

TEACHING INTRO STATISTICS *USING*  
*INQUIRY BASED AND PROJECT BASED*  
*LEARNING STRATEGIES*



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# PROJECT DESCRIPTION



- Explore students' achievements of the course after learning the course based on Inquiry Based and Project Based Learning strategies.
- Those alternative instructional methods enhance students' learning deeply and conceptually.
- Students' satisfaction

# PROJECT PROGRESS



- Each student get assigned in a group, and completed group project.
- At the beginning of the semester, each group decide their project topic
  - Topics must be able to analyze numerically; Height, shoe size, Weight, Commuting time
  - Both genders
- Students get assigned a total of 8 labs
  - Between the Lab 1 through 4: Review the contents
  - Between the Lab 5 through 8: Hypothesis Test Project.

# THIS PRESENTATION



- Demonstrate
  - Labs
  - Activities
    - Creating frequency tables
    - Binomial Probability
    - Normal Distribution / Central Limit Theorem
- Hypothesis Project
  - Activity sheets
  - Students' Presentation
  - Sample students' projects

# EXAMPLE OF LAB



Lab 1:

Throughout the Lab1, you would choose one quantitative topic, such as height, weight, foot size, age, or etc. You would write a short paragraph based on the following steps, and submit by next class.

Step 1: Determine your quantitative topic

Step 2: Collect 30 data at BMCC campus: Your data must include gender.

Step 3: Create Frequency table with 5,6,or 7class. Justify why you choose the number of class

Step 4: Create histogram

Step 5: Find the central Tendency

Step 6: Create Box-Plot

Step 7: Find the variance and standard deviation.

# EXAMPLE OF LAB



## Lab 2

Throughout the Lab 2, you would create probability contents based on your data from the Lab 1. Write a short paragraph based on answering the following steps

Step 1: Create the conditional probability table based on your data

	Male	Female	Total
Greater than or equal to the mean			
Less than the mean			
Total			

Step 2: Answer the following questions

- Find the probability of male
- Find the probability of female
- Find the probability “Greater than or equal to the mean”
- Find the probability “Less than mean”
- Find the probability of male given that “Greater than or equal to the mean”
- Find the probability of “Less than the mean” given that female
- Find the probability of female given that “Less than the mean”
- Are events “Male and Female” mutually exclusive?
- Are events “Greater than or equal to the mean” and “Female” mutually exclusive?
- Are Less than the mean and female independent event?

# EXAMPLE OF LAB



## Lab 3

Using your data from the Lab 1, write a short paragraph based on answering the following questions.

Question 1: Find the probability to select male and female

Question 2: Find the probability to select “Greater than or equal to the mean”, and “Less than mean”

Question 3: Find the probability to select

- a. exactly three females selected
- b. less than four females selected
- c. between 6 and 7 females

Question 4: Find the probability to select

- a. exactly six people selected “Greater than or equal to the mean”
- b. between four and six people selected “Less than the mean”

# EXAMPLE OF LAB



Lab 4: Summary of the previous three labs.

Using your own words, create essay paragraphs of your projects. This short summary must be at least 6 pages with including

1. Frequency Table with the description
2. Minimum three displaying graphs: Histogram, Pie-chart, Bart-charts, line graph and etc.
3. Box-plot with as detail description as possible
4. Input a conditional probability table, and describe at least three conditional probabilities.
5. Define at least one mutually exclusive events
6. Demonstrate two of independent/dependent event, and explain why.

# ACTIVITY: FREQUENCY TABLE



Number of Siblings (x)	Frequency (f)	$x \cdot f$	$x^2 \cdot f$

1. Find the mean of the data using  $\bar{x}$
2. Find the variance and standard deviation

$$s = \sqrt{\frac{n(\sum (x^2 \cdot f)) - (\sum x \cdot f)^2}{n(n-1)}}$$

# ACTIVITY: FREQUENCY TABLE



Number of Siblings (x)	Frequency (f)	$x \cdot f$	$x^2 \cdot f$
0	4	0	0
1	6	6	6
2	3	6	12
3	2	6	18
4	6	24	96
5	1	5	25
	22	47	157

$$\bar{x} = \frac{47}{22} = 2.14$$

$$s^2 = \frac{22 \cdot 157 - 47^2}{22 \cdot 21} = 2.694$$

$$s = \sqrt{2.694} = 1.642$$

1. Find the mean of the data using  $f$
2. Find the variance and standard deviation

$$s = \sqrt{\frac{n(\sum (x^2 \cdot f)) - (\sum x \cdot f)^2}{n(n-1)}}$$



- Prof. Lee gives a multiple choice quiz, which is composed to a total of four choice, and only one answer is correct. James is one of his student, but he did not study, thus, he doesn't understand any question.

