The following pages contain activities that were shared during the presentation at the conference. These activities and many more (40 in total) are available at the following Dropbox link:

https://www.dropbox.com/sh/n3dyjykf79ewqiz/AACuEDorD94GPJ0HDalRHrbwa?dl=0

First to Fifteen

Nine markers are labeled with a digit from 1 to 9 and placed on a table. Each marker gets one digit and no digit is repeated. Taking turns, two players remove the markers from the table. The winner is the **first** player to obtain amongst his/her markers, three that sum to 15. It is possible to have more than three markers, but exactly 3 must sum to 15. [Note: it is possible that neither person wins]

Play the game with a partner at least 20 times and record each person's selections in the exact order chosen.

Game	Markers					Win y/n
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

Game	Markers					Win y/n
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

Game	Markers					Win y/n
21						
22						
23						
24						
25						
26						
27						
28						
29						
30						

Game	Markers					Win y/n
31						
32						
33						
34						
35						
36						
37						
38						
39						
40						

Combine your results with your partner's to determine the number of times each marker was included in the winning hand.

Marker	1	2	3	4	5	6	7	8	9
# of times in winning hand									

Based on your results, can you identify a marker you would most likely want to have in your hand? If so, which one?

Now write down all possible sets of three unique digits (from 1 to 9) that add up to 15.

Make a table showing how often each digit is used.

Digit	1	2	3	4	5	6	7	8	9
# of times used									

Based on this information, what is your best first move? Explain your reasoning.

Arrange the digits 1 to 9 into a magic square so each sum is 15.

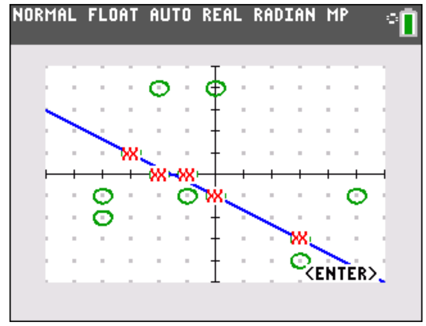
Explain how this game is the same as playing tic-tac-toe, and then determine a “winning” strategy for playing the game.

Green Globs Activity Name(s)________________________________

Materials:

Graphing Calculator with Green Globs program

Lab sheet



Objective:

In this activity, we will use the Green Globs program to find the slope and y-intercept of lines.

Procedure:

Obtain the Green Globs program on your graphing calculator. Run the program and follow the directions on screen. Work with a partner to determine the slope and y-intercept of the line that will hit the most globs remaining on screen. Continue until all 12 globs have been hit. Higher points will be earned by hitting multiple globs with a single line and by using fewer lines to hit the 12 globs. After all 12 globs have been hit, record your results for the round in the grid below. Then play again to try to improve your score.

Round	Number of Lines Used	Total Score	Bonus	Final Score
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

Cryptology and Matrices
Student Activity (modified)

Name ______________
Class ______________

In this activity, you will use matrices and their inverses to encode and decode messages.

Cryptology is the science of developing secret codes and using those codes for encrypting and decrypting data.[1] In June of 1929, an article written by Lester S. Hill appeared in the American Mathematical Monthly. This was the first article that linked the fields of algebra and cryptology.[2]

Today, governments use sophisticated methods of coding and decoding messages. One type of code, which is extremely difficult to break, makes use of a large matrix to encode a message. The receiver of the message decodes it using the inverse of the matrix. This first matrix is called the encoding matrix and its inverse is called the decoding matrix.[3]

In this activity, we will use a simple method for encoding a message by first assigning a numeral to each letter of the alphabet. We will represent the letter A with the numeral 1 and continue to the letter Z which will be assigned the numeral 26. We will also assign the numeral 0 to a space in the message.

For example, using the chart to the right, the word

SYSTEM

can be written using numerals as

19 25 19 20 5 13

and then recorded in a matrix as

$$\begin{bmatrix} 19 & 25 \\ 19 & 20 \\ 5 & 13 \end{bmatrix}.$$

_ = 0	**I** = 9	**R** = 18
A = 1	**J** = 10	**S** = 19
B = 2	**K** = 11	**T** = 20
C = 3	**L** = 12	**U** = 21
D = 4	**M** = 13	**V** = 22
E = 5	**N** = 14	**W** = 23
F = 6	**O** = 15	**X** = 24
G = 7	**P** = 16	**Y** = 25
H = 8	**Q** = 17	**Z** = 26

[1] http://www.answers.com/topic/cryptology

[2] http://www.glassblower.info/cryptosystems-journal/HILL29.HTM

[3] http://aix1.uottawa.ca/~jkhoury/cryptography.htm

1. To protect this message as it is transmitted, it is *encoded* by multiplying the message matrix by an encoding matrix, such as $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$.

