AMATYC

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Linear Functions
New Models for New Times

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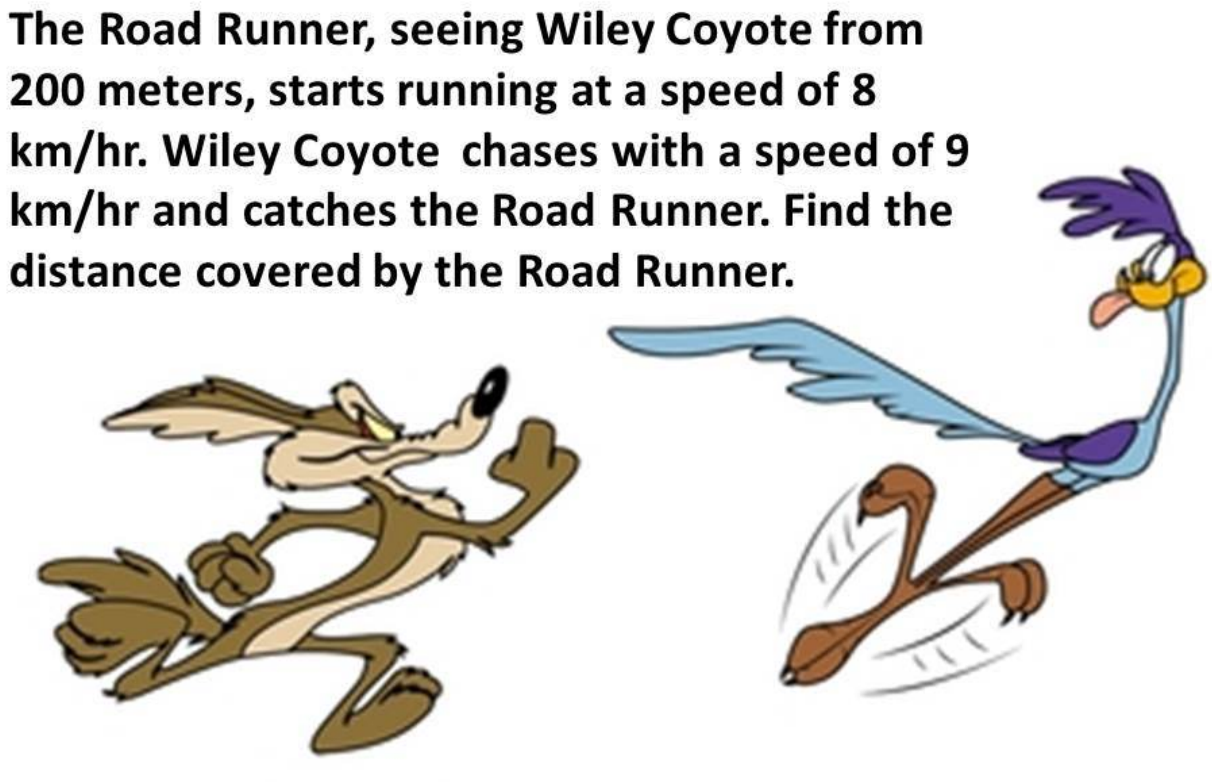
Mathematics and its Applications for Careers (MAC) Committee

Carol Hannahs
Embry-Riddle Aeronautical University Worldwide
Kaplan University Online
channahs@kaplan.edu
Stefan Baratto, Clackamas Community College
sbaratto@clackamas.edu

Linear Functions: Introduction



Classic motion problems $(d = rt)$ can be brought to life.

Choose any pair of your favorite characters, such as Wiley and the Road Runner or Tom and Jerry. Utilizing humor can elicit student attention and engagement.

Solving

Let *RR* be the distance covered by the Road Runner traveling at 8 km/hr.

This means that *WC* covers $RR + 0.2$ km at 9 km/hr.

The time for each is represented by d/r and the times are equal.

$$\frac{RR}{0.8} = \frac{RR + 0.2}{0.9}$$

Use the proportion rule:

$9RR = 8RR + 1.6$ and $RR = 1.6$ (the distance covered by the Road Runner).

