Blending Randomization-Based Approaches and the Central Limit Theorem

Session S144, 1:05-1:55 Saturday, November 11

Matt Rogala, Westchester Community College
Helen Burn, Highline College

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Outline

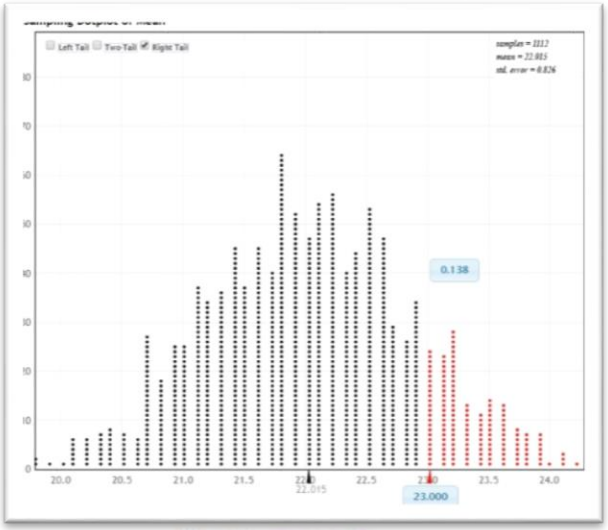
- Brief history of randomization-based approaches
- Helen's approach – low tech
- Matt's approach – high tech
- Questions

Randomization-Based Approaches

Cobb, G. W. (2007). The introductory statistics curriculum: A Ptolemaic curriculum? *Technology Innovations in Statistics, 1(1).*

"Our curriculum is needlessly complicated because we put the normal distribution, as an approximate sampling distribution for the mean, at the center of our curriculum, instead of putting the core logic of inference at the center" (p. 4).

Major names in curriculum that followed:
Beth L. Chance, Nathan Tintle,
Allan J. Rossman, the Lock family, many others



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Helen Burn's Approach

- Simulations to motivate the Central Limit Theorem
- Formal treatment of confidence intervals
- Formal treatment of hypothesis testing
- Randomization based approaches to reinforce logic of hypothesis testing and meaning of p values

Prerequisites

- Experience early on with simulations
- Classroom culture of collaboration and trust

- Properties of normal curve properties
- Binomial Theorem (optional)

Week 1 Simulation to Reinforce Sample Variability

Question: What percentage of your chickens are diseased?

Compute percentage with

n = 10
n = 20
n = 30

Sample variability decreased as sample size increases

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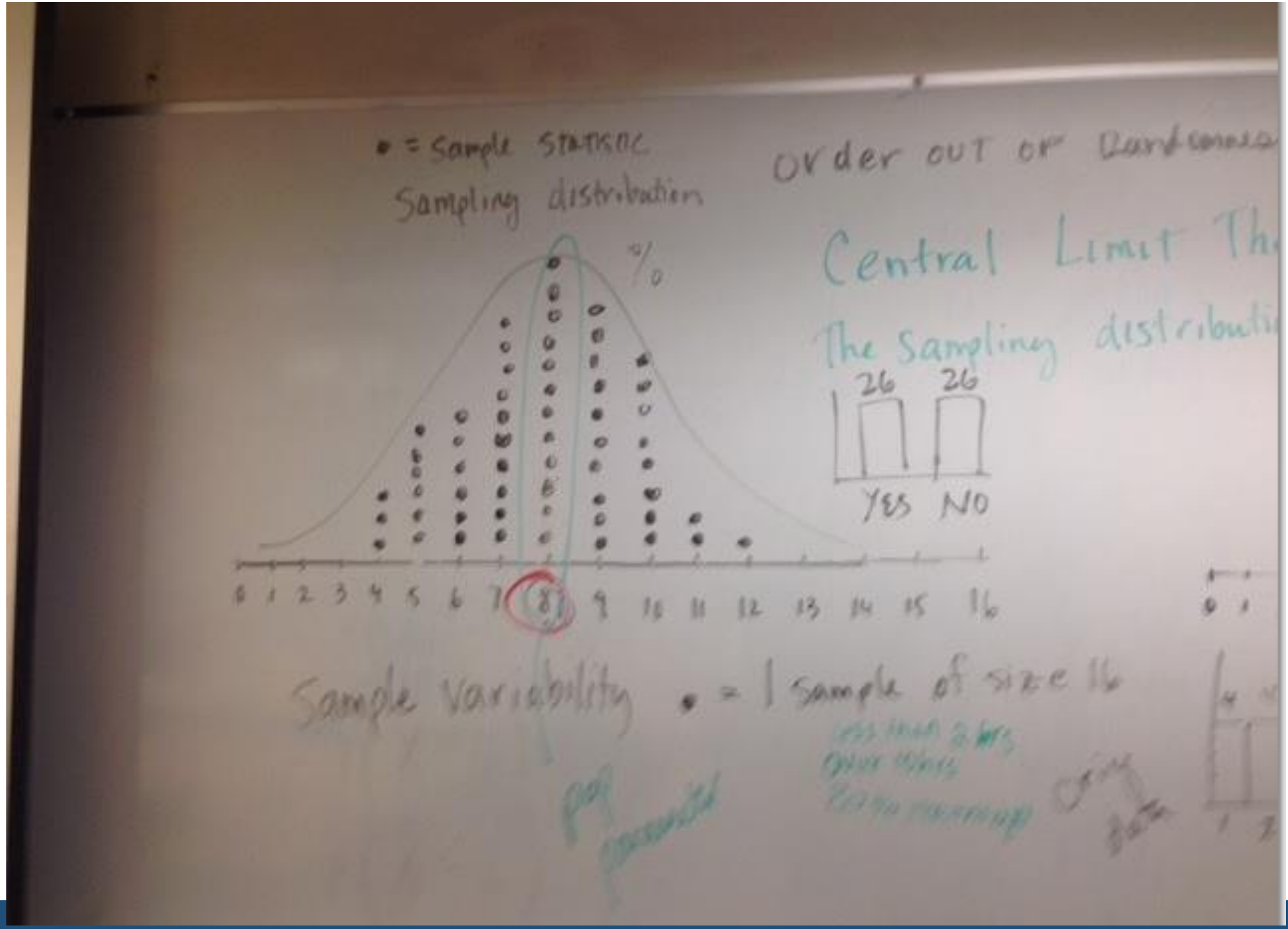
Simulation to Demonstrate Central Limit Theorem: ~30 minutes

Cards represent chickens: Red = infected, Black = healthy

Draw a random sample of 16 chickens and count the number that are infected (e.g., red)

Do this at least five times with your partner and then report out

Keep an open mind---this playful activity is going to show one of the most important theorems of statistics


