that students read the problem-solving template (see pp. 15 – 16 of this packet) (Vicich & Perales, 2014) and become familiar with productive problem-solving behaviors. Require that students make a written conjecture about the answer prior to solving the problem, and have students include a verification that the answer makes sense. Ask members of the class to find something they like about the solution, and then ask for recommendations to improve the presentation.
What research has helped influence your notion of camera ready?

Essentially, camera ready is an extension of paired-board work (Vicich, 2007) using the doc camera rather than the white board as the medium. Read Schoenfeld’s (2013, 1992) theoretical framework for categorizing problem-solving behaviors, Vicich’s Diagnostic Instrument for Analyzing Problem-Solving Behaviors (Vicich & Perales, 2014), and Arcavi et al. (1998) description of Schoenfeld’s problem solving class at Berkeley in which the ultimate authority is the mathematics itself. Additionally, read Kazemi’s (1998) paper on sociomathematical norms and classroom discourse also informed my development of “camera ready”.

What does a teacher need to have (or need to know or be aware of) to effectively use the term camera ready in his/her own classroom?

A teacher must be able to detect various levels of performance including the depth of conceptual & procedural understanding and also be able to diagnose problem solving skills (see Vicich, 2014, found on p. 14 of this paper) to select student work for display. In this way a teacher can accomplish two goals: (1) acknowledge good first steps in problems solving such as initial engagement, planning and making conjectures even though the student may not be able to complete the problem correctly; and (2) to share different solution pathways for the purpose of building students’ resource knowledge.

Silverman and Thompson (2008) describe two aspects of understandings that empower teachers in helping students learn. First, key developmental understandings (KDU) (introduced by Simon (2002)) refer to the deep conceptual understandings of mathematics that carry through an instructional sequence and relate to a network of ideas and mathematical relationships. Secondly, Mathematical Knowledge for Teaching (MKT) (described by Ball, Thames & Phelps (2008)) refer to a teacher’s awareness of actions that empower students’ learning of key developmental understandings and why those actions are effective. A teacher’s own collection of KDU’s combined with her/his MKT are instrumental for implementing any instructional strategy and in particular creating an interactive classroom generated by using the doc camera as a vehicle for students making sense of their mathematical thinking and that of others. Boaler (2016) describes four goals of mathematics instruction for students as including opportunities for active engagement, equity, individual accountability and high expectations for written work.

Stein and Smith (2011) describe a particular set of teacher behaviors that facilitate effective classroom discourse and inquiry-based instruction. These include:

- **Anticipating** what students will do--what strategies they will use--in solving a problem;
- **Monitoring** students’ work as they approach the problem in class;
- **Selecting** students whose strategies are worth discussing in class;
- **Sequencing** those students’ presentations to maximize their potential to increase students’ learning; and
- **Connecting** the strategies and ideas in a way that helps students understand the mathematics learned.

Oct 29, 2016
©Dr. James Vicich  james.vicich@gmail.com,
These materials are selected from a paper under review for publication and may not be duplicated by any method.
Social norms are the “characteristics of the classroom community and document regularities in classroom activity that are jointly established by the teacher and students” (Cobb, et al., 2001, pp 122-123). Classroom social norms include:

- Students question each other’s thinking;
- Students explain their ways of thinking;
- Students work together to solve problems; and
- Students see making mistakes as a natural part of learning.

How do you see camera ready as influencing the social norms of the classroom? What is it about camera ready that helps a teacher establish good social norms?

The presentation of a solution on a large screen (or white board) immediately creates a public forum for sharing one’s thinking publically. The audience fully participates in determining whether or not the mathematical argument is both convincing and correct. Students and presenter all have access to the visual information concurrently. The presenter can point to items in the solution while responding to audience questions or criticisms. So the doc camera/projector/screen calls for the audience’s active participation in critiquing the solution while the validity of the mathematics becomes the focus of the classroom discourse.

Like classroom social norms, sociomathematical norms are “normative aspects of students’ activity that are specific to mathematics” (Cobb, et al., 2001, p 124). Sociomathematical norms include:

- Students ask each other questions that press for mathematical reasoning, justification, and understanding;
- Students explain their solutions using mathematical argumentation;
- Students reach consensus using mathematical reasoning and proof;
- Students solve problems using a variety of approaches;
- Students compare their strategies looking for mathematically important similarities and differences; and
- Students use mistakes as an opportunity to rethink their conceptions of mathematical ideas and examine contradictions. Mistakes support new learning about mathematics.