Build on this:

How long would this take? Does a cartoon last that long?

Since RR/ 8 = time we have (1.6 km) / (8 km/hr) or .2 hour or .2 (60) = 12 mins.

To use in class

Do this or a similar character problem to get attention and creativity in motion.

Discuss ideas for translating this to a similar application

Examples

a. A cop chasing a robber down the interstate (pretty much a straight line) when the cop starts out some set distance behind but traveling at a faster rate.

b. Two truckloads of supplies being shipped to the same destination that are scheduled to arrive at the same time. Somehow the trucks get separated. Using mile markers an estimate can be made of the distance between them. One agrees to speed up and the other slow down.

c. Maybe not so technical but part **(b)** applies to two carloads of students traveling to the spring break destination.

d. What about production $(P = rt)$? The older machine, A, has already produced 4000 widgets at a rate of 1,000 widgets per hour when the newer faster machine, B, begins producing the same widget at rate of 1,500 widgets per hour. Soon Machine B will catch up to the production level of machine A. Find how many widgets have been produced, by machine A at that time. How many total widgets have been produced?

Answer

8,000 widgets; 12,000 widgets

e. Others?

Linear Functions: A Primer

Compute Globally but Apply Locally

- Local data makes for more interesting applications to students
  - It is very easy to find local data in today's digital environment
  - Models based on local information can be used to improve technical communication skills
- Modeling with and using linear functions is the really important learning that needs to happen
- Anything that hinders that learning should be avoided
- Go with what students know

Maintain Focus

- Build models
- Use models

Build on Models

- Exercise parts are helpful
- First build a model, then evaluate the model at points
  - Consider where reality differs from a model's predictions (teachable moments)
- Complicate the model by adding elements

Remember

- The most important outcomes is for students to be able construct linear functions to model applications and that they can use one once it is built
- We model the math, we should also model the interpretations (sentences)

Linear Functions: Business Applications

Instructor Notes

Cost, revenue, and profit models are a standard application of linear functions at the introductory level.

This model is especially relevant to students in states that have seen explosive growth in the microbrew industry.

Profit models are an opportunity to demonstrate the need for grouping symbols and distribution.

Students have difficulty relating the graph of a function to an equation; they have even more trouble using a graph to understand a model.

The fixed cost of producing one batch of *Oktoberfest* is \$140. Each 12-oz bottle brewed costs an additional 41¢. You will sell each bottle for \$2.50.

1. Let x represent the number of bottles produced and sold. Find equations to represent the cost of production C, the revenue from sales R, and the profit P.

Answer

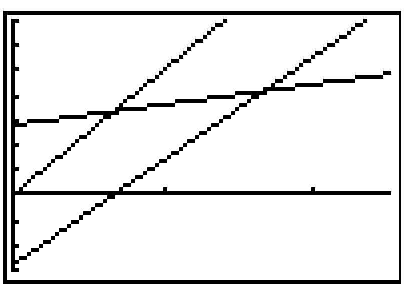
$C = 0.41x + 140$

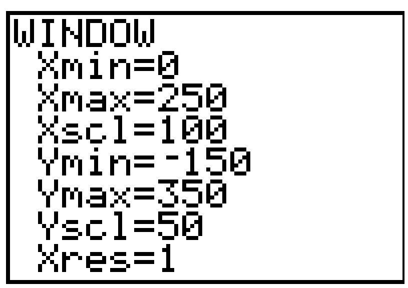
$R = 2.5x$

$P = 2.09x - 140$

2. Graph all three equations in the space provided. Be sure to label and scale your axes appropriately.

Answer





3. Use the *profit* equation to determine the break-even point.

4. At what point do the cost and revenue equations intersect? Interpret this point in the context of the application.

5. Why is it sufficient to graph these equations in Quadrants I and IV, only?

Linear Functions: Mixture Problems

Instructor Notes

Mixture problems come in two flavors. If we simply have $x + y = 200$ so $y = 200 - x$, followed by substitution, we really should work with this as a straightforward model in one variable rather than as a system.