• = Sample statistic
Sampling distribution
%
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Central Limit Th
The Sampling distributi
26 26
YES NO
Sample variability
• = 1 sample of size 16

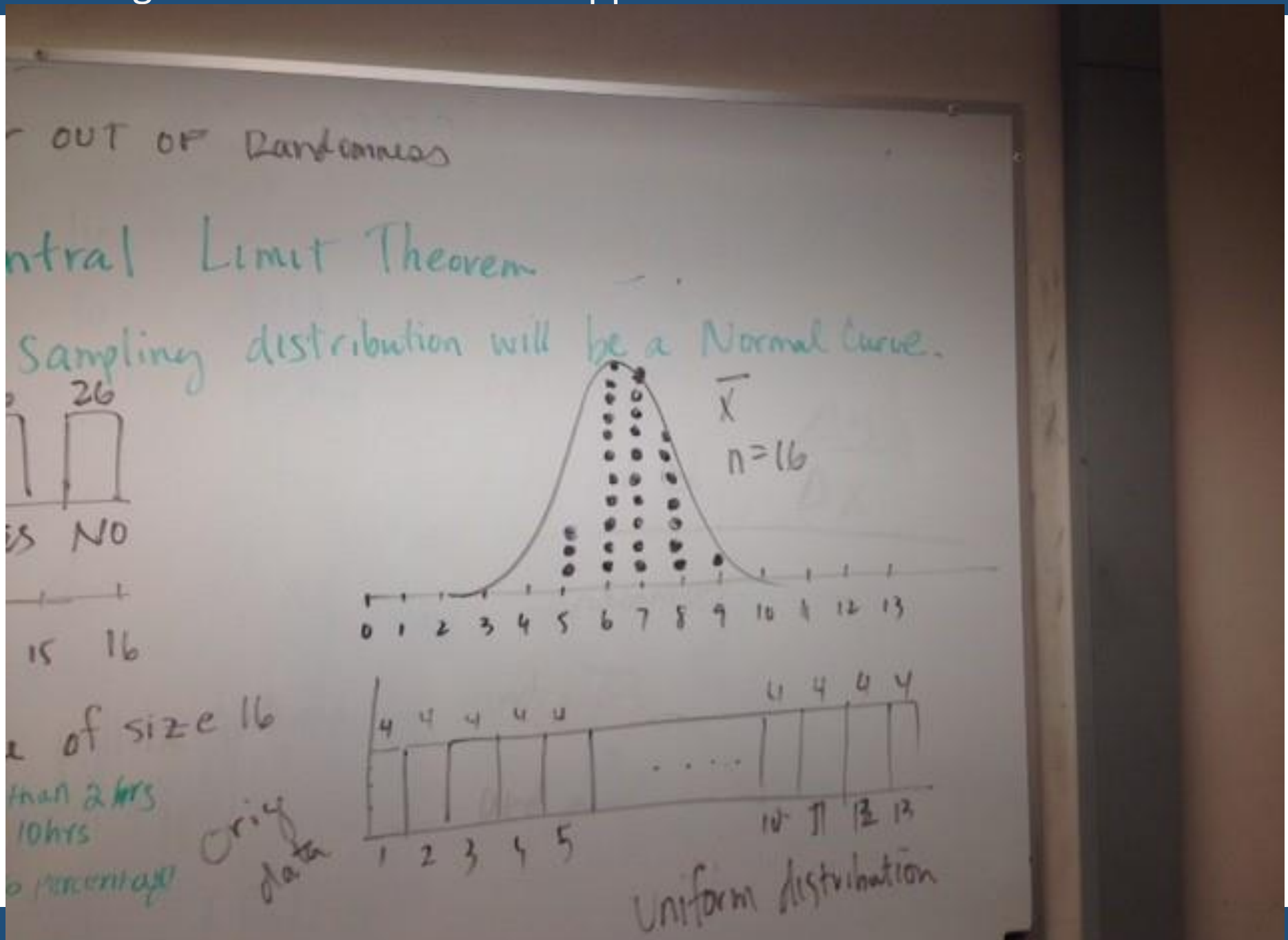
Simulation to Demonstrate Central Limit Theorem: ~30 minutes

Distribution of $\overline{X}$

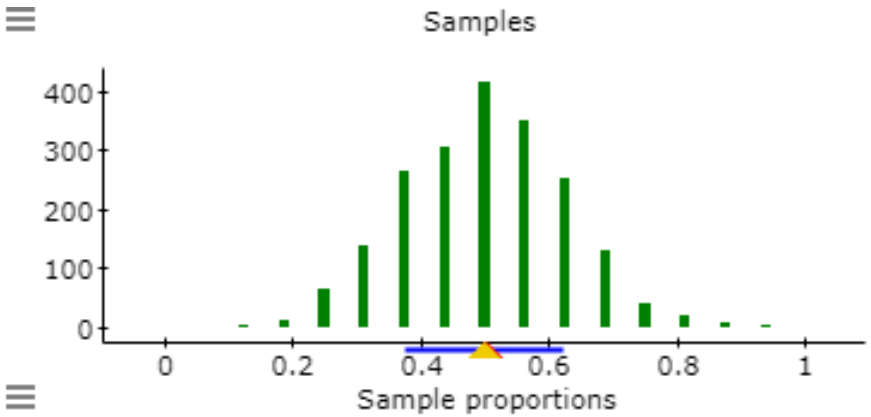
Cards represent children's ages: Ace = 1 . . . Jack = 11, Queen = 12, King = 13

Draw a random sample of 20 children and compute the average age of the sample.

Do this at least five times with your partner and then report out


OUT OF Randomness
ntral Limit Theorem
Sampling distribution will be a Normal Curve.
26
NO
15 16
of size 16
than 2 hrs
10 hrs
percentage
orig data
$\bar{x}$
n=16
0 1 2 3 4 5 6 7 8 9 10 11 12 13
4 4 4 4 4
4 4 4 4
1 2 3 4 5
Uniform distribution

Next Class Session: Review what we did using Software



Sample prop. of 1s

# of Samples	2000
Mean	0.4991
Median	0.5
Std. dev.	0.1236

Easy transition to presenting the Central Limit Theorem

The sample distribution is normal: $N(\mu, \sigma/\sqrt{n})$ or $N\left(p, \sqrt{\frac{pq}{n}}\right)$

