AMATYC Annual Conference
Thursday, November 9th, 2017

Measurement:
What is the right “Angle”?

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Teaching Math for Meaning

• Previously, emphasis was on **WAYS OF DOING** (e.g. blindly following procedures and rules without sense making)

• Today, we aim to emphasize developing **WAYS OF THINKING**

• **WAYS OF THINKING** naturally leads to effective, efficient, flexible, and fluent **WAYS OF DOING** as well as supporting the development of **PROBLEM SOLVING** strategies
What does it mean “to measure”? What is “measurement”?

Measurement – a number that indicates a multiplicative comparison between the attribute of an object being measured and the same attribute of a given unit of measure (Van de Walle, 2001)

When measuring:
1) determine the attribute you want to measure,
2) find something else with the same attribute and use it as the measuring unit, and
3) compare the two multiplicatively (“How many copies of...”).
What can we measure?

- Height
- Weight
- Blood pressure
- Age
- Temperature
- IQ
- Body Mass Index (BMI)
- Strength
- Flexibility
- Surface area
- Speed
A Way of Thinking about Measurement

“How many times as large is his vertical length compared to a foot?”
“How many copies of one foot will his height be equal to?”
The Value of Robust Definitions

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1 D - A Way of Thinking about Measurement
“2D or not 2D...that is the question!”

Measureable attributes of 2 Dimensional Figures:

• Perimeter - the length or distance around the outside of a figure
  • Circumference – the length or distance around the outside of a circle

• Area - the amount of *space enclosed by a two-dimensional object
  *Recall that space is defined as the collection of all points
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2D Measurement - Perimeter

- Triangle: \( P = a + b + c \)
  - 1 toothpick

- Square: \( P_{\text{square}} = s + s + s + s = 4s \)
  - 1 inch

- Rectangle: \( P_{\text{rectangle}} = l + w + l + w = 2l + 2w \)
2D Measurement - Circumference

Measurement: Count the number of "copies" of the unit of measure there are.

6.28 copies of the radius 2 inches

\[ C \approx 6.28(2) \approx 12.56 \]

\[ C \approx 6.28r \approx 2(3.14)r = 2\pi r \]
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2D Measurement - Area

Area - the amount of space * inside an enclosed two dimensional object.
2 D - A Way of Thinking about Measurement
3D Measurement

Measureable attributes of 3 Dimensional Figures:

• Surface Area – the total area that covers the surface of an object
• Lateral Surface Area – the surface area of all faces of a three-dimensional object excluding the base
• Volume – the amount of space enclosed by a three-dimensional object
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A Way of Thinking about Measurement
3 D - A Way of Thinking about Measurement

Image Source: http://www.youtube.com/watch?v=cAxHYFRx1Fs
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A Way of Thinking about Measurement
A Way of Thinking about Measurement

The distribution of heights of young women aged 18 to 24 is approximately normal with a mean of 64.5 inches and a standard deviation of 2.5 inches.

What does this expression measure?

\[ \frac{x - \mu}{\sigma} \]

where \( x \) is a specific woman’s height.

Estimate the % of women 18-24 years old that are shorter than 66 inches.

Estimate the % of women 18-24 years old that are shorter than 66 inches.
What **is** an “angle”?

What are your thoughts about each definition:

a) the space between two intersecting lines or surfaces at or close to the point where they meet.

b) An angle is a combination of two rays (half-lines) with a common endpoint.

c) A shape, formed by two lines or rays diverging from a common point (the vertex).

d) The amount of turn between two straight lines that have a common end point (the vertex).
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Attribute? Unit of Measure? How many copies?

Attribute? Amount of "turn"

Unit of measure?
• Degrees
• Radians
• Gradians

How many copies of 1 _____?
Attribute? Unit of Measure?
How many copies?
Unit conversion

2.3 sextants = _____________ quadrants

15.2 hours = ________________ gradians

100,000 arcseconds = ________________ degrees

7:00 o’clock = ________________ hexacontade
Locate these angles (measured in radians) on the unit circle:

\[
\begin{align*}
1 &= \\
4.3 = \\
\frac{3\pi}{4} &= \\
\frac{2\pi}{3} &= \\
\frac{11\pi}{7} &=
\end{align*}
\]
Applications in Astronomy

The stars appear to be attached to a giant celestial sphere, spinning about the celestial poles, and around us, once every 23 hours and 56 minutes.
Applications in Astronomy

How many hours was this long term exposure taken over?
Applications in Astronomy

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