A key component of student learning is the ability to reason, "if we have a total of 200 lb, and *x* makes up a portion, then $200 - x$ makes up the remainder (all 200 lb except that which is already accounted for by *x*).

The first exercise is a fairly straightforward mixture problem; this allows the instructor to help students with the basics of setting up and solving mixture problems.

In the interest of building on prior learning, the second exercise adds a twist as students must add the water to the 42-gal mixture when setting up the equation.

1. A coffee reseller wishes to mix two types of coffee beans for the House Blend. The Kona bean that she wants to use wholesales for $4.50 per pound; the Sumatran bean wholesales for $3.25 per pound. If she wishes to mix 200 pounds of beans for a wholesale price of $4 per pound, how many pounds of each type of coffee bean should she include in the mix?

2. Currently, 8% of a 42-gal mixture of patching compound is water. Local conditions require the mixture to be 13% water. How much water needs to be added to the mix in order for it to be 13% water (accurate to three decimal places)? What will the total volume of the mixture be after the water is added?

Equation

Let *x* be the amount of water added.

$0.08 \cdot 42 + x = 0.13(42 + x)$

Linear Functions: Flight Application

Instructor Notes

Students appreciate applications that are obviously real.

Landing a Plane

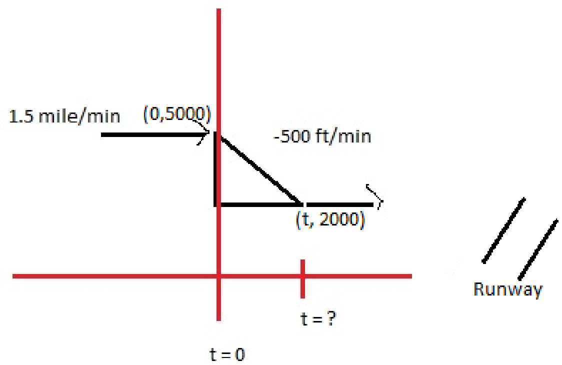
In general, when landing, planes descend at a rate of 500 ft/min until they reach an altitude of 2,000 ft in order to enter into the landing pattern.

For a plane flying at 5,000 ft at 1.5 mi/min (ground speed), the equation that models this is

$$h = -500t + 5{,}000$$

How far away should the pilot start the descent into the landing pattern?

Note: Appendix B of this handout provides descent information, for those interested in more detail.



Using the slope equation, we find $t = 6$.

It takes 6 min to descend and reach the landing pattern.

How far away from the edge of the landing pattern must the pilot start?

If the plane is flying at a ground speed of 1.5 mi/min, then

$$d = (1.5)(6) = 9$$

The pilot should begin the descent at 9 miles out.

To use in class

Solve this problem.

How does the equation change for a plane flying at a higher altitude? Commercial flights are usually up to 30,000 on longer flights. Note that ground speed rates are probably larger as well.

Have students research cruising speeds for different size planes. There will probably be a conversion from knots to miles.

Have the students research flying into their local airport to find the exact level the airport is above sea-level. Add 1,000 to that value to get the approximate landing pattern.

Linear Functions: Proportions

Instructor Notes

Proportions and direct variation are staple elements in Prealgebra content.

Proportions are a good way to ease students into linear applications.

This particular application engages students in several ways. It provides them with an example that many have seen but perhaps have never thought about. This enables an instructor to have a small discussion involving the class before moving into the mathematics.

It is also a multi-step problem, as they must compute the hypotenuse of a triangle prior to constructing a proportion; this provides instructors with an easy way to promote sketching a problem in order to understand it. This is especially important to realizing they are given a hypotenuse.

Several other discussions can be had with this problem: There is a units discussion to be had; there is also a rounding discussion as students should learn to work with $\sqrt{100^2+8^2}=\sqrt{10,064}$ rather than approximating early; they should only round at the end.

Truck drivers are often warned that a hill is steep with a sign that gives the percent grade.

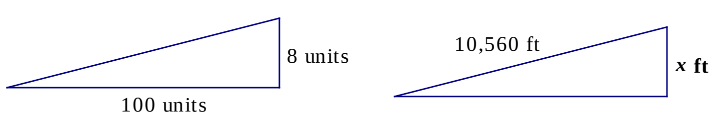
Sorry, couldn't resist.