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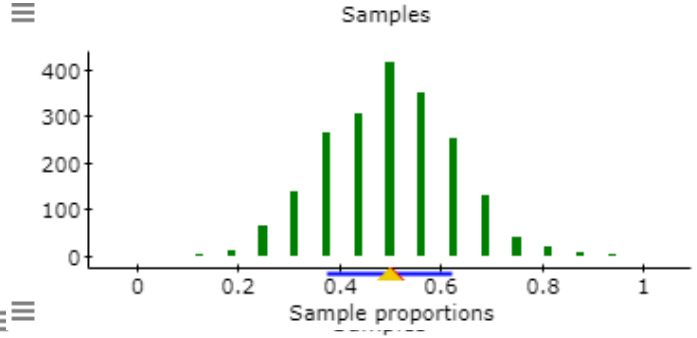
Next Class Session: Emphasize that sample size decreases variability (e.g., standard deviation)



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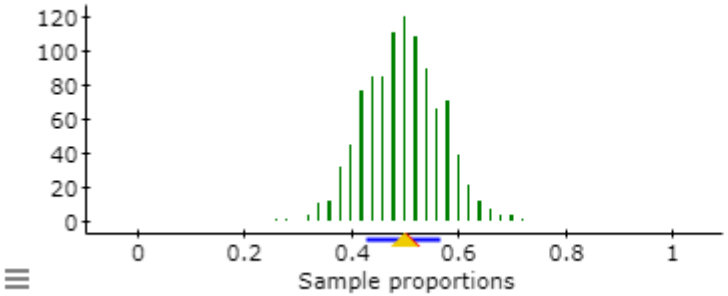

Sample prop. of 1s

# of Samples	1000
Mean	0.4966
Median	0.5
Std. dev.	0.0692

Can emphasize the role of n in the Standard Error : $\sqrt{\frac{.5*.5}{16}} = .125$ $\sqrt{\frac{.5*.5}{50}} \approx .07$

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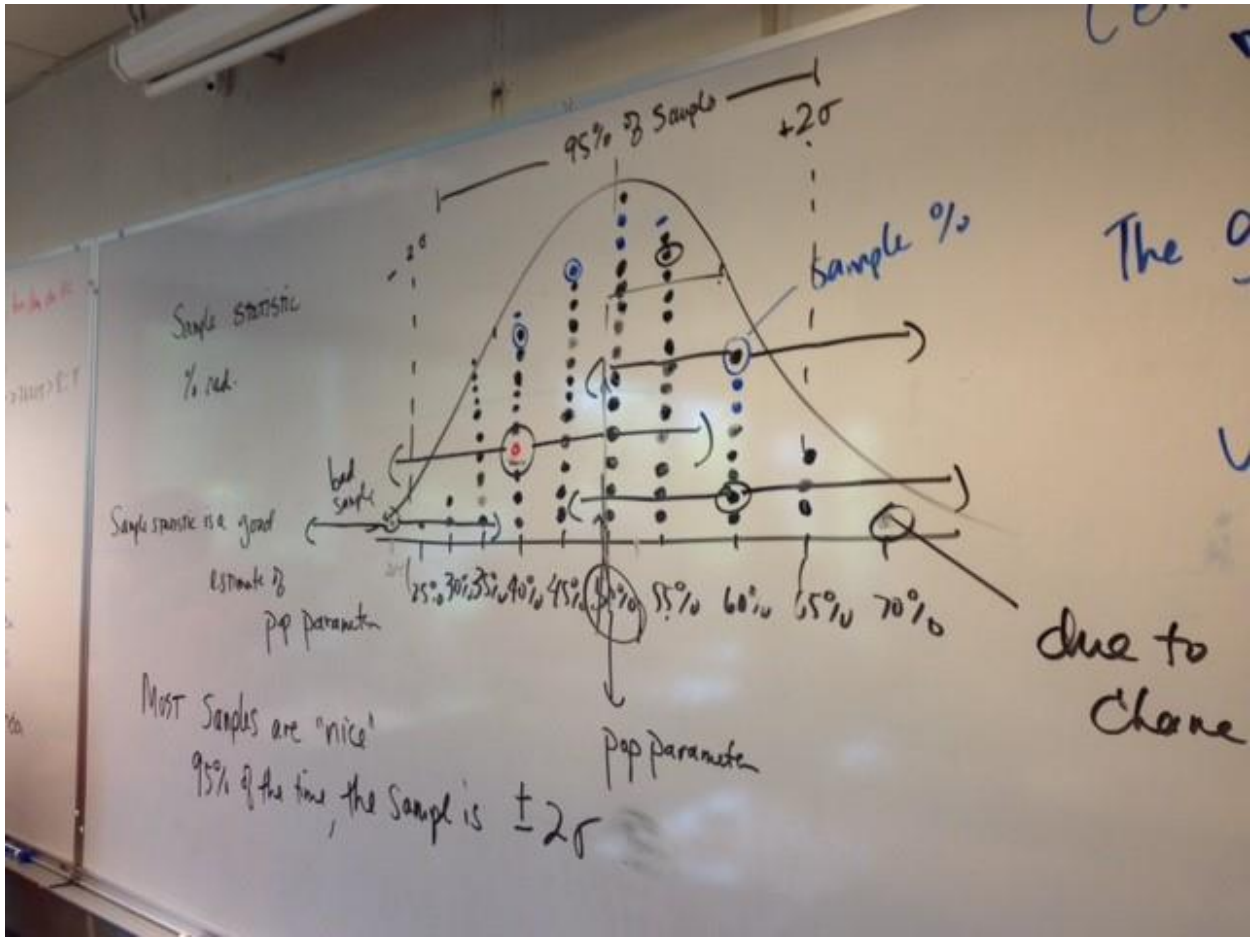
Transition to Confidence Intervals

You have one shot to determine the proportion of chickens that are infected.



Sample Statistic should be a good estimate of population parameter +/- a margin of error

