S145

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1:05 PM–1:55 PM

Making Transformations of Functions

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Transformations of Functions

Function	Transformations
Linear	$y = a\,(cx - h) + k$
Quadratic	$y = a\,(cx - h)^2 + k$
Cubic	$y = a\,(cx - h)^3 + k$
Radical	$y = a\,\sqrt[b]{cx - h} + k$

Transformations of Functions

Function	Transformations
Absolute Value	$y = a\,\lvert cx - h \rvert + k$
Rational	$y = \dfrac{a}{cx - h} + k$
Exponential	$y = a \cdot b^{cx-h} + k$
Logarithmic	$y = a \log_b(cx - h) + k$

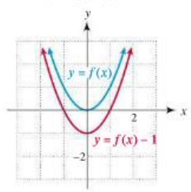
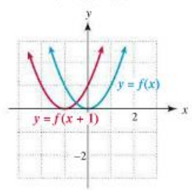
Trigsted

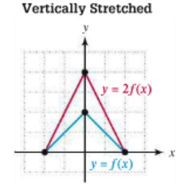
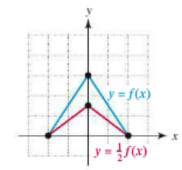
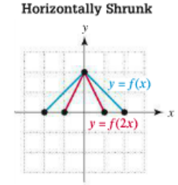
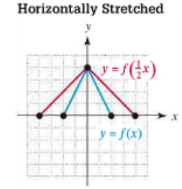
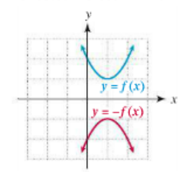
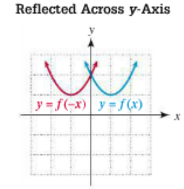
Summary of Transformation Techniques

Given a function $y = f(x)$ and a constant $c > 0$:

1. The graph of $y = f(x) + c$ is obtained by shifting the graph of $y = f(x)$ vertically upward c units.
2. The graph of $y = f(x) - c$ is obtained by shifting the graph of $y = f(x)$ vertically downward c units.
3. The graph of $y = f(x + c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the left c units.
4. The graph of $y = f(x - c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the right c units.
5. The graph of $y = -f(x)$ is obtained by reflecting the graph of $y = f(x)$ about the x-axis.
6. The graph of $y = f(-x)$ is obtained by reflecting the graph of $y = f(x)$ about the y-axis.
7. Suppose a is a positive real number. The graph of $y = af(x)$ is obtained by multiplying each y-coordinate of $y = f(x)$ by a.
   If $a > 1$, the graph of $y = af(x)$ is a vertical stretch of the graph of $y = f(x)$.
   If $0 < a < 1$, the graph of $y = af(x)$ is a vertical compression of the graph of $y = f(x)$.
8. Suppose a is a positive real number. The graph of $y = f(ax)$ is obtained by dividing each x-coordinate of $y = f(x)$ by a.
   If $a > 1$, the graph of $y = f(ax)$ is a horizontal compression of the graph of $y = f(x)$.
   If $0 < a < 1$, the graph of $y = f(ax)$ is a horizontal stretch of the graph of $y = f(x)$.

Rockswold

EQUATION	EFFECT ON GRAPH OF $y = f(x)$
Let $c > 0$. $y = f(x) + c$ $y = f(x) - c$ $y = f(x + c)$ $y = f(x - c)$	The graph of $y = f(x)$ is shifted upward c units. The graph of $y = f(x)$ is shifted downward c units. The graph of $y = f(x)$ is shifted to the left c units. The graph of $y = f(x)$ is shifted to the right c units. **Examples:** **Shifted Down**  **Shifted Left** 

EQUATION	EFFECT ON GRAPH OF $y = f(x)$
Let $c > 0$. $y = cf(x)$	If (x, y) lies on the graph of $y = f(x)$, then (x, cy) lies on the graph of $y = cf(x)$. The graph is vertically stretched if $c > 1$ and vertically shrunk if $0 < c < 1$. **Examples:** **Vertically Stretched**  **Vertically Shrunk** 
Let $c > 0$. $y = f(cx)$	If (x, y) lies on the graph of $y = f(x)$, then $\left(\frac{x}{c}, y\right)$ lies on the graph of $y = f(cx)$. The graph is horizontally shrunk if $c > 1$ and horizontally stretched if $0 < c < 1$. **Examples:** **Horizontally Shrunk**  **Horizontally Stretched** 
$y = -f(x)$ $y = f(-x)$	The graph of $y = f(x)$ is reflected across the x-axis. The graph of $y = f(x)$ is reflected across the y-axis. **Examples:** **Reflected Across x-Axis**  **Reflected Across y-Axis** 

