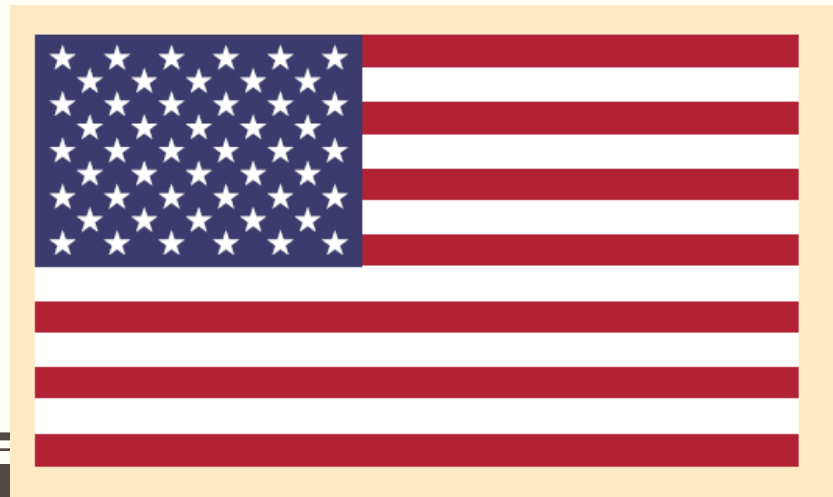
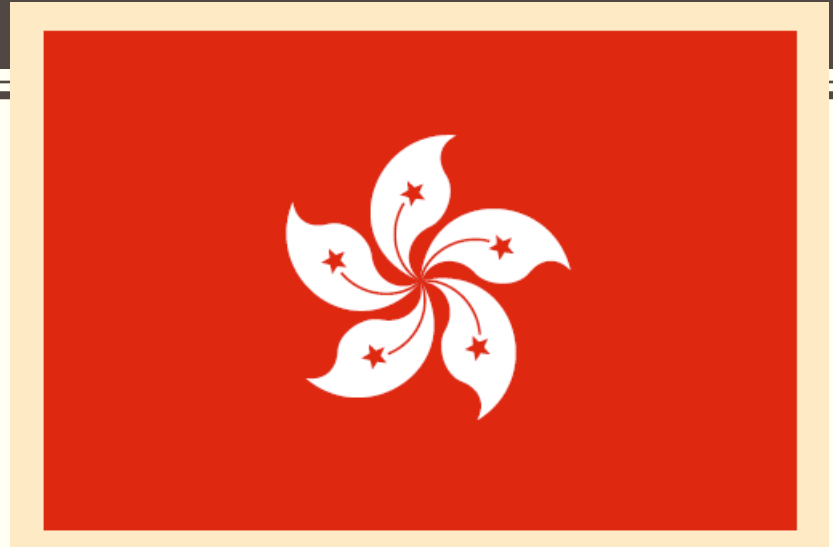


# COMPARISON OF HONG KONG AND U.S. IN MATHEMATICS TEACHING AND LEARNING



# Presenters

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- **Shane Tang**, M.S., Associate Professor of Mathematics,  
Salt Lake Community College
- **Amy Kong**, M.S., Professor of Mathematics,  
Helena College University of Montana
- **Oiyin Pauline Chow**, M.S., Senior Professor of Mathematics (retired),  
Central Pennsylvania's Community College, HACC

## **Synopsis of our presentation**

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All of us received mathematics instruction in Hong Kong and we will share our firsthand learning experience. With our background as the U. S. College mathematics educators, we will select topics to showcase and highlight potential pedagogical advantages and cultural differences. Participants will be engaged in doing sample problems, including applications.

## Goals for our presentation

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- Learn the pedagogy differences between the U.S. and Hong Kong in mathematics education.
- Identify the uniqueness in the mathematics education systems in the U.S. and Hong Kong.
- Recognize how cultures affect students mindset on learning mathematics.
- Discuss ways to enhance mathematics teaching and learning.

# Agenda

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- Infra-Structure: Education System, Curriculum, Assessment
- Cultural Differences
  - Belief in Math Education
  - Homework
  - Tutoring
  - Calculator Use
  - Parental Expectation
- Sample problems from:
  - Hong Kong Textbooks
  - Public Exams
  - Daily Math Homework Assignments



## **Brief History of Hong Kong**

- In 1898, Britain acquired the New Territories on a 99-year lease.
- In 1941, Japanese occupied Hong Kong for 3 years and 8 months.
- Hong Kong received hundred of thousands of refugees from Mainland China and Vietnam in the past.
- Hong Kong was reverted back to Chinese authority in 1997.
- Chinese government will have full authority in 2047.