Introduce the concept of a Confidence Interval

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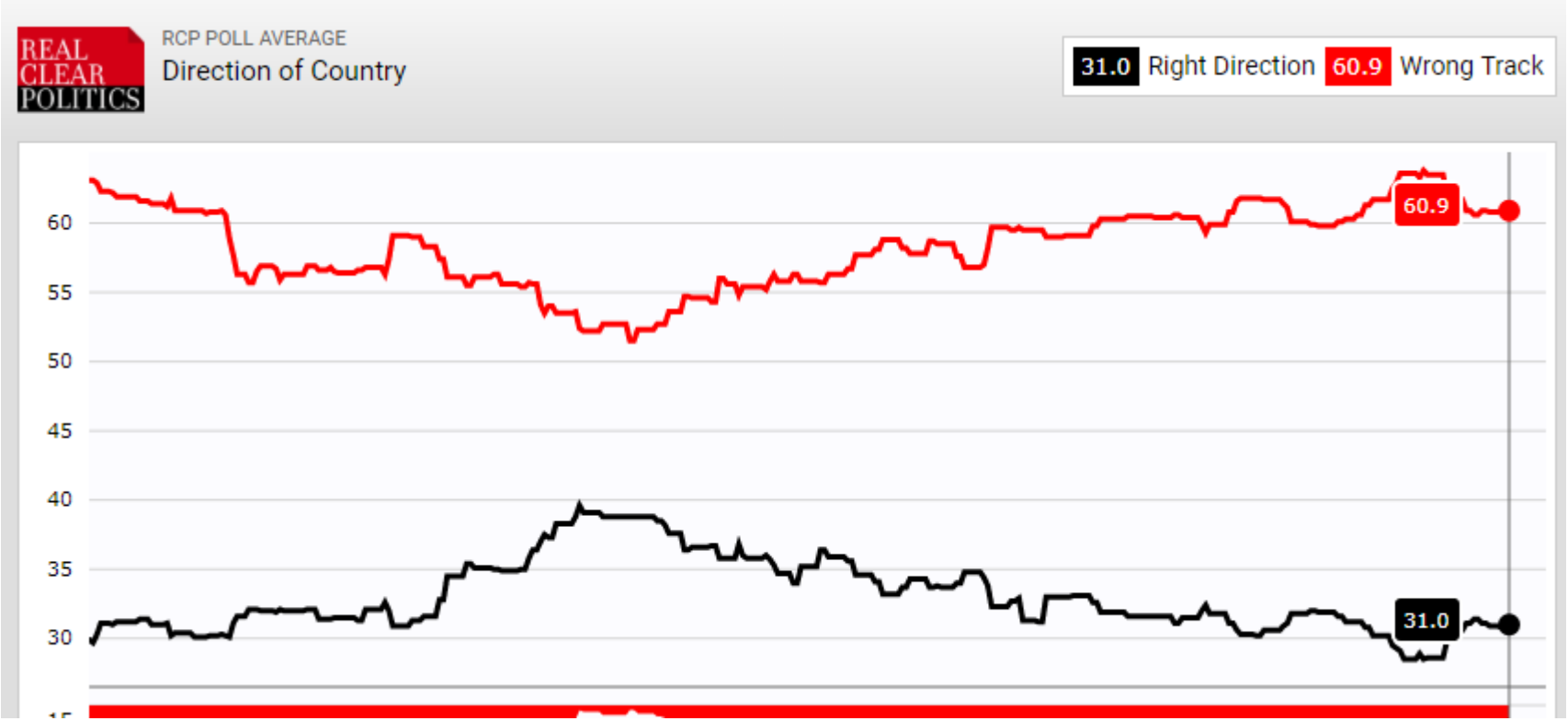
Next Example: Public Opinion Polls



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Next Example: Public Opinion Polls

RCP Average	9/27 - 10/31	--	31.0	60.9	-29.9
CBS News	10/27 - 10/30	1109 A	31	64	-33
Economist/YouGov	10/29 - 10/31	1291 RV	31	58	-27
Reuters/Ipsos	10/27 - 10/31	1798 A	26	63	-37
Rasmussen Reports	10/21 - 10/26	2500 LV	32	62	-30
Harvard-Harris	10/14 - 10/18	2159 RV	29	60	-31
Marist	10/15 - 10/17	928 RV	31	63	-32
Emerson	10/12 - 10/14	820 RV	40	50	-10
IBD/TIPP	9/29 - 10/8	887 A	35	64	-29
USA Today/Suffolk	9/27 - 10/1	1000 RV	24	64	-40

From polling source:

Daily results are based on telephone interviews with approximately 1,500 national adults; Margin of error is ±3 percentage points.

Now move to a formal treatment of confidence intervals and hypothesis testing (~ 1.5 weeks)

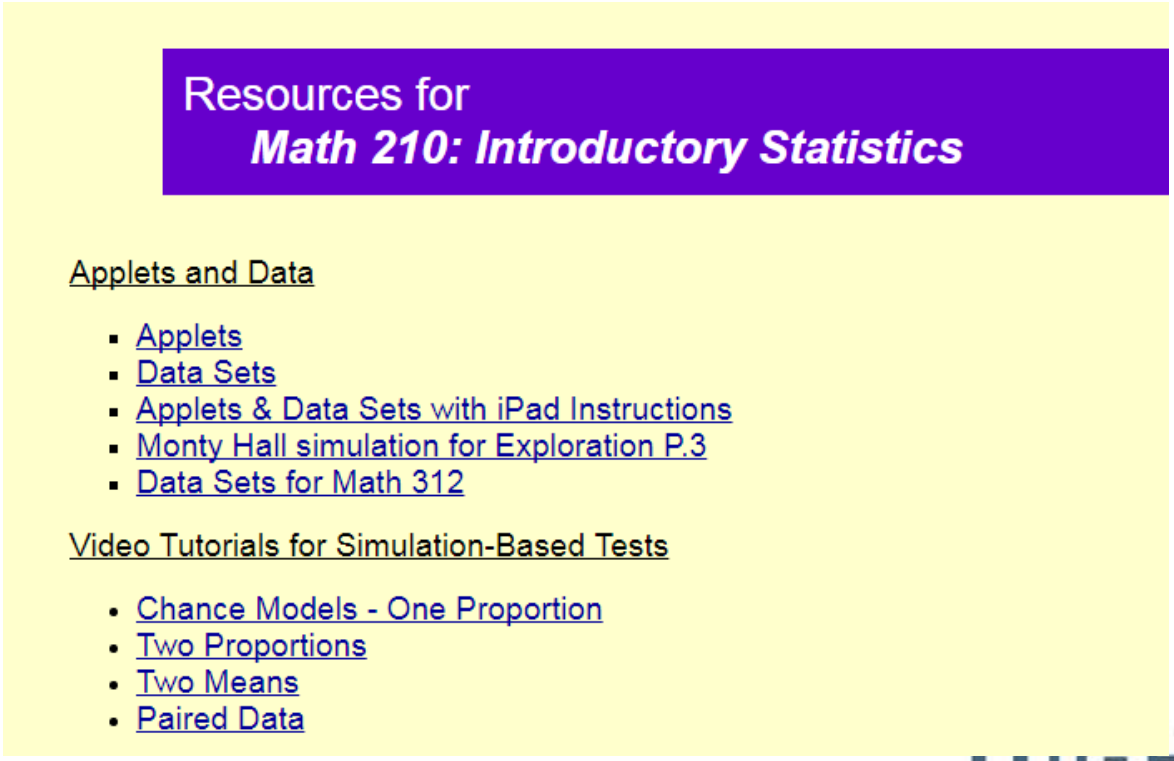
- Assumptions
- How to compute (with calculator or by hand using tables)
- How to interpret

I am 95% confident that . . . . (communicate that this is about the population parameter)

At the $\alpha = .05$ *level,* (make correct rejection decision and conclusion)

Reinforcing Logic of Hypothesis Testing through Randomization-Based Approaches

http://www.math.hope.edu/stats/ (Matt, who should I give credit to here?)