   Find the following product and fill in the spaces provided below.

$$\begin{bmatrix} 19 & 25 \\ 19 & 20 \\ 5 & 13 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \\ \square & \square \end{bmatrix}$$

   c. Fill in the numerals for the new message:

   _____ _____ _____ _____ _____ _____.

The receiver of this message can retrieve the original message by *decoding* it by using the *inverse* of the coding matrix.

The **inverse** of a 2 X 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, provided $\det(A) \neq 0$, is $A^{-1} = \frac{1}{\det(A)} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

2. What is the decoding matrix (the inverse of the encoding matrix)?

   (Recall that $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$.)

   Fill in the information below.

$$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}^{-1} = \frac{1}{\square} \cdot \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

3. Test the decoding matrix to verify that these matrices are inverses. (When you multiply a matrix by its inverse, in either direction, their product will be the identity matrix.) Type the appropriate entries into the matrix given below. Change the order of the multiplication to verify that the two matrices are inverses, and record both of the results below.

$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \cdot \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \cdot \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

4. a. Multiply the encoded message by the decoding matrix. Record the results below.

$$\begin{bmatrix} \square & \square \\ \square & \square \\ \square & \square \end{bmatrix} \cdot \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \\ \square & \square \end{bmatrix}$$

   b. Use the chart given at the beginning of the worksheet to see if the decoded message is the same as the original one.

5. Decode the message **18 27 51 81 37 58 60 100 18 27 85 137 59 93 51 79**, which was encoded with encoding matrix $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$.

   - For this example, you will enter your own matrices into your TI-84 Plus. The first matrix will have 8 rows and 2 columns. The second matrix will have 2 rows and 2 columns.
   - Press [2ND] [MATRIX D x^{-1}] [▶] [▶] then select matrix A.
   - Enter the number of rows, 8, and the number of columns, 2, to create a matrix with 8 rows and 2 columns. Enter the values above.
   - Repeat the process to create a matrix with 2 rows and 2 columns. Enter the values for the **decoding** matrix.
   - Press [2ND] [MATRIX D x^{-1}] [L1 Y 1] [2ND] [MATRIX D x^{-1}] [L2 Z 2] [ENTRY SOLVE ENTER] to find the product.

Name ________________
Class ________________

a. What is the decoding matrix (the inverse of the encoding matrix)? $\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

b. What is the matrix that has been decoded? $\begin{bmatrix} \square & \square \\ \square & \square \\ \square & \square \\ \square & \square \\ \square & \square \\ \square & \square \\ \square & \square \\ \square & \square \end{bmatrix}$

c. To help you decode the message, write out the numerals from the decoded message.

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____

d. What does the message say? (Be sure to use the chart at the beginning of this activity to decipher the results.)

Sines, Sines, Everywhere are Sines.
Can't You Read the Sines?
Using TI-84 Plus C to Study Sine Functions

Name________________________

Materials:

Lab sheet
TI-84 Plus C Graphing Calculator

Objective:

In this lab activity, we will use the TI-83 to study sine curves. At the completion of this lesson, you will be able to describe and to graph the curve of $\boldsymbol{y = c + a\sin b(x - d)}$ for all real *a*, *b*, *c* and *d* with *b* > 0. Also, you will be introduced to the following terms: maximum value, minimum value, amplitude, period, and phase shift.

Procedure:

1. We will begin this activity by looking at the sine values of several angles. Do this as follows:
   a. Change your calculator to "Degree" mode:
      - Press [MODE].
      - Arrow to Degree.
      - Press [ENTER].
      - Choose [2nd] [QUIT].

   b. Use the calculator to find sin 0°, sin 45°, etc. and fill in the chart to the right.

$x°$	$y = \sin x$	$x°$	$y = \sin x$
0		360	
45		405	
90		450	
135		495	
180		540	
225		585	
270		630	
315		675	

2. What do you notice about the sine values? ______________________________
   ____
   ____

3. Use the TI-83 to study even more sine values:
   a. In L_1, enter the values: -360, -345, -330, . . . , 330, 345, 360 as follows:
      - Choose [STAT], Edit.
      - If necessary, clear L_1.
      - Arrow up so that L_1 is highlighted.
      - Choose [2nd], [LIST], OPS, seq(.
      - Punch in: x,x,-360,360,15)
      - Press [ENTER].

   b. Use L_2 to find the sine of the values in L_1.
      - Arrow up so that L_2 is highlighted.
      - Punch in: [SIN], [L_1], [ )].
      - Press [ENTER].