# BINOMIAL PROBABILITY



- 1. If the quiz has only one question, what is the probability that he chooses the correct answer?
- All students found the answer is  $\frac{1}{4}$  easily

# BINOMIAL PROBABILITY



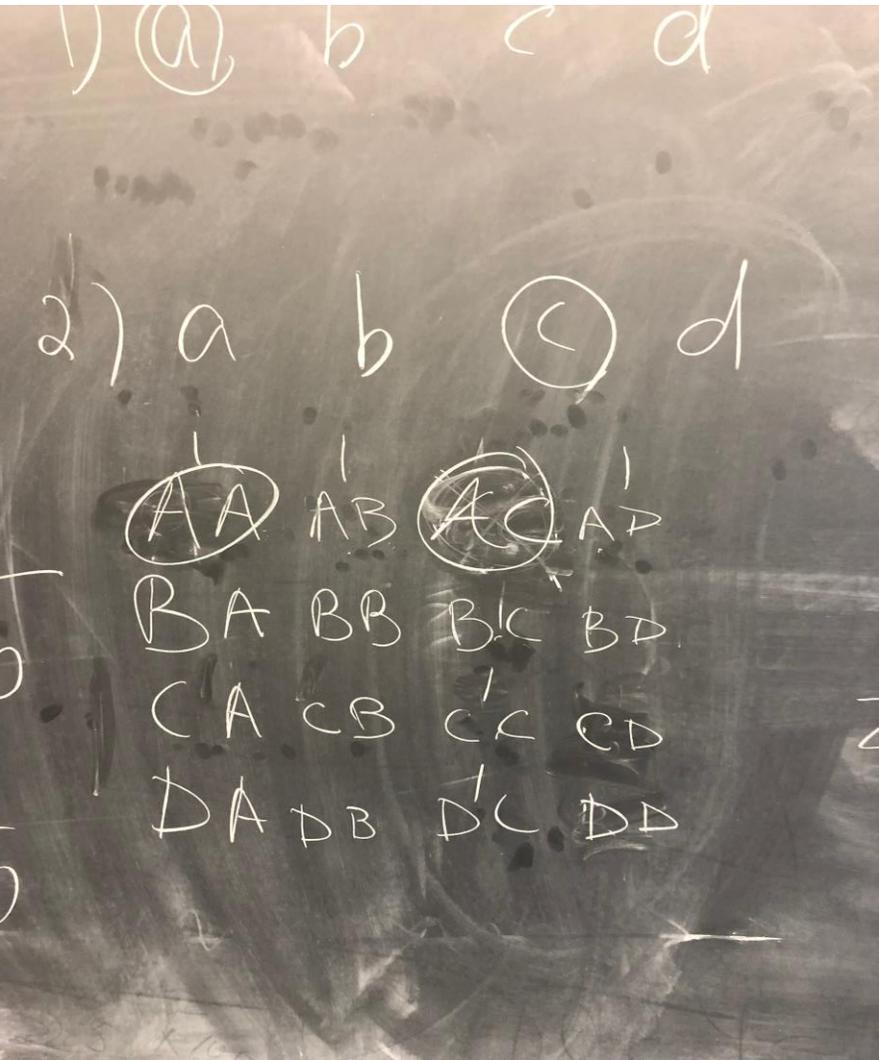
- 2. If the quiz has two questions,
  - A. What is the probability that he choose all correct answers?
  - B. What is the probability that he guess one question correctly?

# BINOMIAL PROBABILITY



- The most of the group thought
  - The answer is  $2/8$ , means  $1/4$
- Instructor: Is the probability getting one question correct among one question and two questions?
  - Students realized that it should not be the same.

# BINOMIAL PROBABILITY



- One group found all the possibilities, and found the answer
- $P(\text{All questions correct}) = 1/16$
- $P(\text{one question correct}) = 6/16 = 3/8$
- **Instructor: Can you find a different way?**

# BINOMIAL PROBABILITY



- 3. If the quiz has three questions,
  - A. What is the probability that he guesses three questions correctly?
  - B. What is the probability that he guesses one question correctly?
  - C. What is the probability that he guesses two questions correctly?
  - D. What is the probability that he guesses at least one question correctly?

