

# Teaching Critical Thinking in Mathematics

AMATYC Conference  
November 9, 2017

# Icebreaker

▶ Facebook Users  
2.1 billion



▶ National Debt  
13.6 trillion



▶ Average NFL Salary  
1.9 million



# What is Critical Thinking?

- ▶ Who?
- ▶ What?
- ▶ Where
- ▶ When?
- ▶ Why?
- ▶ How?

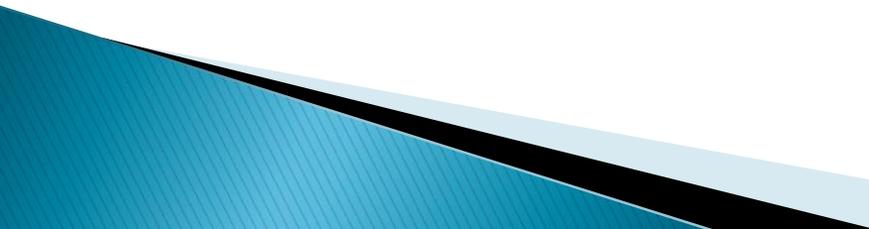
# We Know It When We See It

- ▶ Write the equation of the line through the points  $(-1, 4)$  and  $(7, 6)$ .
- ▶ If 10 shirts cost \$110 and 30 shirts cost \$280, write a linear equation that gives the cost of  $n$  shirts

Intermediate Algebra Connecting  
Concepts Through Applications,  
Clark & Anfinson

# Critical Thinking as Defined by the National Council for Excellence in Critical Thinking, 1987

Critical thinking is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action. In its exemplary form, it is based on universal intellectual values that transcend subject matter divisions: clarity, accuracy, precision, consistency, relevance, sound evidence, good reasons, depth, breadth, and fairness.



**MATH CLASSES TEACH HOW TO:**

**CHOOSE THE BEST APPROACH TO A SITUATION**

**THINK CRITICALLY ANALYZE INFORMATION**

**MAKE INFORMED DECISIONS PERSEVERE**

**PLAN AHEAD APPLY PRIOR KNOWLEDGE**

**INTERPRET INFORMATION PROBLEM SOLVE**

**MAKE CONNECTIONS ENVISION SOLUTIONS**

**DISCOVER PATTERNS REASON ABSTRACTLY**

**USE TOOLS APPROPRIATELY REPRESENT**

**EXPLAIN LEARN FROM MISTAKES**

**REASON QUANTITATIVELY BE PRECISE**

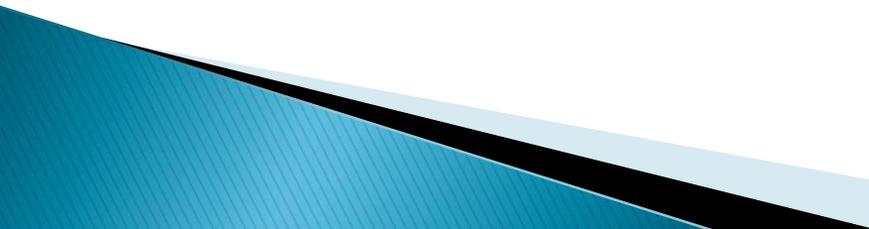
# How do we know if our students our thinking critically in mathematics?

- ▶ A Typical Class ...
  - Listen
  - Copy
  - Memorize
  - Drill
    - “Do #1–30,000 odd for the next class ...”

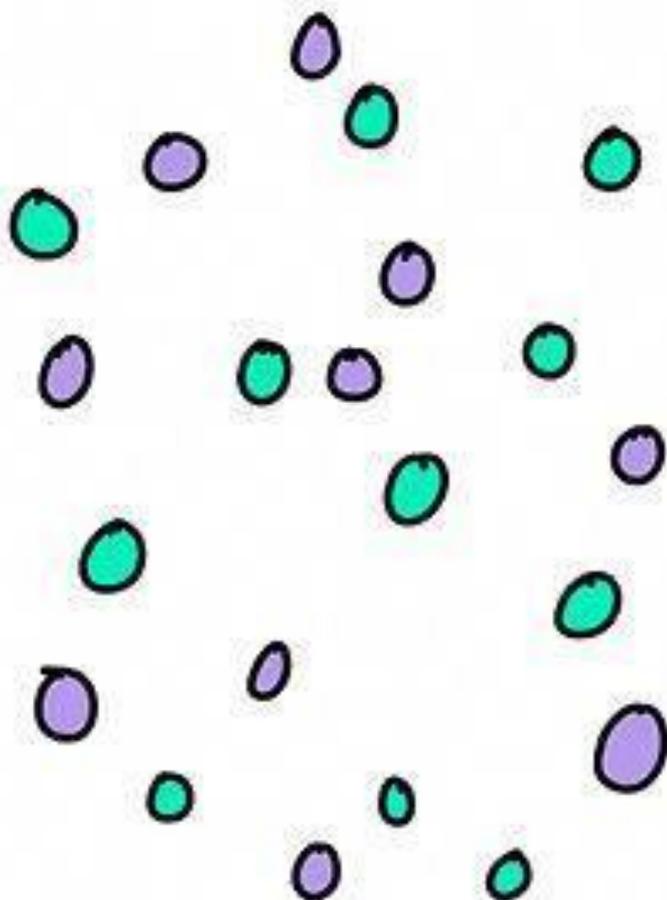
# In the best of all situations, students should:

- ▶ Share ideas – right or wrong – and defend them to others.
  - ▶ Listen to others and try to make sense of their ideas and together come up with a solution.
  - ▶ Decide if the answer was correct without looking at the back of the book or asking the instructor.
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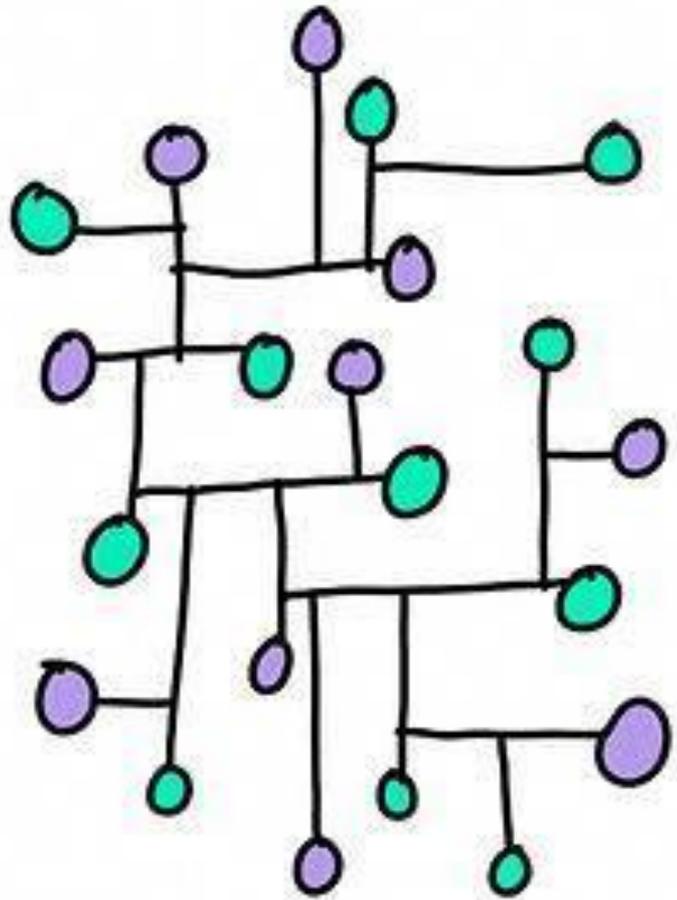
# How Do We Get There?

- ▶ Students need to understand what the problem is asked AND if they have the skills, knowledge, and tools to solve it currently.
  - ▶ Use applications – not just “Dolciani Word Problems”. Use connections to other classes and higher level mathematics classes.
  - ▶ Students need to understand if their answer is reasonable and that a correct answer includes units and is written in complete sentences.
  - ▶ Look for errors in other student’s work.
- 

information:



knowledge:



# Asking Questions

- ▶ What is the problem? What am I trying to figure out?
  - ▶ What do I know? What is the given information?
  - ▶ What do I need to know to solve the problem?
  - ▶ What problems like this have I solved before?
  - ▶ What solution could work? What strategies will work best in this situation?
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# Analytical Thinking

- ▶ Make the following fractions close to 1:  
 $X/24$ ,  $x/7$ ,  $5/y$ ,  $9/y$
- ▶ Make the following fractions close to  $\frac{1}{2}$  :  
 $x/90$ ,  $x/16$ ,  $4/y$ ,  $10/y$
- ▶ Which of the following is larger in each pair and why?
  - a.  $1/5$  or  $1/6$
  - b.  $2/5$  or  $2/7$
  - c.  $2/5$  or  $3/5$
  - d.  $3/4$  or  $2/3$