# Education System

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## Hong Kong





- Secondary School (F.1 - F.6 = 7<sup>th</sup> to 12<sup>th</sup> grade): Six years of compulsory math
- Extended math classes (Module 1 or 2) for STEM students in addition to compulsory math (F.4 - F.6 = 10<sup>th</sup> to 12<sup>th</sup> grade)

## U.S.



- High School (8<sup>th</sup> to 12<sup>th</sup> grade): Min. two years of math
- Options for advanced / honors classes for all students
- Students are assigned to the math classes based on their ability

# Curriculum

<div data-bbox="777 321 1159 399">Hong Kong</div> <div data-bbox="1238 315 1431 439"></div>	<div data-bbox="2071 282 2198 349">U.S.</div> <div data-bbox="2211 344 2397 451"></div>
<div data-bbox="163 482 1809 843"><p><b>F.1 to F.6 - Compulsory Math with three strands :</b></p><ul style="list-style-type: none"><li><b>Number and Algebra</b></li><li><b>Measures, Shapes, and Space</b></li><li><b>Data Handling</b></li></ul></div>	
<div data-bbox="163 872 1775 1408"><p><b>F.4 to F.6</b></p><p><b>Extended Math (by choice)</b></p><p><b>Module 1 (Calculus and Statistics) – for disciplines or careers requiring mathematical application</b></p><p><b>Module 2 (Algebra and Calculus) – for math-related fields and careers</b></p></div>	<div data-bbox="1898 615 2379 939"><p>8th to 12 grade: Algebra I, Algebra II, PreCalculus, Trigonometry, Calculus I, Statistics</p></div>



# Curriculum

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Hong Kong



## F.1 – F.3 – Junior Secondary Compulsory Math

5 class periods per week

40 minutes per period

480 periods available for the three years

320 hours of lesson time

# Curriculum

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Hong Kong



F.4 to F.6 (i.e. 10<sup>th</sup> – 12<sup>th</sup> grades)

Compulsory Math with the same three strands

250 to 313 hours of lesson time

Extended Math (by choice)

Module 1 (Calculus and Statistics) or

Module 2 (Algebra and Calculus)

125 hours of lesson time

# Assessment

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**Hong Kong**



Hong Kong Diploma of Secondary Education (HKDSE) Examination at F. 6 (12<sup>th</sup> Grade) (exam results are generally used for local colleges/universities admission after 12<sup>th</sup> Grade)



**U.S.**



- Graduation Exam as required by each state
- SAT, ACT, AP etc (not necessarily compulsory for students)

# Spending on Education (% of GDP)

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		Expenditures on Education (% of GDP)	Year
	United States	5.6	2012
	Hong Kong	3.4	2012

Source: <http://hdr.undp.org/en/content/expenditure-education-public-gdp>

# International Comparison of Math Skills Among 15-Year-Olds

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## Mean Performance on Mathematics Scale

	Range of Ranks <sup>1</sup>	
	Upper Rank <sup>2</sup>	Lower Rank <sup>3</sup>
Hong Kong	1	3
United States	25	28

1. Because data are based on samples, it is not possible to report exact rank order positions for countries. However, it is possible to report the range of rank order positions within which the country mean lies with 95% likelihood.
2. Rank based on top two levels of proficiency (out of six) on mean scale of mathematical performance.
3. Rank based on lowest two levels of proficiency (out of six) on mean scale of mathematical performance.

Source: Organization for Economic Cooperation and Development, OECD PISA (Program for Student Assessment) 2003 database.

# Cultural Beliefs About One's Abilities to Learn Mathematics

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Schoenfeld (1989) and Tang (2007) both characterized how learners perceive math abilities: Nature versus Nurture

**Nature:** the belief of the abilities in learning math comes from natural endowment and born talent.

**Nurture:** the belief that sufficient facilitation and the learner's own effort can overcome difficulties in learning mathematics.

# **Manifestation of the Cultural Beliefs (U.S.)**

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An examination of the attitude toward homework at the high school level:

*Method:* Focus group discussion with six PA HS math teachers in September 2016.