4. Look through the lists. Does the pattern you discussed in #2 still hold? Explain:

____

____

____

5. Scatter plot the points from L_1 and L_2 on your calculator:
   a. If necessary, turn off all active graphs and stat plots before you begin:
      - Choose [2nd] [STAT PLOT], PlotsOff, [ENTER].
      - Choose [Y=] and clear all entries from the screen.
   b. Activate stat plot for our graph:
      - Choose [2nd] [STAT PLOT], Plot1, [ENTER].
      - Set the Plot1 screen as in Figure 1.
   c. Choose the Window settings:
      - Press [WINDOW]
      - Change the settings as in Figure 2.
   d. Press [GRAPH] to view the graph.
   e. Choose [TRACE]. Arrow to the right and left to see what happens to *y* = sin *x* as *x* changes.

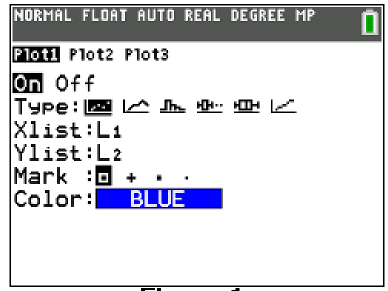


Figure 1

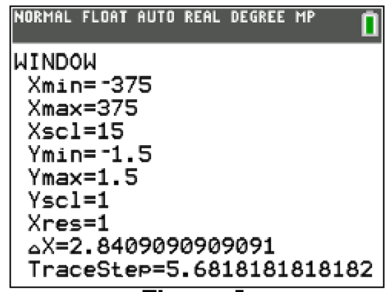


Figure 2

6. Describe what happens to our graph (that is, what happens to *y*) as *x* increases from -360° to 360°. ____

____

____

7. Now you will graph the "whole" curve *y* = sin *x* (not just plot points on the curve as was done above).
   a. Turn off stat plot:
      - Choose [2nd] [STAT PLOT], Plot1, [ENTER].
      - Arrow right to Off, then press [ENTER].
      - Choose [2nd] [QUIT] to get back to the main screen.
   b. Graph the curve *y* = sin *x*:
      - Press [Y=].
      - If necessary, arrow to Y_1=.
      - Press [SIN], [X,T,θ], [ )]. The screen should now look like Figure 3.
      - Choose [ZOOM], ZTrig. (This automatically sets the window for a nice viewing of trigonometric functions.)
      - Press [ENTER] to see the graph.
   c. This graph should have the same shape as the scatter plot. The only difference is that the curve is now "filled in." Functions like this, whose curve consists of a repeated pattern, are called *periodic functions*. The *period* of the function is the length of the shortest interval after which the graph repeats itself.

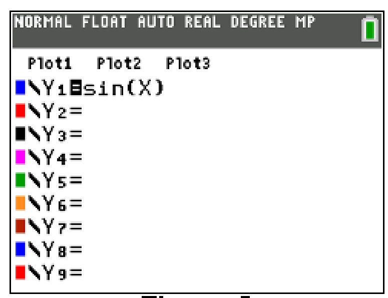


Figure 3

8. Now you will discover changes in *maximum* and *minimum values* and changes in *amplitude*.
   a. Graph the curve $y = 2\sin x$. Leave $y = \sin x$ on the graph as well.
      - Press [Y=].
      - Arrow down to Y_2 and type 2sin(x); that is, press [2], [sin], [X,T,θ], [ )]. The screen should now look like Figure 4.
      - Press [GRAPH].

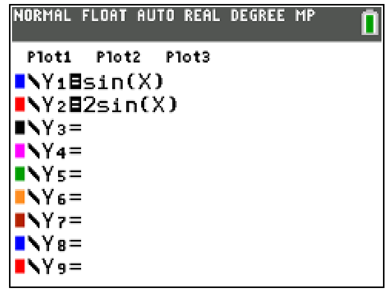


Figure 4

   b. How did the 2 affect the curve? ______________
   ____
   ____
   ____
   ____
   c. Now graph $y = 3\sin x$ (Follow the same steps that you used in part a). How did the 3 affect the curve? ______________________________
   ____
   ____
   d. Graph $y = 4\sin x$. How did the 4 affect the curve? ______________________
   ____
   ____
   e. Graph $y = (1/2)\sin x$. How did the 1/2 affect the curve? ____________________
   ____
   ____
   f. Make a generalization about how a will affect the graph of $y = a \sin x$.__________
   ____
   ____
   g. The *maximum value* (M) of any function is its largest y-value. The *minimum value* (m) of any function is its smallest y-value. The *amplitude* of a periodic function is defined as

$$\frac{M - m}{2}$$

For each of the curves you graphed above, find the maximum, minimum, and amplitude. Then, fill those values in the chart.