Most students understand that the larger the percent, the steeper the road. However, most do not actually know what this means.

A truck is driven two miles up a hill with an 8% grade. What was the change in altitude?

Round your final result to the nearest foot.

Recall: $1 \text{ mi} = 5,280 \text{ ft}$.



Linear Functions: Building Models

Instructor Notes

The first learning objective is helping students understand how to read a problem and recognize that they are given two points.

Students can also be taught the value of letting *x* be the number of years since some point in time, rather than using the actual year.

This has the added benefit of ensuring the *y*-intercept has meaning.

Relating average annual rate of change to slope is important for modeling relationships.

The second exercise provides for several additional teachable moments.

Should *x* be the number of years since 2000 or 2006? Discuss careless errors.

2.e. is a good lead-in to regression analysis which is a good lesson to follow with as it facilitates a discussion about the limitations of a model; if desired, one can discuss interpolation and extrapolation.

The Bureau of Labor Statistics (bls.gov) provides an awesome site for data geeks.

1. According to the BLS, the average urban price for a pound of Red Delicious apples in January 2000 was $0.952. By January 2016, the price had risen to $1.450.

- **a.** During this time period, what was the average annual rate of change in the cost of a pound of Red Delicious apples?
- **b.** Write a **linear equation** (in slope-intercept form) that approximates the cost of a pound of apples, by year.
- **c.** Use the equation in part (b) to predict the cost of a pound of apples in the year 2020.
- **d.** Graph your equation and mark the points associated with 2000, 2016, and 2020.

2. According to the BLS, the average price for a pound of white bread in January 2006 was $1.046 By January 2016, the price had risen to $1.425.

- **a.** During this time period, what was the average annual rate of change in the cost of a pound of white bread?
- **b.** Write a **linear equation** (in slope-intercept form) that approximates the cost of a pound of bread, by year.
- **c.** Use the equation in part (b) to predict the cost of a pound of bread in the year 2020.
- **d.** Use the equation in part (b) to predict the cost of a pound of bread in the year 2013.
- **e.** In fact, the Jan 2013 price was $1.422 for a pound of bread. How does that compare to your answer in part (d)?
- **f.** Give the *y*-intercept of your equation.
- **g.** Write a sentence interpreting the *y*-intercept, in the context of this application.
- **h.** Graph your equation and mark points to indicate the predicted price in 2006, 2013, 2016, and 2020.

Linear Functions: Regression Analysis

Instructor Notes

This first example of regression makes a good exam question as instructors can construct two versions of an exam simply by using the other six months for the second exam version.

In class and homework, having students actually construct scatter plots; using their technology to perform regression analyses is appropriate; it may be less appropriate to require that on an exam (at least at the Algebra I level).

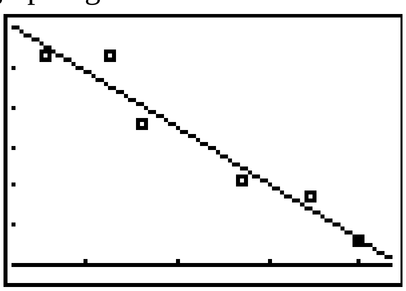
Using local data makes applications more “real” to students.

1. The average maximum temperature (°F) and rainfall (in.) for selected months is shown below.

Month	Jan	Mar	May	Jul	Sep	Nov
Temperature	45.3°	55.9°	67.1°	79.9°	74.5°	52.5°
Rainfall	5.3	3.6	2.1	0.6	1.7	5.3

Source: World Climate; Portland International Airport; 30-year period (1986-2015)

A graphing calculator sketch of a scatter plot of this data along with the line-of-best-fit:



According to a graphing calculator, the line-of-best-fit (to two decimal places) is given by

$$y = -0.14x + 11.92$$

where x represents the average maximum temperature in the month and y gives the amount of rainfall that month.

a. What is the y-intercept of the best-fit line?

b. Interpret the y-intercept of the best-fit line in the context of this application.

c. How much rain would you expect in a month if the maximum temperature were 60°F?