*Findings:*

- Homework given to students is largely procedural, similar to Schoenfeld's (1989) description.
- Minimal opportunities for learners to experience disequilibrium when doing homework.

# **Manifestation of the Cultural Beliefs (U.S.) - Continued**

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## **Findings: (Continued)**



- Learners tend to resist HW when confronted with disequilibrium.
- Parents also tend to resist when they observe their children “stuck” on homework problems.
- School administrators do not generally support the math teachers when learners and parents are pushing back.

## **Interpretation:**

The findings are consistent with the belief of natural endowment of math abilities for many U.S. students.



## Differences

<b>Hong Kong</b> 	<b>U.S.</b> 
<ul style="list-style-type: none"><li>- Political uncertainty</li><li>- Believe that (STEM) education is the key to success.</li><li>- Believe that learning math trains the brain.</li><li>- Parents generally support school teaching.</li><li>- Invest additional resources and time in tutoring.</li></ul>	<ul style="list-style-type: none"><li>- Politically stable</li><li>- Believe that education is one of the ways to success.</li><li>- Believe that learning math is to fulfill the requirement.</li><li>- Parents generally involve in school teaching.</li><li>- Invest additional resources and time in education, sports and music.</li></ul>

## Parental Expectations – Hong Kong (USA Today)

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“Throughout much of Asia, education is seen as the only path to success. Parental demands, fear of failure, competition and pride are fueling Asia’s academic ascension.

Simply put, children in Asia study with a purpose.  
...typical Asian Student: committed, diligent, competitive, passionate, focused and ambitious.”

Source: <http://www.usatoday.com/story/news/nation/2013/08/04/asian-students-carry-high-expectations-for-success/2615483/>

## **Cultural Differences: Homework (Hong Kong)**

1000 students participated in a survey.

- A primary school student spends an average of 2.38 hours per day on home work
- A Form 4 or 5 student (i.e. Grade 10 or 11) spends an average of 2.22 hours per day on home work

## Cultural Differences: Homework (U.S. vs Hong Kong)

15-year olds spend about

- 6.1 hours per week on homework in U.S.
- 6 hours per week on homework in Hong Kong.

## Cultural Differences: Tutoring (Hong Kong)

1,016 students from 14 primary and 27 secondary schools were polled in 2015.

- 67.6% of Primary Four (4<sup>th</sup> grade) and Five (5<sup>th</sup> grade) students took tutorial classes after school
- 40.8% of Form Four (10<sup>th</sup> grade) or Five (11<sup>th</sup> grade) students took tutorial classes after school

## **Cultural Differences: Tutoring (Hong Kong)**

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### Private tutoring in 2013:

- 85% of Form Five (11<sup>th</sup> grade) students have private tutoring
- US\$255 million private tutoring industry or cram school
- Top cram schools can have 10,000 students per month
- Most popular tutors can earn at least HK\$3 million (US\$385,000) a year

## **Cultural Differences: Tutoring (Hong Kong)**

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- HK\$400 (~U.S. \$52) per hour 10 years ago
- Math/Science tutors may charge between HK\$700 to 900 per hour 10 years ago

## **Cultural Differences: Tutoring (Hong Kong)**

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- 1/3 secondary school students spent about HK\$18.9 million (US\$2.42 million) per month on private tutoring in 2004-2005
- Private Tutoring Industry is worth at least HK\$400 million (US\$51.3 million)
- Other reports claimed that the industry generated more than HK\$3.6 billion a year (US\$464 million)



The New York Times August 18, 2013

## “In Hong Kong, the Tutor as Celebrity”



The banner for Modern Education (現代教育) features a large group of people, likely tutors and students, posing in a studio setting. The text on the banner includes:

- 現代教育 (Modern Education)
- 加入我們 facebook | 分校地址 | 相關網站 | 聯絡我們 | 現代書院 | 網站地圖 | 返回首頁
- 最新消息 | 星級名師 | 課程資料 | 下載章程 | 網上交易系統 | 誠邀導師加盟
- 33425
- 常規課程 現正招生
- 新舊生半價
- 詳情按此

Source: <http://www.nytimes.com/2013/08/19/world/asia/In-Hong-Kong-the-Tutor-as-Celebrity.html>

# The New York Times August 18, 2013

*“Advertisements for star tutors in Hong Kong can be seen all over here: on billboards that loom over highways and on the exteriors of shopping malls. Invariably, the local teaching celebrities are young, attractive and dressed in designer outfits befitting pop stars. But beyond the polished shine, the advertisements also claim that their celebrity tutors can help students ace Hong Kong’s university entrance exam.”*



## Cultural Differences: Tutoring (United States)

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“The tutoring market is fragmented. Some online tutoring marketplaces aggregate a large number of private tutors. One site has over 34,000 registered tutors in California. The hourly rate is, in average \$44.77.”