# Integrative Thinking (Synthesis)

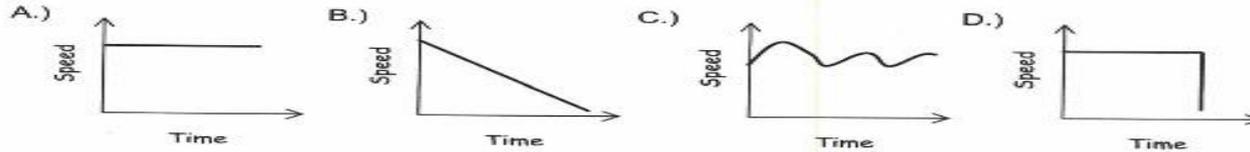
- ▶ Explain the difference between the directions “solve”, “evaluate”, and “simplify”. Write an example using each direction with the expression  $3(x + 2) - x$ .
- ▶ Create three quadratic equations: one have two distinct real solutions, one having two complex solutions, and one have exactly one real solution.

# Balanced Thinking

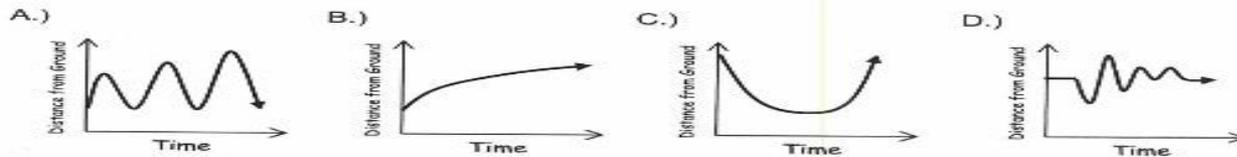
- ▶ The inequality  $x^2 + 1 < -5$  has no solutions. Explain why.
- ▶ Is the inequality  $x^2 + 1 > 1$  true for all real numbers? Explain.

# Applied Thinking (Applications)

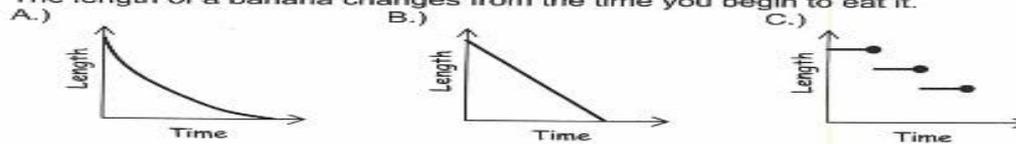
1. A train pulls into a station and lets off its passengers.



2. A child swings on a swing.



3. The length of a banana changes from the time you begin to eat it.



# Applications (con't.)

- ▶ The number of volunteers in the US can be modeled by  $V(t) = 1.3t + 51.6$  where  $V(t)$  is the number of volunteers in millions since 2000.
  - What is the  $t$ -intercept and what does it represent?
  - Find  $V(10)$  and explain its meaning.
  - What is the slope of this model and what does it represent?

# Multidimensional Thinking

1. State the circumstances that causes the graph of a quadratic function to have no  $x$ -intercepts.
  2. Can a quadratic function have a range of all real numbers? Justify your answer.
  3. Can the graph of a polynomial function have no  $y$ -intercept? Can it have no  $x$ -intercepts?
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# Creative Thinking

HOW DO YOU KNOW ...

- ▶ ... that a triangle with vertices  $(2, 6)$ ,  $(0, -2)$ , and  $(5, 1)$  is an isosceles triangle?
- ▶ ... that  $x^2 + y^2 + 6x - 10y = 12$  is an equation of a circle and not a parabola?
- ▶ ... which way a parabola opens?
- ▶ ... whether an ellipse's center is the origin or another point?

Some classroom  
assessment techniques  
that promote critical  
Thinking

# Chain Problems

Groups of 4 – One equation (trig identity, derivative, integral, diff eq., matrix, etc.) per piece of paper.

Each person does the *first* step only and then passes paper to the right.

Next person checks the previous work and then does the *next* step only.

Etc.



# Math Jeopardy

The answer is 3. What is the question?

- What is the name of the variable in  $16z^2 - 3z + 5$ ?
- What is the degree of the polynomial  $2mn + 6m - 3$  ?
- How many terms are in the polynomial  $2mn + 6m - 3$ ?
- What is the coefficient of  $b$  in the polynomial  $3a^2b - 9a + 5b$ ?

# Find the Error/Correct the Quiz

- ▶ In this activity students can work as a team or individually to grade a student quiz.
  - ▶ If the answer is correct, students can mark correct. If the answer is wrong, mark it wrong and show the correct answer.
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# One Final Thought

Access provides  
education, but  
completion changes  
lives.



# Contact Information

Thanks for attending! Enjoy the rest of the conference!

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