# BINOMIAL PROBABILITY



$$\textcircled{b) \frac{27}{64}}$$

$$P(C, I, I) = \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{64}$$

$$P(I, C, I) = \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{9}{64}$$

$$P(I, I, C) = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{9}{64}$$

$$\frac{27}{64}$$

- After discussing using the probability, students started using the probabilities.
- Some students started thinking a shortcut using the combination property:  $nCr$
- Instructor: Why do you think to use the combination for this problem?
- Student A: Because  $P(C, I, I)$ ,  $P(I, C, I)$ , and  $P(I, I, C)$  are all the same cases, so the question did not matter with the order.

# BINOMIAL PROBABILITY FORMULA



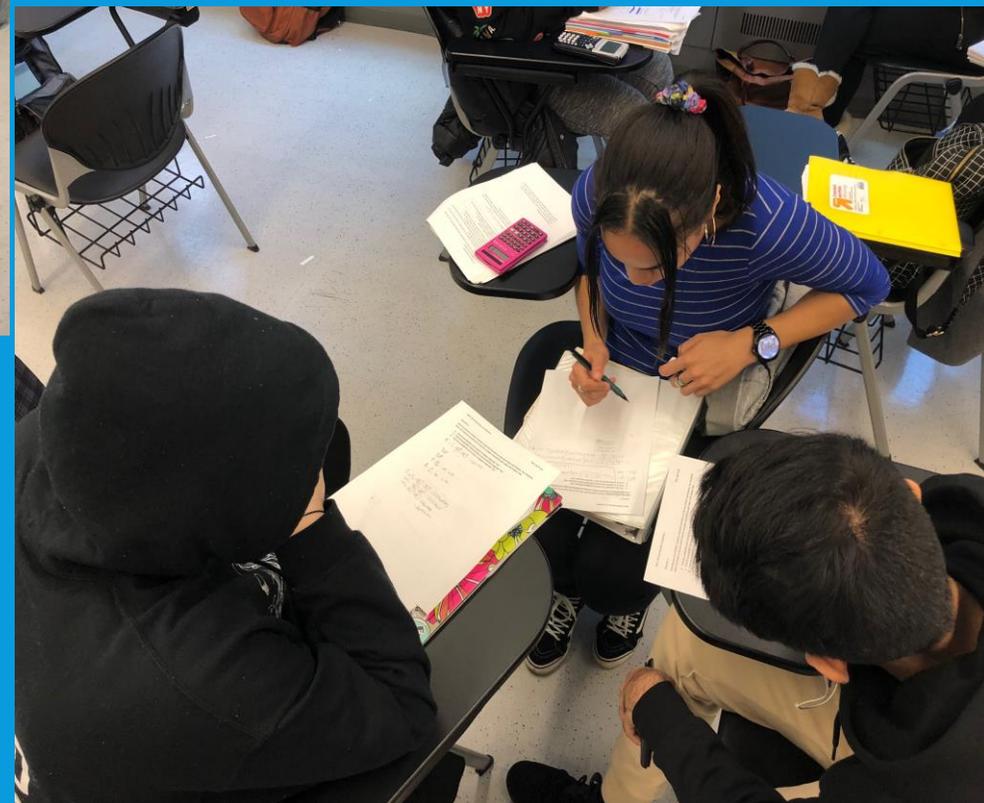
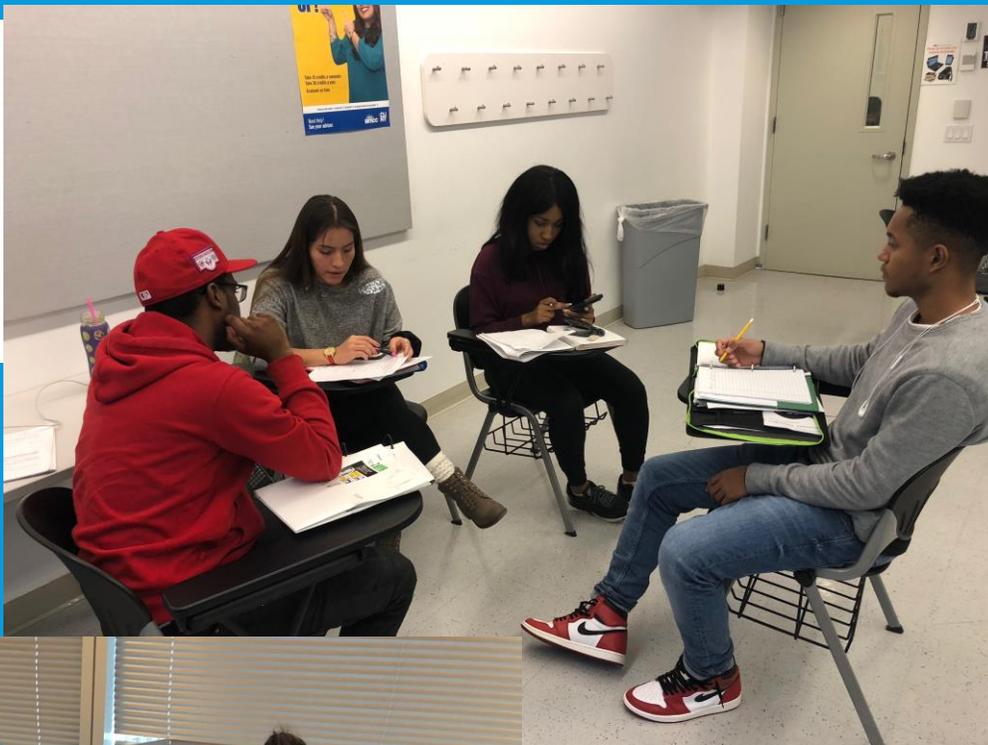
- After the activity, introducing the binomial probability theorem:

$$P(x) = {}_n C_x p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

# SOLVING PROBLEMS USING BINOMIAL PROBABILITY THEOREM



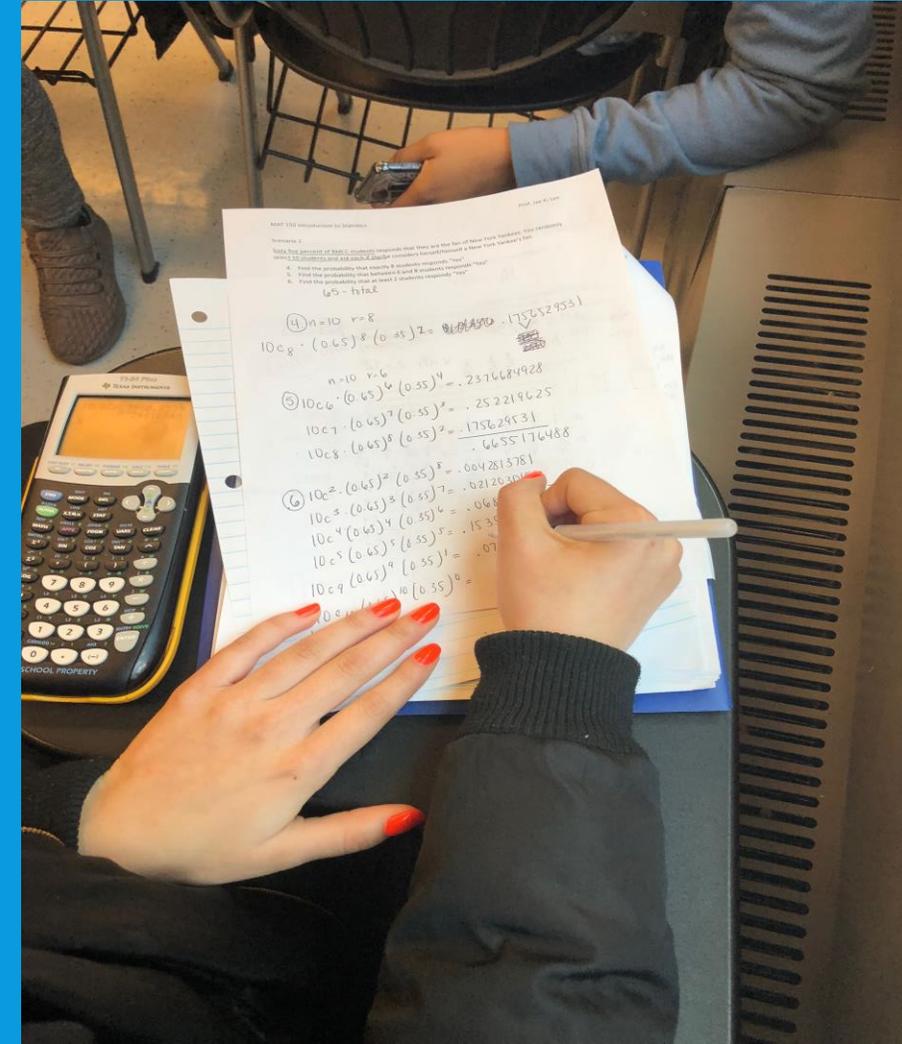
- Sixty five percent of BMCC students responds that they are the fan of New York Yankees. You randomly select 10 students and ask each if she/he considers herself/himself a New York Yankee's fan.
  - 1. Find the probability that exactly 8 students responds "Yes"
  - 2. Find the probability that between 6 and 8 students responds "Yes"
  - 3. Find the probability that at least 2 students responds "Yes"