Resources for
Math 210: Introductory Statistics

Applets and Data

- Applets
- Data Sets
- Applets & Data Sets with iPad Instructions
- Monty Hall simulation for Exploration P.3
- Data Sets for Math 312

Video Tutorials for Simulation-Based Tests

- Chance Models - One Proportion
- Two Proportions
- Two Means
- Paired Data

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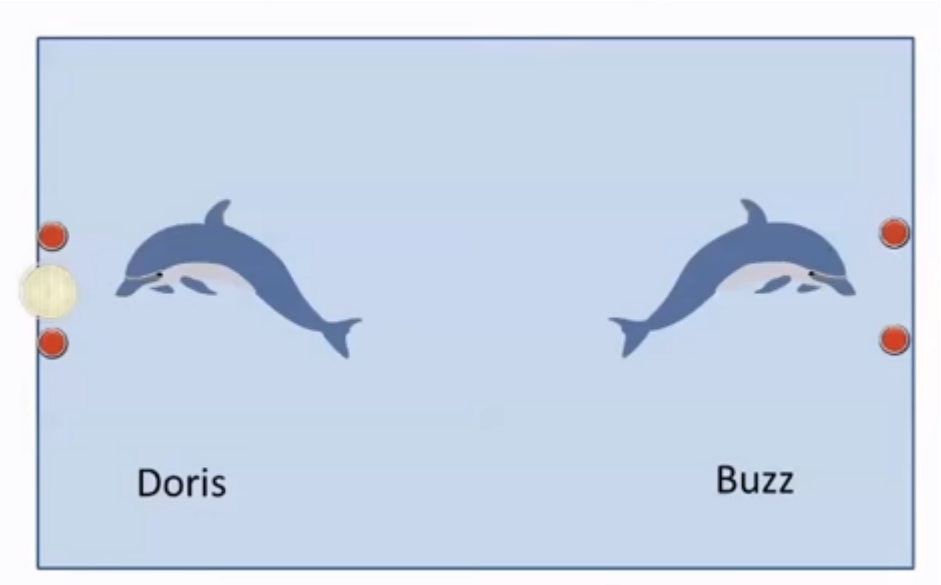
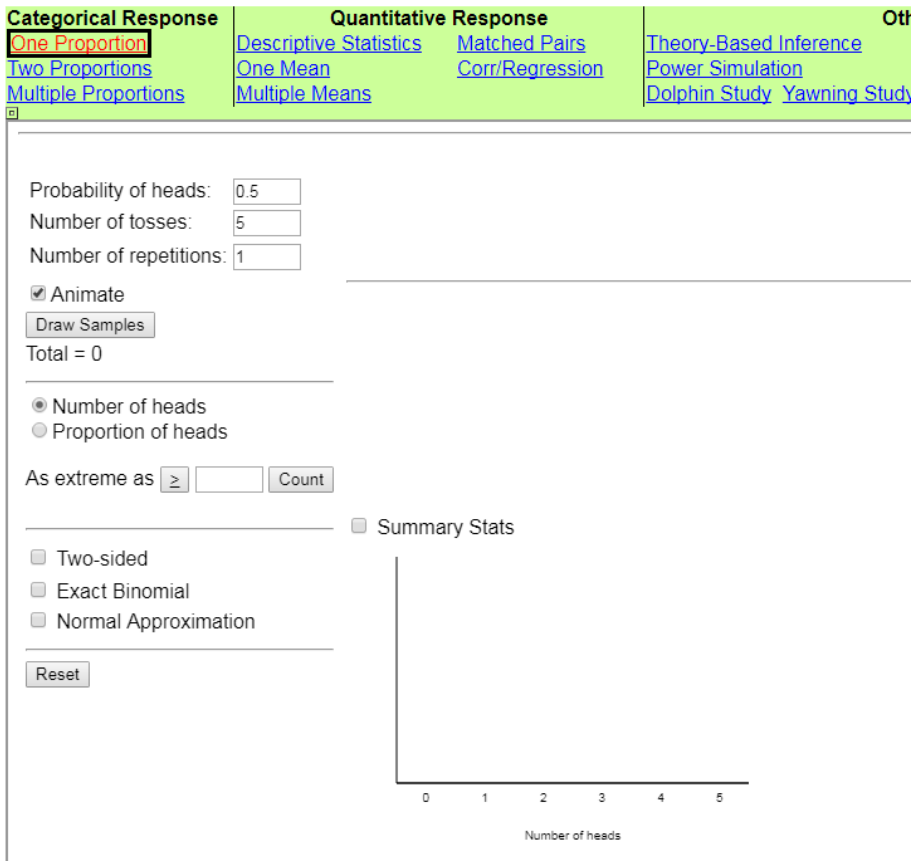
The Dolphin Study



Buzz gets 15/16 correct after some conditioning. What is the probability he is just guessing?

Reinforces the meaning of the p value



Assessment Results on Confidence Intervals (n = 30)

CI for a population proportion given summary statistics
(e.g., 26 out of 50)
100% computed the confidence interval
80% (n = 24) interpreted correctly
87% (n = 26) correctly computed the margin of error

CI for a population mean from raw data (X = times per week person used gym membership)
93% (n = 28) correctly computed the confidence interval
83% (n = 25) correctly interpreted the meaning of the confidence interval
77% (n = 23) correctly computed the Margin of Error

Assessment Results Hypothesis Testing for Mean

Expectation

- State sample data
- Compute test statistic
- Make reject decision
- Write conclusion

Class 1:
Completely correct: 55%
All good except conclusion: 23%

Class 2:
Completely correct: 76%
All good except conclusion: 15%

Example: At the $\alpha = .05$ level, I fail to reject the null hypothesis. The sample data are not statistically significant. There is insufficient evidence to conclude that the mean age of an employee is more than 40.