Private tutoring is not as common in the U.S.

Source: <https://en.wikipedia.org/wiki/Tutor>



## Cultural Differences: Calculator Use (H.K. vs U.S.)

- In 2007, **52%** of 4<sup>th</sup> grade teachers in Hong Kong did not permit calculators and **31%** in U.S.
- In 2014, use of graphing calculator is **12%** in Hong Kong and **77%** in U.S.
- In 2014, use of scientific calculator is **88%** in Hong Kong and **23%** in U.S.
- Calculators are not allowed in tests for nine- and 11-year-olds in Hong Kong

## **Cultural Differences: Calculator Use (Hong Kong)**

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- Elementary students learn how to perform basic arithmetic operations without using a calculator
- When students take the Hong Kong Diploma of Secondary Education Examination (HKDSE), they can only bring a calculator from a list of permitted scientific calculators

## Sample Problems: School Exams (Hong Kong)

Form 4 (10<sup>th</sup> grade) Mid-Year Exam (2015-2016)

1. Let  $g(x) = x^2 + 4x - a$ , where  $a$  is a constant. If  $g(3) = 16$ , find the value of  $b$  such that  $g(b) = -9$ .
2. (a) Convert each of the following recurring decimals into a fraction.  
(i)  $0.1\bar{4}$     (ii)  $0.\bar{3}7$   
(b) Using the results of (a), solve  $0.\bar{3}7x + 1.1\bar{4} = 0.1\bar{4}x$ . Give the answer in fraction.

## Sample Problems: School Exams (Hong Kong)

3. The vertex of the graph of a quadratic function  $y = f(x)$  is  $(3, -1)$  and the graph passes through the point  $(7, 3)$ . Find
- (a)  $f(x)$ ,
  - (b) the direction of opening,
  - (c) the  $y$ -intercept of the graph,
  - (d) the  $x$ -intercept of the graph.

## Sample Problems: School Exams (Hong Kong)

4. Natalie bought a car for \$320,000. She then sold the car to Henry at a loss of 24%. What price should Henry sell the car in order to make a profit of 5%?

A. \$225,340

B. \$231,620

C. \$243,200

D. \$255,360



## Sample Problems: School Exams (Hong Kong)

5. Which of the following quadratic equations in  $x$  has roots  $18m$  and  $-32m$ ?

A.  $x^2 - 18mx + 32m = 0$

B.  $x^2 + 18mx - 32m = 0$

C.  $x^2 + 14mx - 576m^2 = 0$

D.  $x^2 - 14mx + 576m^2 = 0$

## Sample Problems: School Exams (Hong Kong)

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6. If  $g(2x + 1) = 4x^2 - 8x$ , then  $g(x) =$

