Algebra Top 5 List
(Lessons from Differential Equations)

Don Hickethier Flathead Valley Community College

AMATYC Denver,CO Fall 2016

1. Implicit to Explicit
   (a) Solve for x (as a function of t).
      i. Given $x(0) = 4$, find C. Solve for x.
      $$(x)^2 = t + C$$
      ii. Repeat with $x(0) = -4$.

   Recall: $\sqrt{a^2} = |a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$

   (b) Quadratic Formula
      i. Given $x(0) = 2$, find C. Solve for x.
      $$x^2 + 2x - 2(e^t + C) = 0$$
      ii. Repeat with $x(0) = -4$.
   (c) Absolute Value
      i. Given $x(2) = -4$, find C. Solve for x.
      $$|x| + |t| = C$$
      ii. Repeat with $x(-2) = 4$.
      iii. Repeat with $x(-2) = -4$.

2. Logarithms
   (a) From (Malthusian) population model
      i. Given $N(0) = 2$, find C. Solve for N.
      $$\ln|N| = -\lambda t + C$$
      ii. Repeat with $x(0) = -2$.

   Recall: $\log_b(N) = k \Leftrightarrow N = b^k$

   (b) From projectile motion with air resistance
   Solve for ω.
   $$\frac{1}{2}\left(\ln|1 + \omega| - \ln|1 - \omega|\right) = C - t$$

3. Partial Fractions (Undo the Common Denominator)

   (a) From projectile motion with air resistance.
   Find A and B such that

$$\frac{1}{1-\omega^2} = \frac{1}{(1+\omega)(1-\omega)} = \frac{A}{1+\omega} + \frac{B}{1-\omega}$$

   (b) From Logistic Population Model
   Find A and B such that

$$\frac{K}{P(K-P)} = \frac{A}{P} + \frac{B}{K-P}$$

4. Projectile Motion

   The Equation for Launching an object from the Earth's surface

   Let M be the mass of the earth, R the radius of the Earth, G the universal gravity constant and v_0 the initial velocity of the object, then using Newton's 2nd Law of Motion and Universal Law of Gravitation it can be shown that

$$v^2 - v_0^2 = -2GM\left(\frac{1}{R} - \frac{1}{R+y}\right)$$

   where v is the velocity of the object and y is the distance above the surface of the earth.

   (a) Maximum Height
   Show the maximum height of the object is

$$y = \frac{v_0^2 R}{2GM/R - v_0^2}$$

   Hint: What is the velocity at a maximum height?

   (b) Escape Velocity
   Show the minimum velocity required for the object to "escape" earth's gravitational field is

$$v_0 = \sqrt{\frac{2GM}{R}}$$

   Hint: What is the potential max height if one escapes the earth's gravitational field?

5. One More: Projectile Motion, no air resistance

   A body is released from rest and travels the last half of the total distance fallen in precisely one second. How far did the body fall and how long did it take to fall the complete distance?

$$y(t) = -\frac{1}{2}g\,t^2 + v_0\,t + y_0$$