Matt's Approach – Background

- Use the fully randomization-based approach in Introduction to Statistical Investigations (Tintle et al.) during Fall/Spring semesters
- Previously (2014-15) compared randomization-based sections to traditional (CLT) sections in the same semester
- Experimented with blended approach during Summer 2016 and Summer 2017 for quicker iteration

Matt's Approach – Topic Outline

- Unit 1: Statistical Studies and Study Design
  - Statistical process, sampling methods, observational studies vs. experiments, other study design considerations

- Unit 2: Data and Descriptive Statistics
  - Types of data, managing data, presenting and summarizing data

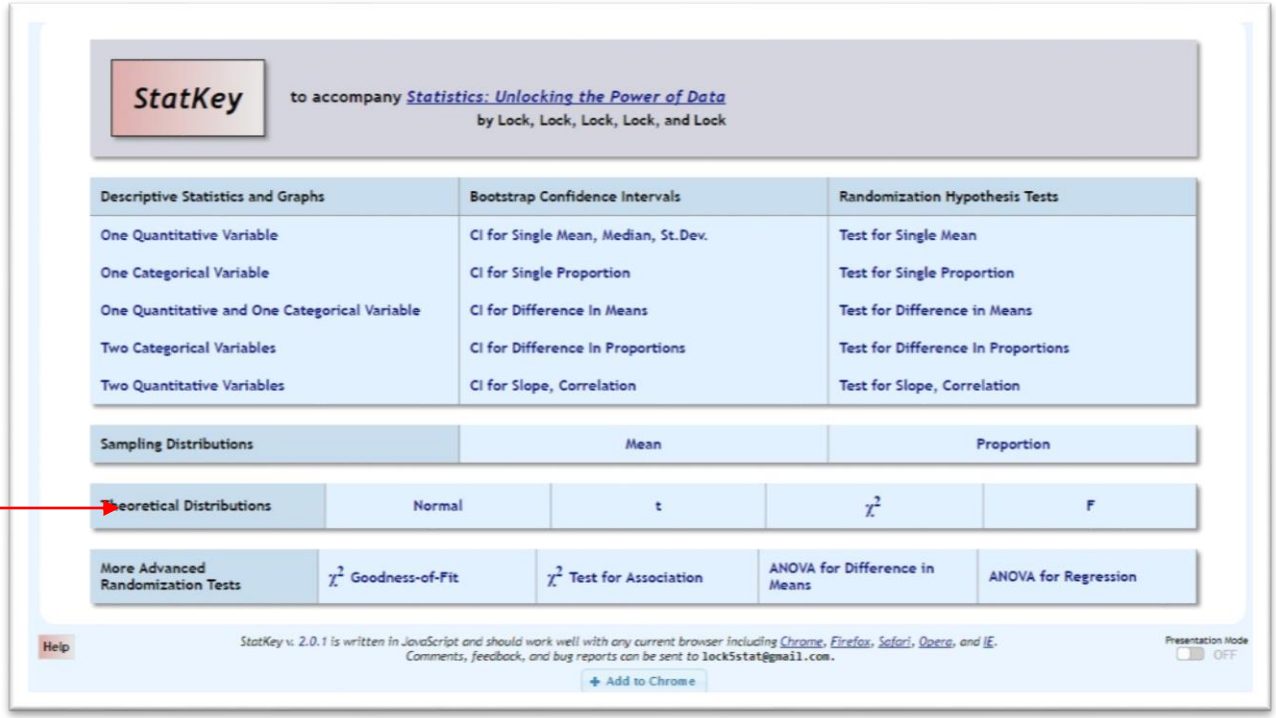
Topic Outline (Cont.)

- Unit 3: Probability and Random Processes
  - Basic probability, modeling random processes, sampling distributions, Central Limit Theorem and continuous probability distributions
- Unit 4: Statistical Inference
  - Significance tests and confidence intervals for each type of study covered by the course

Sampling Distribution Lesson/Activity

- Follows lesson on modeling random processes
- Students given envelopes with data from pre-class survey written on index cards
- Students randomly choose five different samples of size 10 from the data, compute sample mean for each
- Students plot their sample means on a dotplot on the board

Sampling Distribution Activity (Cont.)



StatKey to accompany *Statistics: Unlocking the Power of Data* by Lock, Lock, Lock, Lock, and Lock

Descriptive Statistics and Graphs	Bootstrap Confidence Intervals	Randomization Hypothesis Tests
One Quantitative Variable	CI for Single Mean, Median, St.Dev.	Test for Single Mean
One Categorical Variable	CI for Single Proportion	Test for Single Proportion
One Quantitative and One Categorical Variable	CI for Difference in Means	Test for Difference in Means
Two Categorical Variables	CI for Difference in Proportions	Test for Difference in Proportions
Two Quantitative Variables	CI for Slope, Correlation	Test for Slope, Correlation

Sampling Distributions	Mean		Proportion	
Theoretical Distributions	Normal	t	χ^2	F

More Advanced Randomization Tests				
	χ^2 Goodness-of-Fit	χ^2 Test for Association	ANOVA for Difference in Means	ANOVA for Regression

Help

StatKey v. 2.0.1 is written in JavaScript and should work well with any current browser including Chrome, Firefox, Safari, Opera, and IE.
Comments, feedback, and bug reports can be sent to lock5stat@gmail.com.

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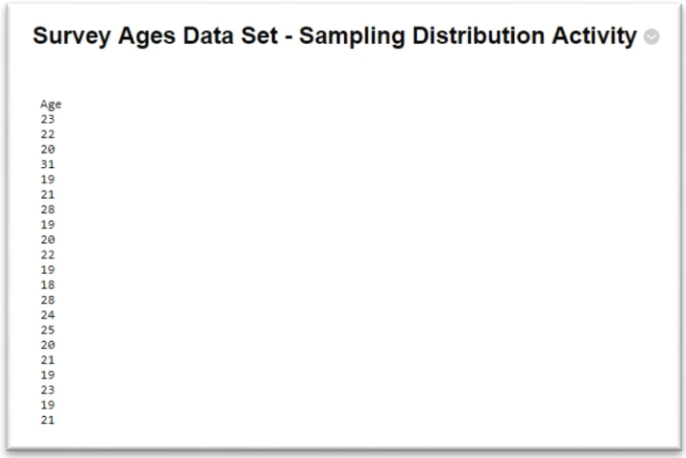
Sampling Distribution Activity (Cont.)