Instructor Notes

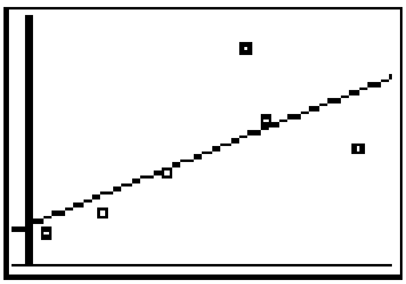
The second example of regression also makes a good exam question as instructors can construct two versions of an exam simply by using different cities from their local airport.

Part **(c)** can be used to facilitate a discussion about the limitations of models.

2. A Clackamas CC math instructor used a website to find the lowest nonstop roundtrip airfares from Portland, OR, to select U.S. cities. Prices do not include taxes and fees and are rounded to the nearest dollar.

City	**Distance** (mi)	**Cost** ($)
Chicago, IL	1,742	401
Dallas, TX	1,619	563
New York City, NY	2,439	336
Phoenix, AZ	1,010	280
San Francisco, CA	550	188
Seattle, WA	130	144

A graphing calculator sketch of a scatter plot of this data along with the line-of-best-fit:



According to a graphing calculator, the line-of-best-fit (to two decimal places) is given by

$$y = 0.12x + 163.18$$

where x represents the distance to the city and y gives the cost of a ticket.

a. What is the slope of your model?

b. Interpret the slope in the context of this application.

c. The nonstop flight from Portland to Boston is 2,531 mi. How much does the regression model predict the cost to be (to the nearest dollar)?

Instructor Notes

The final regression example does not make a good exam question.

This example requires strong reading and technical communication skills in order to complete part **(e)**.

The National Center for Education Statistics (nces.ed.gov) provides a great site for data geeks.

3. An NCES survey of public school libraries and media centers allowed us to compare the per-student spending on library materials with the number of books acquired during the year (per 100 students). Some of this data is shown in the table.

State	**Expenditures**	**Acquisitions**
Arizona	$15.30	121
Georgia	$14.20	76
Minnesota	$15.20	111
Ohio	$10.90	75
Virginia	$16.20	88

a. Construct a scatter plot of the data with each per-student spending along the x-axis and the number of books acquired per 100 students along the y-axis.

b. Find the line-of-best-fit for the data set. Sketch this line on your scatter plot.

c. Give the equation of the line-of-best-fit (round to two decimal places).

d. What is the slope of the best-fit line?

e. Interpret the slope in the context of this application.

Answer

For every additional dollar spent per student, libraries will increase their holdings by 5.73 books per 100 students.

f. How many books would you expect to be acquired (per 100 students) if a state's per-student expenditures were $17 (round to the nearest whole number)?

g. What level of expenditures should policy makers approve if they want their state's school libraries to acquire 100 books (per 100 students) in a given year (round to the nearest cent)?

Appendix A: Additional Exercises

Percent Applications

A multitude of linear applications can be created utilizing percents.

If someone can't work with percents, the only thing they can do with a business is run it into the ground.

Utilize interesting applications, products students know or want, current pricing, and other techniques.

Build from straightforward to those that require an additional step.

1. An LG Electronics 4.2 Cu Ft IEC Ultra-Capacity Front Load Washer is being offered at a special price of $809.10 after a 10% discount. What is the original price of washing machine?
2. A state adds a 7.25% sales tax to the price of most goods. If a 30-GB iPod is listed for $299, how much will it cost after the sales tax has been added?
3. In order to make room for the new fall line of merchandise, a proprietor offers to discount all existing stock by 15%. How much would you pay for a Fendi handbag that the store usually sells for $229?
4. A store sells a certain Kicker amplifier model for a car stereo system for $249.95. If the store pays $199.95 for the amplifier, what is their markup percentage for the item (to the nearest whole percent)?
5. A toy store is selling the Fisher-Price Rollin' Rumblin' Dump Truck at a 10% discount for $16.19. How much does the toy normally sell for?
6. A grocery store adds a 30% markup to the wholesale price of an item to determine the selling price. If the store sells a half-gallon container of orange juice for $2.99, what is the wholesale price of the orange juice?