# AS A RESULT



- Students understood the Binomial Probability
- Realized that Binomial Probability generate one case each time
- Finding more than one case, they need to find all different cases, and combined them.



Scenario 2

Sixty five percent of BMCC students responds that they are the fan of New York Yankees. You randomly select 10 students and ask each if she/he considers herself/himself a New York Yankee's fan.

4. Find the probability that exactly 8 students responds "Yes"
5. Find the probability that between 6 and 8 students responds "Yes"
6. Find the probability that at least 2 students responds "Yes"

$0.65$  - total

④  $n=10$   $r=8$   
 $10c_8 \cdot (0.65)^8 (0.35)^2 = \cancel{0.175629531} \cdot 175629531$

$n=10$   $r=6$   
 ⑤  $10c_6 \cdot (0.65)^4 (0.35)^4 = .2376684928$

$10c_7 \cdot (0.65)^7 (0.35)^3 = .252219625$

$10c_8 \cdot (0.65)^8 (0.35)^2 = \frac{.175629531}{.6655176488}$

⑥  $10c^2 \cdot (0.65)^2 (0.35)^8 = .0042813781$

$10c^3 \cdot (0.65)^3 (0.35)^7 = .021203015$

$10c^4 (0.65)^4 (0.35)^6 = .068$

$10c^5 (0.65)^5 (0.35)^5 = .1535$

$10c^9 (0.65)^9 (0.35)^1 = .07$

$10c^{10} (0.65)^{10} (0.35)^0 =$



- Showing the complementary event:
- $P(b) = 1 - P(\text{not } b)$

# SOLVING PROBLEMS USING BINOMIAL PROBABILITY THEOREM



MAT 150 Introduction to Statistics

Scenario 2

Sixty five percent of BMCC students responds that they are the fan of New York Yankees and ask each if she/he considers herself/himself a New York Yankee's fan.

4. Find the probability that exactly 8 students responds "Yes"
5. Find the probability that between 6 and 8 students responds "Yes"
6. Find the probability that at least 2 students responds "Yes"

$$n=10, r=8$$

$$4. \quad 10C8 \left(\frac{65}{100}\right)^8 \left(\frac{35}{100}\right)^2 \quad \text{or} \quad 10C8 \left(\frac{65}{100}\right)^8 \left(\frac{35}{100}\right)^2$$

$$= 0.175652921$$

$$5. \quad 10C6 \left(\frac{65}{100}\right)^6 \left(\frac{35}{100}\right)^4 \quad \text{or} \quad 10C7 \left(\frac{65}{100}\right)^7 \left(\frac{35}{100}\right)^3$$

$$= 0.252219625 \quad \text{or} \quad 0.252219625$$

Total = 0.665541071

$$6. \quad n=10, r=2$$

$$10C2 \left(\frac{65}{100}\right)^2 \left(\frac{35}{100}\right)^8$$

$$= 0.0042813781$$

$$n=10, r=3$$

$$10C3 \left(\frac{65}{100}\right)^3 \left(\frac{35}{100}\right)^7$$

$$= 0.0212030153$$

$$r=4$$

$$10C4 \left(\frac{65}{100}\right)^4 \left(\frac{35}{100}\right)^6$$

$$= 0.0681097812$$

$$r=5$$

$$10C5 \left(\frac{65}{100}\right)^5 \left(\frac{35}{100}\right)^5$$

$$= 0.1535204107$$

$$r=6$$

$$10C6 \left(\frac{65}{100}\right)^6 \left(\frac{35}{100}\right)^4$$

$$= 0.0274916949$$

$$r=10$$

$$10C10 \left(\frac{65}{100}\right)^{10} \left(\frac{35}{100}\right)^0$$

$$= 0.0131627483$$

Scenario 2

100

Sixty five percent of BMCC students responds that they are the fan of New York Yankees. You randomly select 10 students and ask each if she/he considers herself/himself a New York Yankee's fan.