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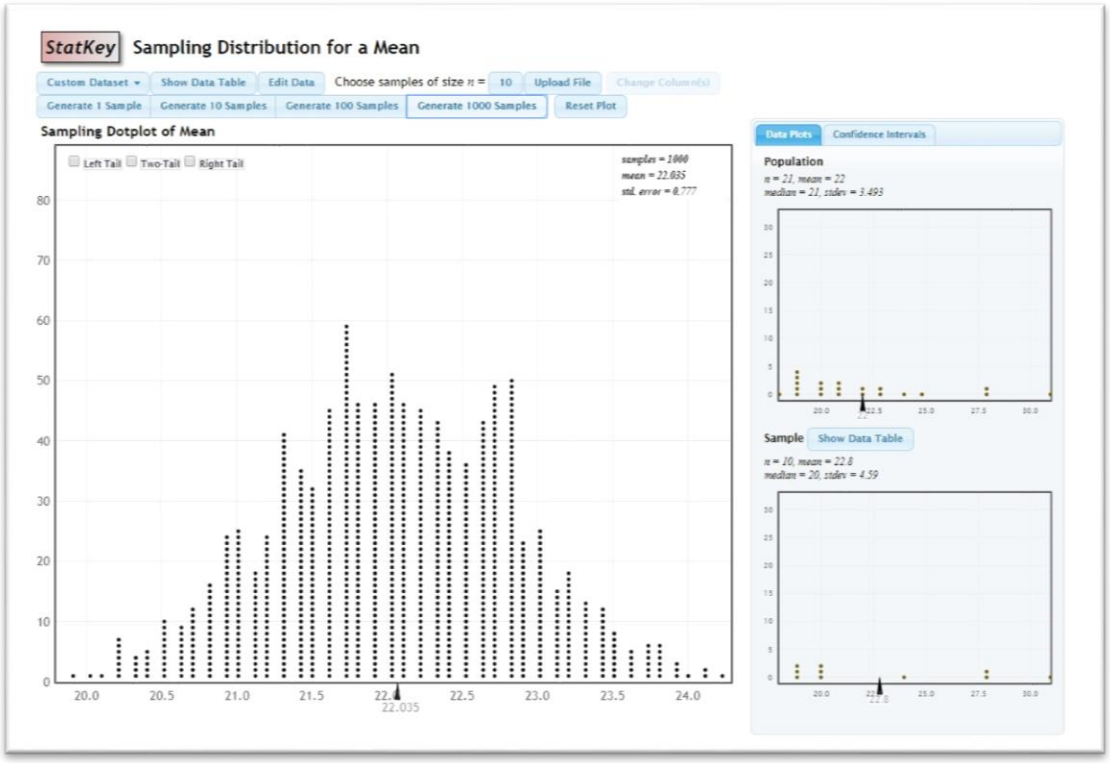
Sampling Distribution Activity (Cont.)



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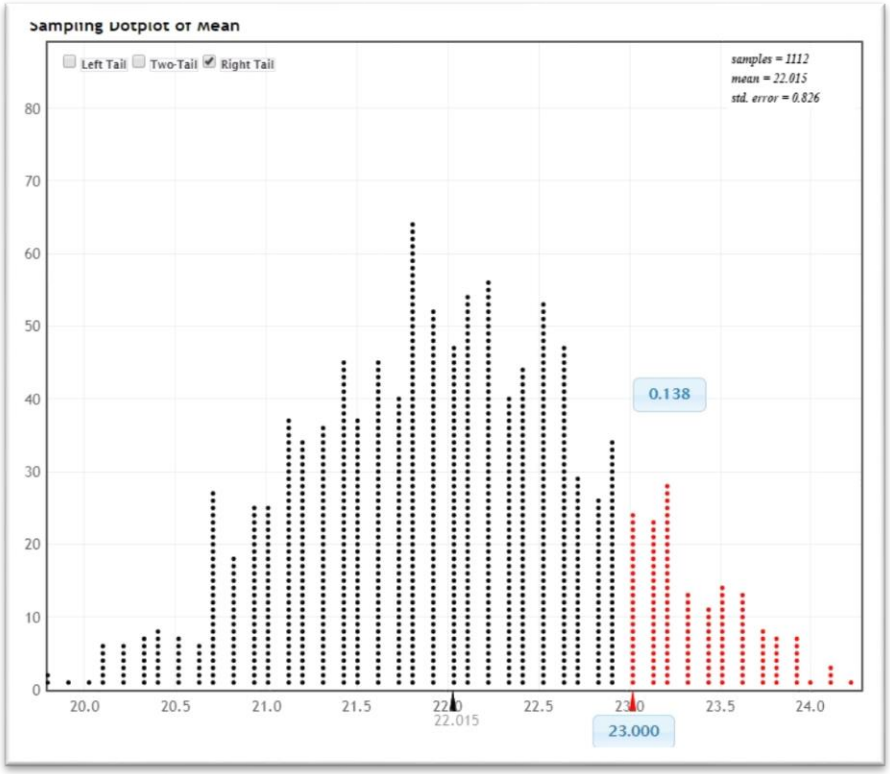
Sampling Distribution Activity (Cont.)



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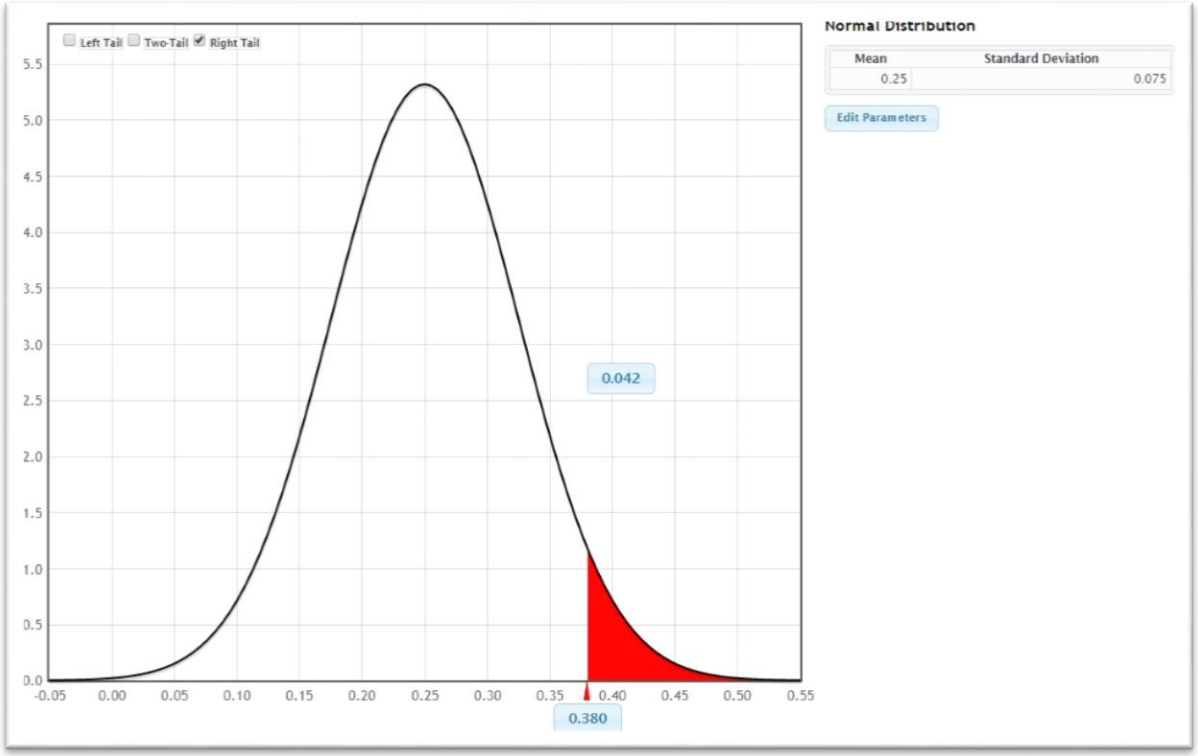
Central Limit Theorem Lesson

