Blending Randomization-Based Approaches and the Central Limit Theorem

Session S144, 1:05-1:55 Saturday, November 11

Matt Rogala, Westchester Community College
Helen Burn, Highline College
Outline

• Brief history of randomization-based approaches
• Helen’s approach – low tech
• Matt’s approach – high tech
• Questions
Randomization-Based Approaches


“Our curriculum is needlessly complicated because we put the normal distribution, as an approximate sampling distribution for the mean, at the center of our curriculum, instead of putting the core logic of inference at the center” (p. 4).

Major names in curriculum that followed: Beth L. Chance, Nathan Tintle, Allan J. Rossman, the Lock family, many others
Helen Burn’s Approach

- Simulations to motivate the Central Limit Theorem
- Formal treatment of confidence intervals
- Formal treatment of hypothesis testing
- Randomization based approaches to reinforce logic of hypothesis testing and meaning of p values
Prerequisites

- Experience early on with simulations
- Classroom culture of collaboration and trust

- Properties of normal curve properties
- Binomial Theorem (optional)
Week 1 Simulation to Reinforce Sample Variability

Question: What percentage of your chickens are diseased?

Compute percentage with

n = 10
n = 20
n = 30

Sample variability decreased as sample size increases
Simulation to Demonstrate Central Limit Theorem: ~30 minutes

Cards represent chickens: Red = infected, Black = healthy

Draw a random sample of 16 chickens and count the number that are infected (e.g., red)

Do this at least five times with your partner and then report out

Keep an open mind---this playful activity is going to show one of the most important theorems of statistics
Blending Randomization-Based Approaches

Sample variability: \( \bar{x} = 1 \) sample of size 16

Order out or randomize

Central Limit Theorem

The sampling distribution

\[ \begin{array}{cc}
26 & 26 \\
YES & NO
\end{array} \]

Sample = 16, less than 8.0, over 8.0

\( \bar{x} \) generated
Simulation to Demonstrate Central Limit Theorem: ~30 minutes

Distribution of $\bar{X}$

Cards represent children’s ages: Ace = 1 . . . Jack = 11, Queen = 12, King = 13

Draw a random sample of 20 children and compute the average age of the sample.

Do this at least five times with your partner and then report out
Central Limit Theorem

Sampling distribution will be a Normal Curve.

\[
\bar{X} \\
N = 16
\]

Out of Randomness

Data of size 16

More than 2 hrs

10 hrs

0 percentage

Uniform distribution
Next Class Session: Review what we did using Software

Easy transition to presenting the Central Limit Theorem

The sample distribution is normal: \( N(\mu, \sigma/\sqrt{n}) \) or \( N\left( p, \sqrt{pq/n} \right) \)
Next Class Session: Emphasize that sample size decreases variability (e.g., standard deviation)

Can emphasize the role of $n$ in the Standard Error:

$$\sqrt{\frac{.5 \times .5}{16}} = .125$$
$$\sqrt{\frac{.5 \times .5}{50}} \approx .07$$
Transition to Confidence Intervals
You have one shot to determine the proportion of chickens that are infected.

Sample Statistic should be a good estimate of population parameter +/- a margin of error

Introduce the concept of a Confidence Interval
Next Example: Public Opinion Polls
Next Example: Public Opinion Polls

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<th>RCP Average</th>
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<td>-40</td>
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</table>

From polling source:

Daily results are based on telephone interviews with approximately 1,500 national adults; Margin of error is ±3 percentage points.
Now move to a formal treatment of confidence intervals and hypothesis testing (~ 1.5 weeks)

- Assumptions
- How to compute (with calculator or by hand using tables)
- How to interpret

*I am 95% confident that . . . .* (communicate that this is about the population parameter)

*At the $\alpha = .05$ level,* (make correct rejection decision and conclusion)
Reinforcing Logic of Hypothesis Testing through Randomization-Based Approaches

http://www.math.hope.edu/stats/ (Matt, who should I give credit to here?)