A.  $x^2 - 2x - 3$

B.  $x^2 - 6x + 5$

C.  $4x^2 - 4x - 3$

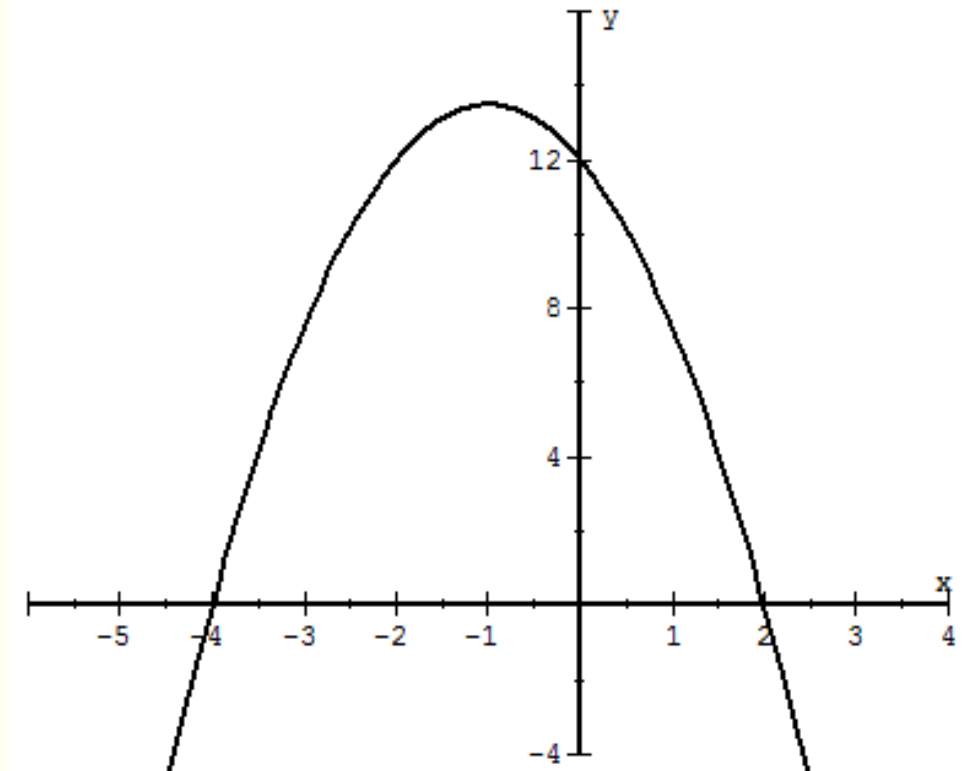
D.  $4x^2 - 12x + 5$

## Sample Problems: School Exams (Hong Kong)

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7. The figure shows the graph of a quadratic function  $y = p(x)$  and its intercepts.  $p(x) =$

- A.  $(x + 4)(x - 2)$
- B.  $12 - (x + 4)(x - 2)$
- C.  $12 - 1.5(x + 4)(x - 2)$
- D.  $-1.5(x + 4)(x - 2)$



## Sample Problems: School Exams (Hong Kong)

Form 3 (9<sup>th</sup> grade) Mid-Year Exam (2015-2016)

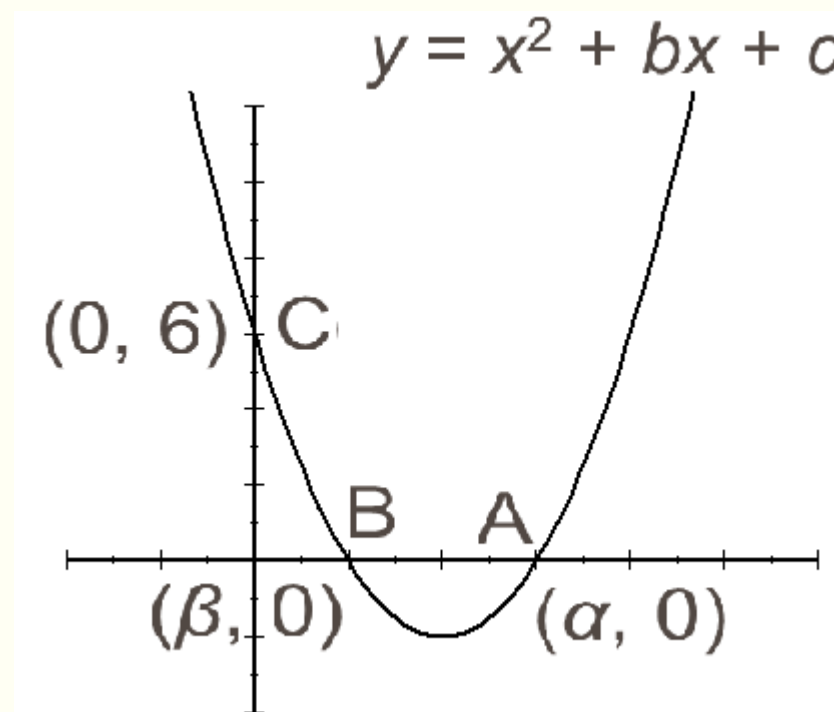
8. There were 1200 students in a school last year, 60% of them were boys. This year, the number of boys is increased by 5%, and the number of girls is decreased by 10%.

- (a) Find the number of boys and girls in the school this year.
- (b) Find the percentage change in the total number of students.