- Motivate the discussion using sampling distributions from previous section
- Introduce continuous random variables, probability density functions, normal distribution
- Compute probabilities using normal distribution, note similarity to computing probabilities in sampling distribution applet

Central Limit Theorem Lesson



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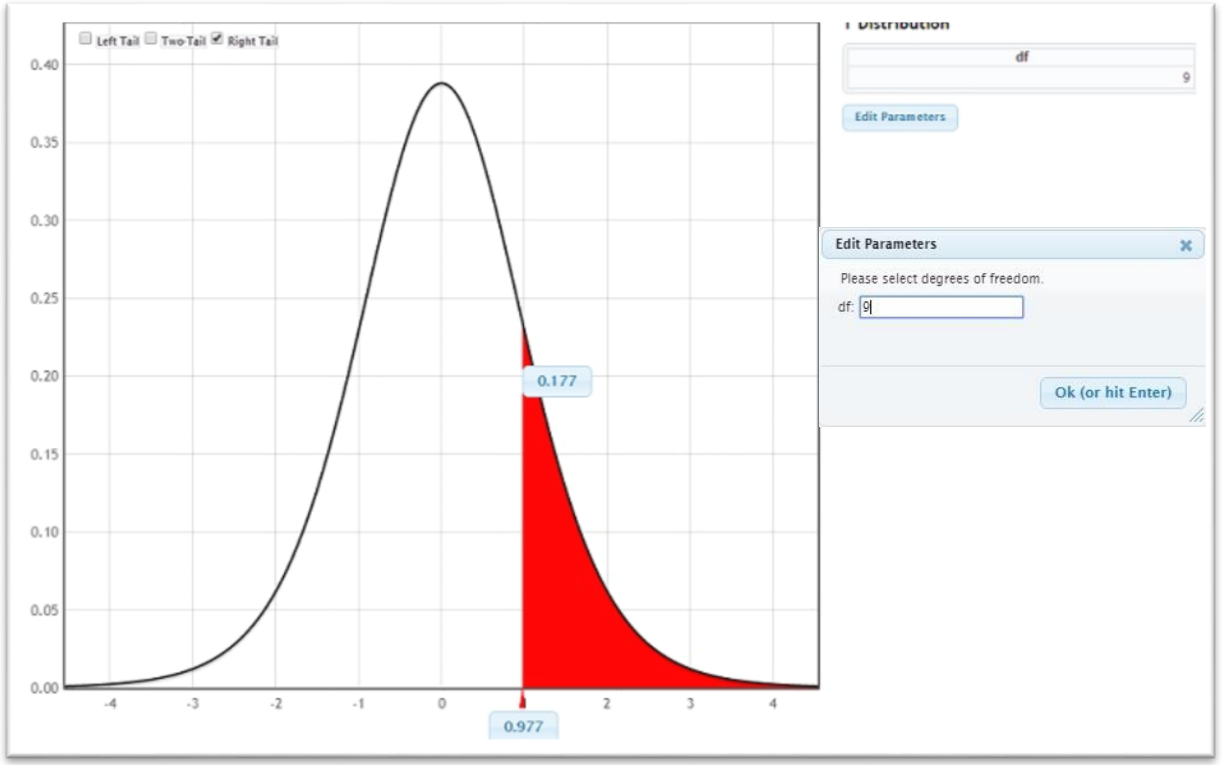
Central Limit Theorem Lesson

- State the Central Limit Theorem for One Proportion and use it to compute probabilities for One Proportion sampling distributions
- State the Central Limit Theorem for One Mean and note the problem of needing the population standard deviation
- Introduce t-distribution, degrees of freedom, and standardizing variables

Central Limit Theorem Lesson



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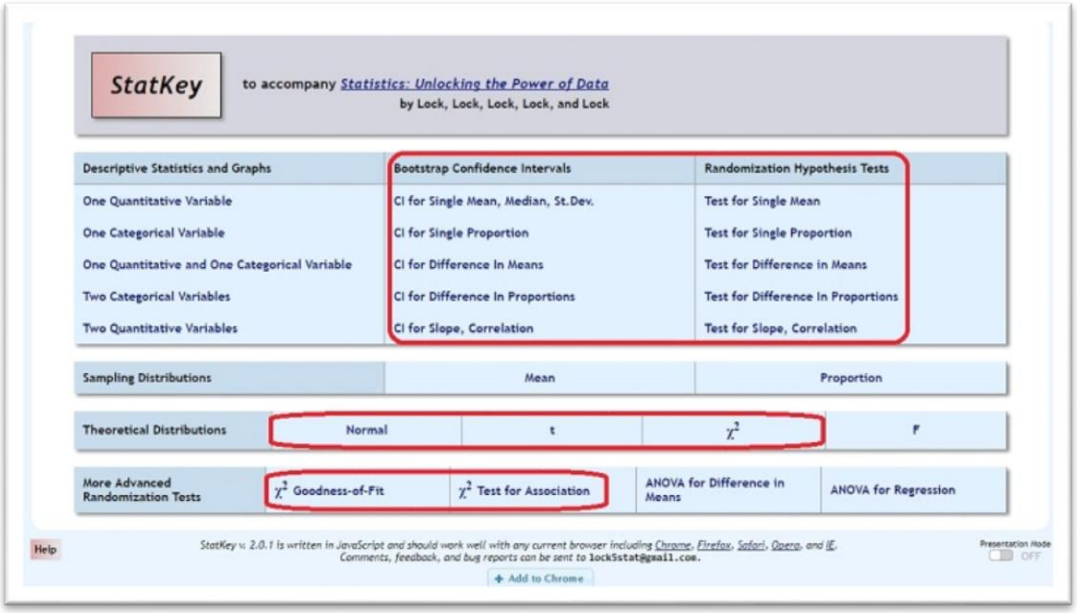
Applying Methods to Inference

- Applets offer a choice between simulation-based inference and CLT-based inference.



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Assessment Results

Spring 2017 Final (n = 25)	Summer 2017 Final (n = 21)
Study Design: 81.9%	Study Design: 90%
Variables and Data: 85.4%	Variables and Data: 94.3%
Descriptive Statistics: 83.5%	Descriptive Statistics: 96.6%
Hypothesis Statements: 79.0%	Hypothesis Statements: 89.1%
Significance Test Conclusions: 67.8%	Significance Test Conclusions: 92.1%
Confidence Intervals: 77.2%	Confidence Intervals: 86.5%

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