Curve	Maximum	Minimum	Amplitude
y = 2sin x			
y = 3sin x			
y = 4sin x			
y = (1/2) sin x			

9. What happens to $y = a \sin x$ when a is negative?
   a. Graph each curve below (Following the same steps you used in 8a.):
      - $y = -\sin x$
      - $y = -2\sin x$.
      - $y = -3\sin x$.
      - $y = -4\sin x$.
      - $y = (-1/2)\sin x$
   b. How did the negative sign affect the curve? ________________________
   ____
   ____

c. For each of the curves you graphed above, find the maximum, minimum, and amplitude. Then, fill those values in the chart.

Curve	Maximum	Minimum	Amplitude
y = - sin x			
y = -2sin x			
y = -3sin x			
y = -4sin x			
y = (-1/2)sin x			

d. How did the negative affect the amplitude? ________________________________

____

____

e. Describe an "easier" way to find the amplitude without using the formula from 8g.

____

____

____

10. The *period* of a curve is the "time" it takes the function to complete one cycle. This will be studied next.

a. Graph $y = \sin 2x$ (Follow the same steps that you used in 8a). How did the 2 affect the curve? ____

____

____

b. Now graph $y = \sin 3x$ How did the 3 affect the curve? ________________________

____

____

d. Graph $y = \sin 4x$. How did the 4 affect the curve? ___________________________

____

____

e. Graph $y = \sin (1/2)x$. How did the 1/2 affect the curve? _____________________

____

____

f. Make a generalization about how b will affect the graph of $y = \sin bx$. __________

____

____

g. The *period* of a wave can be determined by finding the distance between two adjacent maximum points. (Recall that distance is always positive.) For example, using the function y = sin *x,* the distance between the maximum point (90°, 1) and (450°, 1) is 450° - 90° = 360° (See Figure 5). That is, the *period* of the function $y = \sin x$ is 360°. In other words, it takes 360° for the curve to complete one full cycle.

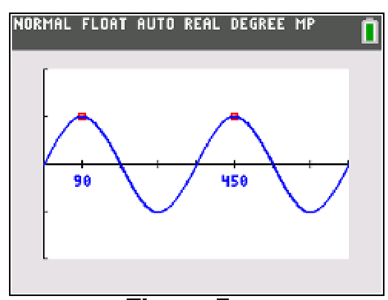


Figure 5

Similarly, the *period* of the function y = sin 2x is the distance between the maximum point (45°, 1) and (225°, 1), which is 225° - 45° = 180° (See Figure 6).

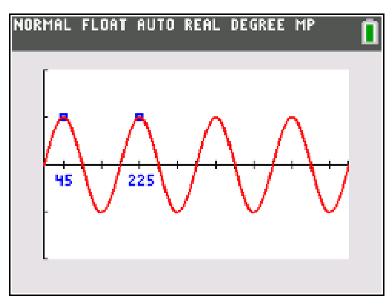


Figure 6

Note that 180° = 360° / 2.

It's NERF or NOTHIN'

Gun name: __________________

Establish a fixed distance over which you will time the flight of a Nerf dart. Record the speed (feet/sec) for 40 shots with a Nerf gun using blue darts, then repeat using orange darts. To get the speed, divide the fixed distance by your recorded time; round to 1 decimal place. Pay attention to units!

Blue darts

Orange darts

What type of sampling plan was used? Explain.

What is the variable in this case? What are the units, if any?

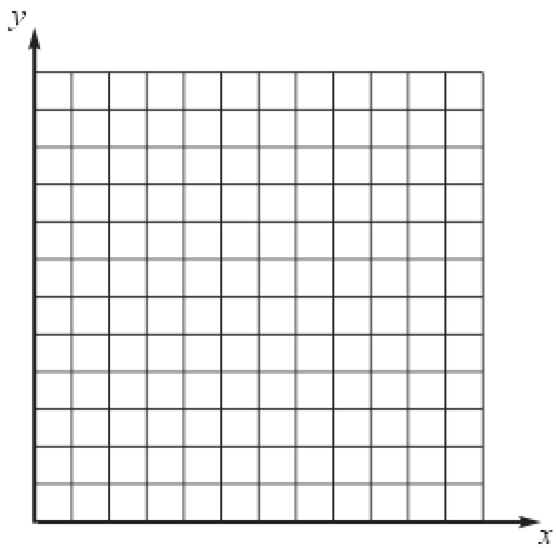
Is the variable discrete or continuous?