## Sample Problems: Public Exams (Hong Kong) 1984-1995

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1. In the figure, the curve  $y = x^2 + bx + c$  meets the  $y$ -axis at  $C(0, 6)$  and the  $x$ -axis at  $A(\alpha, 0)$  and  $B(\beta, 0)$ , where  $\alpha > \beta$ .
- (a) Find  $c$  and hence find the value of  $\alpha\beta$ .
- (b) Express  $\alpha + \beta$  in terms of  $b$ .
- (c) Using the results in (a) and (b), express  $(\alpha - \beta)^2$  in terms of  $b$ . Hence find the area of  $ABC$  in terms of  $b$ .



## Sample Problems: Public Exams (Hong Kong) 1984-1995

2. If  $f(n) = \frac{1}{2}n(n-1)$ , then  $f(n+1) - f(n) =$

A.  $f(1)$

B.  $f(n)$

C.  $\frac{n}{2}$

D. 1

E.  $n$

## Sample Problems: Public Exams (Hong Kong) 1984-1995

3. (a) Solve the simultaneous equations

$$\begin{cases} x + 2y = 5 \\ 5x - 4y = 4 \end{cases}$$

(b) Given that 
$$\begin{cases} \frac{a}{c} + \frac{2b}{c} = 5 \\ \frac{5a}{c} - \frac{4b}{c} = 4 \end{cases}$$
 , where a, b, and c

are non-zero numbers, using the result of (a), find a:b:c.

## Sample Problems: Public Exams (Hong Kong) 1984-1990

4. The equation of circle centered at  $(a, b)$  and tangential to the  $x$ -axis is

A.  $x^2 + y^2 - 2ax - 2by + a^2 = 0$

B.  $x^2 + y^2 - 2ax - 2by + b^2 = 0$

C.  $x^2 + y^2 - 2ax - 2by + a^2 + b^2 = 0$

D.  $x^2 + y^2 + 2ax + 2by + a^2 = 0$

E.  $x^2 + y^2 + 2ax + 2by + b^2 = 0$



## Sample Problems: Daily Math H.W. Assignments (Hong Kong)

Mathematics Today A Programmed Course 5B  
by Leung, Lai, and Lai, 1997

1. Find the range of values of  $k$  such that the line  $y = x$  cuts the curve  $y = x^2 + kx + 1$  at two points.

## Sample Problems: Daily Math H.W. Assignments (Hong Kong)

2. If  $\frac{5x-3y}{x+y} = 2$ , find

(a)  $x : y$

(b)  $\sqrt{x^2 - y^2} : x$

## Sample Problems: Daily Math H.W. Assignments (Hong Kong)

3. (a) Solve each of the following equations for  $0^\circ \leq \theta < 360^\circ$ .

(i)  $5 \sin \theta - 2 = 0$

(ii)  $4 - 7 \cos \theta = 0$

(b) Let  $f(x) = 8 - 20 \sin \frac{x}{3} - 14 \cos(x - 20^\circ)$

(i) Factor  $f(x)$

(ii) Solve  $f(x) = 0$  for  $0^\circ \leq x < 360^\circ$ .

$$+ 35 \sin \frac{x}{3} \cos(x - 20^\circ)$$

## Sample Problems: Keystone Exam 2015 – Algebra I in PA

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1. Raul has \$640 saved, and Jaime has \$320 saved. They each begin a new job on the same day and save all of their money. Raul earns \$180 per day, and Jaime earns \$200 per day. In how many days will they have an equal amount of money?

A. 8   B. 16   C. 24   D. 32

## Sample Problems: Keystone Exam 2015 – Algebra I in PA

2. Two expressions are shown below.

$$\pi x \qquad x^2$$

For which value of  $x$  is the value of  $\pi x$  greater than the value of  $x^2$ ?

A.  $x = -2$    B.  $x = 0$    C.  $x = 1.5$    D.  $x = 9$

## Sample Problems

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Refer to the Question Handout:

<i>Question (Circle One)</i>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	<b>H</b>
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<i>Estimate the number of minutes for an average Calc. I student to complete the question correctly.</i>
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..... **MINUTES**

<i>From which country was the question used in a standardized exam? (Circle One)</i>
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**U.S.** *or* **HONG KONG**

# Answers:

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- A** – Daily Math H.W. Assignments (Hong Kong)
- B** – Public Exams (Hong Kong) 1984-1995
- C** – CA CAHSEE Practice
- D** – Public Exams (Hong Kong) 1994
- E** – FL Algebra I 2015 Sample Problem
- F** – NY Algebra I 2016 Q23
- G** – PA Algebra I Practice Q7
- H** – TX Algebra I 2016 Q24

QUESTIONS?