Rules for Transformation of Linear Functions

Transformation	Function	Description
Horizontal Shift	$f(x + h)$	Shift left h units
	$f(x - h)$	Shift right h units
Vertical Shift	$f(x) + k$	Shift up k units
	$f(x) - k$	Shift down k units
Reflection	$-f(x)$	Reflect across x-axis
	$f(-x)$	Reflect across y-axis
Vertical Stretch/Compress	$a\,f(x),\ a > 1$	Stretch vertically by a factor of a
	$a\,f(x),\ 0 < a < 1$	Compress vertically by a factor of a
Horizontal Stretch/Compress	$f(ax),\ a > 1$	Compress horizontally by a factor of $\frac{1}{a}$
	$f(ax),\ 0 < a < 1$	Stretch horizontally by a factor of $\frac{1}{a}$

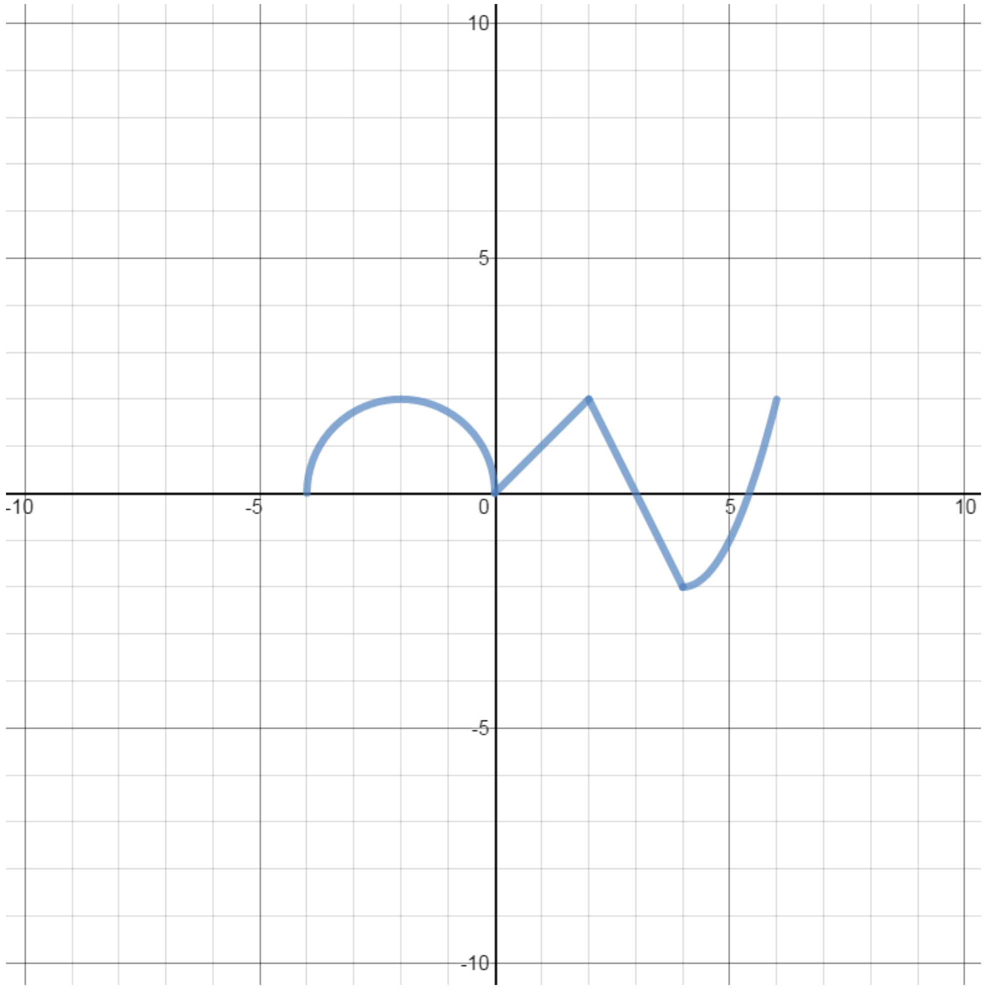
Only $2 \times 3 = 6$ things to remember!