Resources for
Math 210: Introductory Statistics

**Applets and Data**
- Applets
- Data Sets
- Applets & Data Sets with iPad Instructions
- Monty Hall simulation for Exploration P.3
- Data Sets for Math 312

**Video Tutorials for Simulation-Based Tests**
- Chance Models - One Proportion
- Two Proportions
- Two Means
- Paired Data
**The Dolphin Study**

Buzz gets 15/16 correct after some conditioning. What is the probability he is just guessing?

Reinforces the meaning of the p value
Assessment Results on Confidence Intervals (n = 30)

CI for a population proportion given summary statistics (e.g., 26 out of 50)
100% computed the confidence interval
80% (n = 24) interpreted correctly
87% (n = 26) correctly computed the margin of error

CI for a population mean from raw data (X = times per week person used gym membership)
93% (n = 28) correctly computed the confidence interval
83% (n = 25) correctly interpreted the meaning of the confidence interval
77% (n = 23) correctly computed the Margin of Error
Assessment Results Hypothesis
Testing for Mean

Expectation
• State sample data
• Compute test statistic
• Make reject decision
• Write conclusion

Class 1:
Completely correct: 55%
All good except conclusion: 23%

Class 2:
Completely correct: 76%
All good except conclusion: 15%

Example: At the $\alpha = .05$ level, I fail to reject the null hypothesis. The sample data are not statistically significant. There is insufficient evidence to conclude that the mean age of an employee is more than 40.
Matt’s Approach – Background

- Use the fully randomization-based approach in *Introduction to Statistical Investigations* (Tintle et al.) during Fall/Spring semesters

- Previously (2014-15) compared randomization-based sections to traditional (CLT) sections in the same semester

- Experimented with blended approach during Summer 2016 and Summer 2017 for quicker iteration
Matt’s Approach – Topic Outline

• Unit 1: Statistical Studies and Study Design
  • Statistical process, sampling methods, observational studies vs. experiments, other study design considerations

• Unit 2: Data and Descriptive Statistics
  • Types of data, managing data, presenting and summarizing data
Topic Outline (Cont.)

• Unit 3: Probability and Random Processes
  • Basic probability, modeling random processes, sampling distributions, Central Limit Theorem and continuous probability distributions

• Unit 4: Statistical Inference
  • Significance tests and confidence intervals for each type of study covered by the course
Sampling Distribution Lesson/Activity

• Follows lesson on modeling random processes

• Students given envelopes with data from pre-class survey written on index cards

• Students randomly choose five different samples of size 10 from the data, compute sample mean for each

• Students plot their sample means on a dotplot on the board
Sampling Distribution Activity (Cont.)

<table>
<thead>
<tr>
<th>Sampling Distributions</th>
<th>Mean</th>
<th>Proportion</th>
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<tbody>
<tr>
<td>Theoretical Distributions</td>
<td>Normal</td>
<td>t</td>
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<tr>
<td>More Advanced Randomization Tests</td>
<td>$\chi^2$ Goodness-of-Fit</td>
<td>$\chi^2$ Test for Association</td>
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</tbody>
</table>
Sampling Distribution Activity (Cont.)
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Central Limit Theorem Lesson

• Motivate the discussion using sampling distributions from previous section

• Introduce continuous random variables, probability density functions, normal distribution

• Compute probabilities using normal distribution, note similarity to computing probabilities in sampling distribution applet
Central Limit Theorem Lesson
Central Limit Theorem Lesson

• State the Central Limit Theorem for One Proportion and use it to compute probabilities for One Proportion sampling distributions

• State the Central Limit Theorem for One Mean and note the problem of needing the population standard deviation

• Introduce t-distribution, degrees of freedom, and standardizing variables
Central Limit Theorem Lesson

[Graph showing a normal distribution with shaded areas for left, right, and two-tailed tests.]

Edit Parameters

Please select degrees of freedom.

df: 9

Ok (or hit Enter)
Applying Methods to Inference

- Applets offer a choice between simulation-based inference and CLT-based inference.
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<th></th>
<th>Spring 2017 Final (n = 25)</th>
<th>Summer 2017 Final (n = 21)</th>
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<tr>
<td>Confidence Intervals</td>
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<td>86.5%</td>
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