Is the variable quantitative or qualitative?

Why is there variability in your data? List some possible sources of variability (at least six) and tell whether each is an example of sampling error or non-sampling error. Also tell how you might reduce that particular variability.

For the remaining questions, use only the data for the **BLUE** darts.

Construct a well-labeled histogram or stemplot for the data using a lower class limit of 20 and a class width of 10.



Describe the shape of the distribution and note any interesting features.

What is the advantage of a stemplot over a histogram?

Compute the following measures of center (use correct notation and units):

Mean:

Median:

Mode:

Which measure of center must always be one of the data values?

What does it mean for a measure to be resistant?

Which measure of center is the most resistant?

Support or refute your description of the distribution by using the mean and the median.

Compute the following measures of variability:

Range:

Variance:

Standard Deviation:

Which of these three measures of variability are resistant?
What does it mean to be a biased estimator?

Do you think your data may be biased? Why or why not?

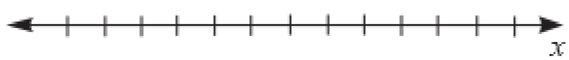
Find the three quartiles, Q_1, Q_2, and Q_3, for this data.

Compute the Interquartile Range for the data.

Is the IQR a resistant measure of variability? Explain why or why not.

Test the data for outliers. Are there any outliers? If so, what are they?

Construct a well labeled boxplot (box and whisker plot) for the data.



Is the shape of the boxplot consistent with what you know about the distribution so far?

Based on the shape of the distribution, what summary statistics ($\bar{x}$ & s, or the 5-Number Summary) should be reported? Explain.

SAVE THIS DATA! We will use it again later with confidence intervals and hypothesis testing.

Barbie Bungee

With her perfect skin, perfect hair, unrealistically perfect waistline, extensive plastic surgery, and huge royalties from Mattel (check out that Malibu dream house), Barbie seems to have it all. She has mastered every profession from airline pilot to zookeeper and has now turned to thrill-seeking to give her life meaning.

You have been contracted to construct a bungee jump for Barbie. You need to construct the jump so Barbie will come within 10 centimeters of hitting the ground (so it is thrilling) but must ensure that she does not hit the ground at all (imagine the lawsuits, particularly from Mattel for loss of revenue). Safety first! However, if the ride is not thrilling enough, Barbie will sue you for breach of contract and take everything you own (including your pet hermit crab).

You decide to run some preliminary tests to help with design construction. You attach an anchor to a simulated Barbie and attach a piece of bungee rope (i.e. one rubber band) to the anchor. Dropping the simulated Barbie, you make note of the farthest distance she fell. [NOTE: this is **not** the final resting distance; we are talking about the farthest point reached ***before*** coming to rest.] You repeat the process several times, each time making the bungee rope longer.

Complete the following table to summarize data you will use to construct the bungee jump.

Number of Rubber Bands, x	Distance Fell (cm), y

Measure the jump height (i.e. railing height) in centimeters. ________________

How far must Barbie fall in order for your ride to be successful? ________________

Use the data from the table to compute the least squares regression line (line of best fit) for the data. Round coefficients to 4 decimal places if needed.

Use your regression line to estimate the number of rubber bands needed to construct the ride. According to your line, the ride will require _________ rubber bands.

Construct your bungee jump ride and let "simulated" Barbie try it out. Were you successful, or did you lose it all?

On the back of this sheet, write up a summary of your results. Be sure to comment on any potential sources of variability in your design process and how these could be reduced. After all, you may need this for the lawsuit. Also compare your results with other groups. Discuss any differences and possible reasons.

Folding Frenzy

One day, while walking through the park, you find a long thin strip of newspaper on the ground. Fascinated by this unusual find, you wonder what the paper could be used for. You hold the paper horizontally with one end in each hand. Then you fold the paper lengthwise and flatten the strip so it has a crease. Inspired by the dramatic result, you decide to find out how many creases are created by folding the strip in a similar manner 19 additional times.

1. Make a sample model by cutting a strip of paper that measures about $2" \times 11.5"$.

2. Fold the strip as described above. How many creases are there after 1 fold?

3. Fold the folded strip in the same way. How many creases are there after 2 folds?

4. Fold two more times in the same manner and record the number of creases in each case.

5. Do you see a pattern? If so, what is it?

6. How many creases would there be if you folded the original strip of newspaper 20 times? How do you know?

7. Can you come up with a general expression for the number of creases resulting from n folds?