Transformation	Verticality (y)	Horizontality (x)
	"outside" $f(x)$	"inside" $f(x)$
Shift	$y = f(x) + c$	$y = f(x + c)$
	$y = f(x) - c$	$y = f(x - c)$
Stretch or compress	$y = af(x)$	$y = f(ax)$
	$y = \frac{1}{a} f(x)$	$y = f\left(\frac{1}{a} x\right)$
Reflect	$y = -f(x)$	$y = f(-x)$

“Outside” vs “Inside”

- Transformations performed on the “outside” of the function affect the y-coordinates (or the *verticality*) of the graph. These transformations are done the way they appear.
- Transformations performed on the “inside” of the function affect the x-coordinates (or the *horizontality*) of the graph. These transformations are done **opposite** from the way they appear.

Example 1



Example 1

Perform each transformation in Desmos.

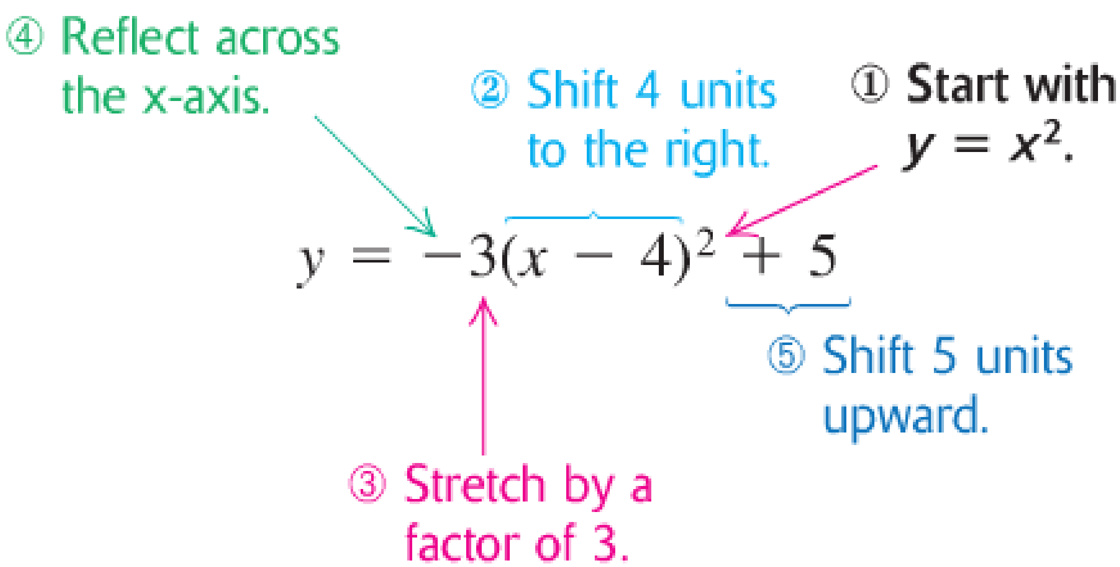
a. $f(x-2)$

b. $f(2x)$

c. $-2f(x)$

d. $f(x)+3$

High **S**peed **R**ecreational **V**ehicle



H	Horizontal shift
S	Stretch/compress
R	Reflect
V	Vertical shift

Example 2

Using the function in Example 1, perform the transformations in Desmos.

$$-2f(2x-2)+3$$

Example 3

Perform each transformation in Desmos.

a. $f(x) = -\sqrt{-3x}$

b. $g(x) = 2(x-3)^2 + 1$

c. $h(x) = 3 + \frac{1}{x+1}$

Final Thoughts

- Transformations of functions need to be taught to students in ways that **engage** them and make it **easier** for them to understand.
- Simplified “rules” for transformations of functions can be extended to **trigonometric** functions with appropriate tweaks to accommodate their periodic nature.
- “Practice makes ~~perfect~~ **permanent**.”