How do you see camera ready as influencing the sociomathematical norms of the classroom? What is it about camera ready that helps a teacher establish good sociomathematical norms?

Sociomathematical norms are developed in the classroom when the teacher acts as a facilitator of discourse rather than a fountain of knowledge and where the mathematics itself is the ultimate authority rather than the teacher (see Arcavi et al., 1998). The perfect opportunity to address students’ affective needs (such as building confidence) exists with each presentation by asking class members to identify elements of the presentation they liked, and by offering a way to “put the plus on you’re A”. Students in the class are prompted to offer suggestion for improvement. In this way, a student earns
positive feedback but also identifies ways to improve as does every audience member. Students learn from others’ experiences doing mathematics and communicating their mathematical thinking (Boaler, 2016). A few examples of feedback include:

- I liked the organization;
- I like that her graph supported her algebraic results;
- I think the horizontal axis needed a label for units;
- I liked that his initial result was highlighted in a box it makes it easier to find;
- I think the variable for area was an upper case A not a lower case a, etc..

Classroom mathematical practices are the “taken-as-shared mathematical practices established by the classroom community” (Cobb & Yackel, 1998, p. 171). Classroom math practices evolve from classroom discussions regarding problems and solutions. They involve “means of symbolizing, arguing and validating in specific task situations” (Bowers, et al., 1999, p.28) that do not require any justification. An example of common mathematical practices are specific procedures and algorithms that the teacher and students agree upon to solve certain math problems.

How would you describe the relationship between camera ready and classroom mathematical practices? Does one influence the other? Is camera ready a classroom math practice?

Oral presentations made with the aid of a doc camera offer several opportunities for shared practices. These include use of precise language and vocabulary, use of proper notation, a verification component in a solution, reinforcement of common procedural skills and algorithms but also opportunities to present alternative solution pathways that may be clever or elegant. Often times students remark when seeing an alternative solution pathway, “I didn’t think of solving the problem that way myself.”

The camera ready technique provides a nice forum for addressing desired classroom practices as described by Gleason, Livers, and Zelkowski (2015) in their observation protocol and also facilitates opportunities to develop mathematical integrity and mathematical intimacy (see p. 10 of this packet).

The Mathematics Classroom Observation Protocol for Practices (MCOP2) is a K-16 mathematics classroom instrument designed to measure the degree of alignment of the mathematics classroom with the various standards set out by the corresponding national organization that focus on conceptual understanding in the mathematics classroom including:

- Common Core State Standards in Mathematics: Standards for Mathematical Practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010),
- Mathematical Association of America (MAA): CUPM Curriculum Guide (Barker, et al., 2004),
- American Mathematical Association of Two-Year Colleges (AMATYC): “Crossroads” (AMATYC, 1995) and “Beyond Crossroads” (AMATYC, 2006), and

1) Students engaged in exploration/investigation/problem solving.
2) Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts.
3) Students were engaged in mathematical activities.
4) Students critically assessed mathematical strategies.
5) Students persevered in problem solving.

Oct 29, 2016
©Dr. James Vicich james.vicich@gmail.com
These materials are selected from a paper under review for publication and may not be duplicated by any method.
6) The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.
7) The lesson promoted modeling with mathematics.
8) The lesson provided opportunities to examine mathematical structure. (Symbolic notation, patterns, generalizations, conjectures, etc.)
9) The lesson included tasks that have multiple paths to a solution or multiple solutions.
10) The lesson promoted precision of mathematical language.
11) The teacher’s talk encouraged student thinking.
12) There were a high proportion of students talking related to mathematics.
13) There was a climate of respect for what others had to say.
14) In general, the teacher provided wait-time.
15) Students were involved in the communication of their ideas to others (peer-to-peer).
16) The teacher uses student questions/comments to enhance conceptual mathematical understanding.

Connections for teachers to the classroom instruction described in this paper.

