1   **Exploring Exponential Relationships:**
2   ***The Case of Ms. Culver[1]***
3   Ms. Culver wanted her students to understand that exponential functions grow by equal factors over
4   equal intervals and that in the general equation $y = b^x$, the exponent (x) tells you how many times to
5   use the base (b) as a factor. She also wanted students to see the different ways that the function
6   could be represented and connected. She selected the Pay It Forward task because it provided a
7   context that would help students in making sense of the situation, it could be model in several ways
8   (i.e., diagram, table, graph, and equation), and it would challenge students to think and reason.
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10   In the movie "Pay It Forward", a student, Trevor, comes up with an idea that he thought could
11   change the world. He decides to do a good deed for three people and then each of the three
12   people would do a good deed for three more people and so on. He believed that before long there
13   would be good things happening to billions of people. At stage 1 of the process, Trevor completes
14   three good deeds. How does the number of good deeds grow from stage to stage? How many
15   good deeds would be completed at stage 5? Describe a function that would model the Pay It
16   Forward process at *any* stage.

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18   Ms. Culver began the lesson by telling students to find a function that models the Pay It Forward
19   process by any means necessary and that they could use any of the tools that were available in the
20   classroom (e.g., graph paper, chart paper, colored pencils, markers, rulers, graphing calculators).
21   As students began working in their groups, Ms. Culver walked around the room stopping at different
22   groups to listen in on their conversations and to ask questions as needed (e.g., How did you get
23   that? How do the number of good deeds increase at each stage? How do you know?). When
24   students struggled to figure out what to do she encouraged them to try to visually represent what
25   was happening at the first few stages and then to look for a pattern to see if there was a way to
26   predict the way in which the number of deeds would increase in subsequent stages.

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28   As she made her way around the room Ms. Culver also made note of the strategies students were
29   using (see reverse side) so she could decide which groups she wanted to have present their work.
30   She decided to have the strategies presented in the following sequence. Each presenting group
31   would be expected to explain what they did and why and to answer questions posed by their peers.
32   Group 4 would present their work first since their diagram accurately modeled the situation and
33   would be accessible to all students. Group 3 would go next because their table summarized
34   numerically what the diagram showed visually and made explicit the stage number, the number of
35   deeds, and the fact that each stage involved multiplying by another 3. Groups 1 and 2 would then
36   present their equations, one after the other. At this point Ms. Culver decided that she would give
37   students 5 minutes to consider the two equations and decide which one they thought best modeled
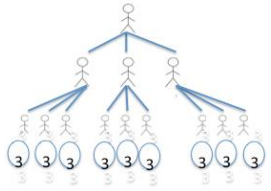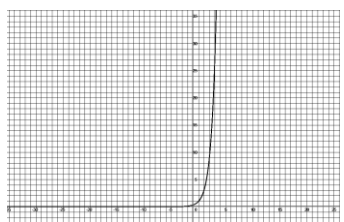38   the situation and why.

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40   Below is an excerpt from the discussion that took place after students in the class discussed the
41   two equations that had been presented in their small groups.
42   Ms. C.:   So who thinks that the equation y = 3x best models the situation? Who thinks that the
43   equation $y = 3^x$ best models the situation? *(Students raise their hands in response to*
44   *each question.)*
45   Ms. C.:   Can someone explain why y = 3x is the best choice? Missy, can you explain how you
46   were thinking about this?
47   Missy: Well Group 1 said that at every stage there are three times as many deeds as the one
48   that came before it. That is what my group (4) found too when we drew the diagram. So
49   the "3x" says that it is three times more."
50   Ms. C.:   Does everyone agree with what Missy in saying? *(Lots of heads are shaking back and*
51   *forth indicating disagreement.)* Darrell, why do you disagree with Missy?
52   Darrell: I agree that each stage has three times more good deeds that the previous stage, I just
53   don't think that y = 3x says that. If x is the stage number like we said then the equation
54   says that the number of deeds is three times the stage number – not three times the

55		number of deeds in the previous stage. So the number of deeds is only 3 more not 3
56		times more.
57	Ms. C.:	Other comments?
58	Kara:	I agree with Darrell. y = 3x works for stage 1 but it doesn't work for the other stages. If
59		we look at the diagram it shows that stage 2 has 9 good deeds. But, If you use the
60		equation you get 6 not 9. So it can't be right.
61	Chris:	y = 3x is linear. If this function were linear then the first stage would be three, the next
62		stage would be 6, then the next stage would be 9. This function can't be linear – it gets
63		really big fast. There isn't a constant rate of change.
64	Ms. C.:	So let's take another look at Group 3's poster. Does the middle column help explain what
65		is going on? Devon?
66	Devon:	Yeah. They show that each stage has 3 times more deeds that the previous one. For
67		each stage there is one more three that gets multiplied. That makes the new one three
68		times more than the previous one.
69	Angela:	So that is why I think $y = 3^x$ best models the situation. Stage 1 had 3 good deeds, stage 2
70		people had three each doing three deeds so that is 3^2, stage 3 had 9 people (3^2) each
71		doing 3 good deeds, so that is 3^3. The x tells how many 3's are being multiplied. So as
72		the stage number increases by 1 the number of deeds gets three times larger.
73	Ms. C.:	If we keep multiplying by another three like Angela described, it is going to get big really
74		fast like Chris said. Chris also said it couldn't be linear, so take a minute and think about
75		what the graph would look like.
76		
77		At this point Ms. Culver asked Group 5 to share their graph and proceeded to engage the class in a
78		discussion of what the domain of the function should be given the context of the problem. The
79		lesson concluded with Ms. Culver telling the students that the function they had created was called
80		exponential and explaining that exponential functions are written in the form of $y = b^x$. She told
81		students that in the five minutes that remained in class they needed to individually explain in writing
82		how the equation related to the diagram, the table, the graph and the problem context. She thought
83		that this would give her some insight regarding what students understood about exponential
84		functions and the relationship between the different ways the function could be represented.
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Group 1 (equation - incorrect)		Group 2 (table like Group 6 & 7 and **equation**)		Group 3 (diagram like Group 4 and **table**)		
y = 3x At every stage there are three times as many good deeds as there were in the previous stage.		y = 3^x		x (stages)		y (deeds)
				1	3	3
				2	3 x 3	9
				3	3 x 3 x 3	27
				4	3 x 3 x 3 x 3	81
				5	3 x 3 x 3 x 3 x 3	243
Group 4 (diagram)		Group 5 (table like Group 6 & 7 and **graph**)		Groups 6 and 7 (table)		
So the next stage will be 3 times the number there in the current stage so 27 x 3. It is too many to draw. You keep multiplying by 3.				X (stages)		Y (deeds)
				1		3
				2		9
				3		27
				4		81
				5		243

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