Numerous other types of linear applications exist that are useful.

For instance, conversions (measurement, monetary, and others).

Appendix B: Landing a Plane

Background for Landing a Plane Example

You're flying at 5,000' MSL, going 90 knots groundspeed, and you need to descend to a pattern altitude of 2,000' MSL. You plan to descend at 500 feet per minute. **How far out should you start your descent?**

After all, nobody *intends* on diving their plane at 2,000 feet per minute to make it to pattern altitude. But if you're not planning ahead, it can happen.

Starting with the Basics

There are a few basic things you need to understand to use the 60:1 rule.

- If you travel at 1 knot, you'll cover 1 nautical mile (NM) in 1 hour
- 1 hour contains 60 minutes

- If you travel at 60 knots, you'll cover 1 NM in 1 minute (which is 1 mile-per-minute, or 1 MPM)

Figuring out how many miles you're traveling each minute is really the key. Here are some examples that will help you down the road in this article, as well as the next time you fly. Remember, these speeds are **ground speed** when it comes to figuring out your MPM, ground speed is the only speed that matters.

- 60 knots = 1 MPM
- 90 knots = 1.5 MPM
- 120 knots = 2 MPM
- 150 knots = 2.5 MPM
- 180 knots = 3 MPM

Back To Our Descent Planning

Now that we have the miles-per-minute stuff out of the way, let’s get back to the descent planning question.

In the question, we had to descend from 5,000' to pattern altitude at 2,000', for a total of 3,000' of descent. We planned to descend at the standard rate of 500 FPM. And we need to figure out how many miles out from the airport we need to start that descent.

Step 1: First, we need to figure out how many minutes it's going to take us to descend, and that's pretty straight forward. If we need to descend 3,000', and we're doing it at 500 FPM, we divide 3,000 by 500, and we get 6 (3000/500 = 6). **It will take us 6 minutes to descend to pattern altitude**.

Step 2: Next, we need to figure out how many miles away from the airport we need to start that descent.

Since we're traveling at 90 knots ground speed, it means we're traveling 1.5 miles per minute (MPM). Now all we need to do is multiply our MPM by the number of minutes we need to descend, which was 6. So we'll multiply 1.5 X 6, which gives us 9 NM. **We need to start our descent 9 NM out to make it to 2,000' at the airport**.

Keep in mind, doing a calculation like this would put you at 2,000' *right over the top* of the airport. Chances are, you want to get to pattern altitude at least a mile or two before the airport, so you can make a pattern entry and not have to 'chop and drop'.

To do that, simply add a mile or two to your calculation. In this example, if you started your 500 FPM descent 11 miles from the airport, you'd reach pattern altitude 2 miles prior to the airport, which would probably work out well.

Descent Planning Any Pilot Can Use

It doesn't matter if you're a VFR pilot or IFR pilot, the 60:1 rule makes descent planning easy.

Whether you're trying to impress your passengers with a smooth descent to the airport, or you're trying to make sure you meet an altitude restriction with ATC, the 60:1 rule takes the guesswork out of descending, and makes you look like a pro.

BOLDMETHOD.COM

Appendix C: Attendee Examples

Lemonade Stand at Fair
Booth cost gives cost function (homework)
Build profit model
Make it real: Which fair? Look up costs for local fairs.

Unit Prices
Useful products such as diapers
Demo in class with online shopping

Percent App: Snacks
15% More: Is that real? Is ad real?

Better Deals: Decision Sciences
Rental cars
Shopping: Cheaper price but longer drive
Cell Plans (minutes to data usage)
General: Different pricing structures.

Transportation
Uber, Taxi, Parking

Solutions to Systems
Ordered pairs in the context of a problem

Environmentally Conscious and Socially Relevant Examples
NOAA, US Fish and Wildlife, HHS
CO_2 Emissions; Bottle Water; HS Cigarette Smoking; Ice Cover over Arctic; Diabetes & Obesity