- The type of resource knowledge described by Schoenfeld’s (2013) is represented as a well-connected network of ideas (see p. 13 of this packet).
- Schoenfeld’s criteria for task selection (for engaging students) is found on p. 9 of this packet.
- Guidelines a teacher may give to students for oral presentations is found on p. 15 of this packet.
- Vicich’s Diagnostic Instrument (used in professional development for teachers) is found on p. 14 of this packet.
- A detailed description of mathematical integrity and mathematical intimacy (as described by DeBellis and Goldin (2006) is found on p. 10 of this packet.
- A problem-solving template, developed by Perales and Vicich (2014), (see pp. 16 – 17 of this packet) can be used by students to become familiar with productive problem-solving behaviors.
Open-Ended Tasks

1. Students from Mr. Reyes’ Problem Solving class are standing (equally spaced) in a circle. Each student is given a whole number then stands in numerical order. Consecutive numbering begins with one. The student holding number 4 is directly opposite the student holding number 19. How many students are in Mr. Reyes’ class? *(Make a labeled drawing and use precise vocabulary words.)* Note: Verify that your solution makes sense. Some problem statements contain mathematically impossible conditions.

*(Note to Instructor: Challenge students that a diameter passes through the center of a circle. Exploit symmetry as a useful/alternate strategy.)*
2. Matt Weber’s Right Triangle Problem

Given: The outer triangle is an isosceles 6 by 6 right triangle, and the margin between the two right triangle’s parallel sides is one inch. Find the area of the inner (shaded) right triangle. Build a convincing mathematical argument. Note: The drawing below may not be drawn to scale. Begin this problem with a shared labeling system of the vertices so that solution pathways can be compared with precision and accuracy.
A 747 jet needs to attain a speed of 200 mph to take-off. If it can accelerate from 0 to 200 mph in 30 seconds, how long must the runway be? Assume constant acceleration.

Note to Instructor: Be sure students include three types of labels per axis when using a graph: meaning/title, variable, and units.)

Schoenfeld’s Criteria for problem selection (found in Arcavi et al. (1998)):

- Problems should be accessible on the basis of prior knowledge.
- Problems should be solvable or at least approachable in more than one way.
- Problems should illustrate important mathematical ideas in terms of either the content or solution strategies.
- Problems should be constructible without tricks.
- Problems should serve as first steps toward mathematical explorations and springboards for further problem posing.
Definitions

Recall: (CCSSM) **MP1 Make sense of problems and persevere in solving them.**

From DeBellis & Goldin (2006) (p. 138)

*Mathematical Integrity*: An insistence that a solution is mathematically adequate and makes sense.

*Mathematical Intimacy*: Willingness to take risks, persevere, and have confidence (Goldin & DeBellis, 1999).

We conjecture that students with strong mathematical integrity structures have the potential to engage in powerful learning and problem solving especially if their mathematical integrity is interacting with their capability for mathematical intimacy.

Excerpt from (pp. 138 – 139)

Important components of this affective construct [i.e. mathematical integrity] include: (1) recognition of an insufficiency of mathematical understanding or achievement, (2) the decision to take further action, and (3) the nature of the action. In any particular mathematical situation, there may be no recognition of lack of understanding, partial recognition, or full awareness; whatever the circumstance, the individual may or may not take further action. If action is taken, mathematical performance may be helped or hindered depending on the action - e.g., looking for a deeper structure in a problem, solving a related problem, making a surface level adjustment ('mathematical bluffing'), or stopping work.

From p. 143: Observed aspects of mathematical integrity include her identifying errors in her thinking and computation, and verbalizing a strong desire to 'get the problem right'.
References


Oct 29, 2016  
©Dr. James Vicich  
james.vicich@gmail.com.
These materials are selected from a paper under review for publication and may not be duplicated by any method.


Cognitive Networks

Hiebert and Carpenter (1992) suggest a theoretical framework for learning and teaching mathematics with understanding. Their framework is based upon several assumptions: knowledge is represented internally; these internal representations are structured; there exists a relationship between internal and external representations (i.e. spoken language, symbols, pictures, and physical objects); and internal representations can be related or connected to one another in useful ways.

Hiebert and Carpenter (1992) define mathematical understanding in the following way: “a mathematical idea or procedure or fact is understood if it is part of an internal network” (p. 67). A network may be a vertical hierarchy where one concept may be represented as a special case of another idea. The vertical network may be narrow and deep showing many hierarchical relationships. Sfard (1991) describes a network that is shallow and wide known as the "mother with too many sons" model (p. 27). A network may also be thought of as a web showing multiple links allowing one to identify similarities and differences between ideas (Hiebert and Carpenter, 1992, p. 67) (See Figure 1 below).

For the web model, an instructional goal is to help students build a coherent mental network in which all pieces are joined to others with multiple links. The degree of understanding is determined by the number and strength of the connections that a student is able to make within her/his network.

![Figure 1: Cognitive Schemata](image)

Figure 1: Cognitive Schemata

---


## Diagnostic Instrument for Problem-Solving Behaviors

Vicich (2014)
(Adapted from Geiger & Galbraith, 1998)

### Engagement

<table>
<thead>
<tr>
<th>Problem is read</th>
<th>Key words underlined</th>
<th>Givens and goals established</th>
<th>Givens and goals represented symbolically</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### *Executive Behaviors*

*Planning:* Did you make a **plan** or “jump into” this problem? Did you make any **conjectures regarding the answer or possible solution path**? Can you identify subgoals or subproblems?

### *Monitoring/Control*

<table>
<thead>
<tr>
<th>Recognition that a solution pathway will lead to a dead end</th>
<th>Changing from one solution pathway to a different solution pathway</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### *Heuristic Strategies*

<table>
<thead>
<tr>
<th>Appropriate strategy initially selected</th>
<th>Data organized</th>
<th>Multiple Strategies used to make progress or clarify</th>
<th>No heuristic used</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Verification

<table>
<thead>
<tr>
<th>Checked if answer was reasonable</th>
<th>checked correctness of answer</th>
<th>Checked for errors in solution</th>
<th>No verification used</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Mathematical Practices and Habits: Solution (is)

<table>
<thead>
<tr>
<th>Coherent, based on reason/logic</th>
<th>Thorough/Complete/ Viable Argument</th>
<th>Neatly organized</th>
<th>Attended to Precision</th>
<th>Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Resources: Knowledge is

<table>
<thead>
<tr>
<th>Complete</th>
<th>Sound with minor errors</th>
<th>Some but significant faults appear</th>
<th>No knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Beliefs and Attitudes: Problem Solver Exhibited

<table>
<thead>
<tr>
<th>Persistence</th>
<th>Confidence</th>
<th>Curiosity: Willingness to Explore</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Oct 29, 2016
©Dr. James Vicich j james.vicich@gmail.com, These materials are selected from a paper under review for publication and may not be duplicated by any method.
Guidelines for Oral Presentations

To the teacher:

- Use daily;
- Assign problems and partners during the previous session;
- First, find something positive to acknowledge; or ask the class, “What did you like/appreciate about this presentation?”
- Ask the class and the presenter for ways to improve.

To the student:

- Provide a neatly organized, thorough written solution;
- Be poised, look and sound confident;
- Use proper body position so that the audience can easily see both your written work and your face;
- Maintain eye contact with the audience;
- Use a voice that is clear and appropriately loud for the setting.

Common Unproductive Beliefs Held by Students*

1) Experts move directly from the problem statement to the solution.
2) There is only one way to solve a math problem (usually the teacher’s way).
3) It is not OK to stop and start over using a different approach once I’ve started.
4) I just need to get an answer; it does not have to make sense to me.
5) If I cannot solve a problem in 5 minutes or less then I cannot solve it at all.
6) The best way to learn math is to memorize.
7) Every problem uses a formula to arrive at an answer.
8) Making unsuccessful solution attempts is not a natural part of doing mathematics.

* All of these beliefs are counterproductive and are not held by expert mathematicians.


Original Problem: Underline the givens and circle the goals.

Conjecture (reasonable guess):

Questions for getting unstuck:
- Have I tried the strategies and followed my plan?
- Am I getting closer to solution?
- Can I break this problem into smaller pieces?
- Other questions I asked myself:

Strategy Selection:
1. Draw and label a diagram.
2. Examine special cases.
3. Simplify the problem.
4. Consider equivalent problems.
5. Consider slightly modified goals and subgoals.
6. Act out the problem/ use manipulatives.
7. Identify what does NOT work.
8. Work backwards.
(Adapted from Schoenfeld (1998))
Solution Pathway (Not just what you did, but WHY you did it. Remember, your work is for everyone!)

Answer (Stated in a complete sentence, referring to the question, units included as necessary):

Check (verification): Use mathematical reasoning to prove that your answer is correct.

I know my answer is right because …

<table>
<thead>
<tr>
<th>Underline the givens and circle the goals (1)</th>
<th>Conjecture and plan are appropriate and make sense in context of the problem (3)</th>
<th>Solution pathway is complete; questions are present; solution is checked for reasonableness and verified. (4)</th>
<th>Answer is correct, and is stated in a way that responds directly to the question asked. (2)</th>
<th>Total (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Template: Peralles and Vicich, 2013

©Dr. James Vicich james.vicich@gmail.com,
These materials are selected from a paper under review for publication and may not be duplicated by any method.