4. Find the probability that exactly 8 students responds "Yes"
5. Find the probability that between 6 and 8 students responds "Yes"
6. Find the probability that at least 2 students responds "Yes"

65% - YES

$$4. \quad 10C8 \left(\frac{65}{100}\right)^8 \left(\frac{35}{100}\right)^2 = 45(0.031864481)(0.1225)$$

$$= 0.175652921$$

$$5. \quad 10C6 \left(\frac{65}{100}\right)^6 \left(\frac{35}{100}\right)^4 = 210(0.07541889)(0.01500625) = 0.237668423$$

$$10C7 \left(\frac{65}{100}\right)^7 \left(\frac{35}{100}\right)^3 = 120(0.049022278)(0.042875) = 0.252219625$$

$$+ 0.237668423$$

$$+ 0.252219625$$

$$+ 0.175652921$$

$$= 0.665541071$$

$$6. \quad 10C2 \left(\frac{65}{100}\right)^2 \left(\frac{35}{100}\right)^8 = 10(0.35)(0.020711912) = 0.072491634$$

100

$$100$$

$$- 7.2491634$$

$$= 92.7508366$$

# NORMAL DISTRIBUTION



- Claire and Susan are very close friends with each other. They are going to the same college, and have been taking many classes together. Although, they are best friends, on the other hand, they are always rivals. One semester, they are taking statistics classes, but different classes with different professors. After the first exam, Claire got 75, and Susan got 77 on their first exam. The average of the Claire's class is 68 with the standard deviation 8.2, and the average of the Susan's class is 70 with the standard deviation 6.7
- Questions: **Who got the higher grade on the first exam based on the data distribution?**

# NORMAL DISTRIBUTION



Claire and Susan are very close friends with each other. They have been taking many classes together. Thus, they are always rivals of each other's performance, and evaluate who gets a higher grade (in statistics classes; but different classes with different professors). Susan got 77 on their first exam. The average of the Claire's class is 68 and the average of the Susan's class is 70 with the standard deviation of 8.2 and 6.7 respectively.

Question: Who got the higher grade on the first exam?

$\mu - \sigma$

$\mu + \sigma$

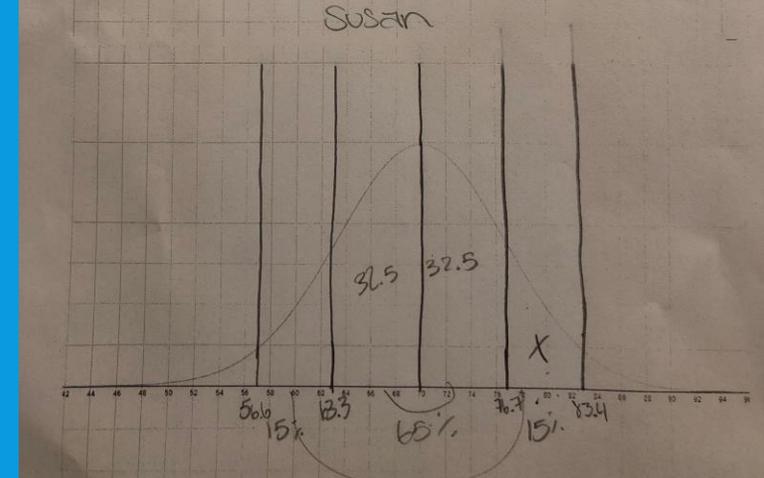
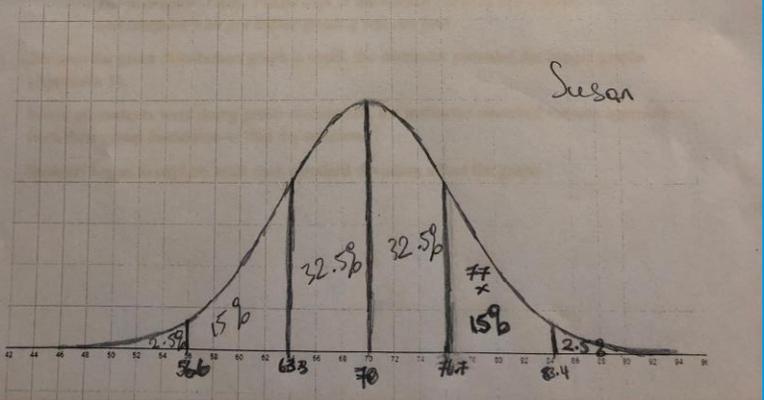
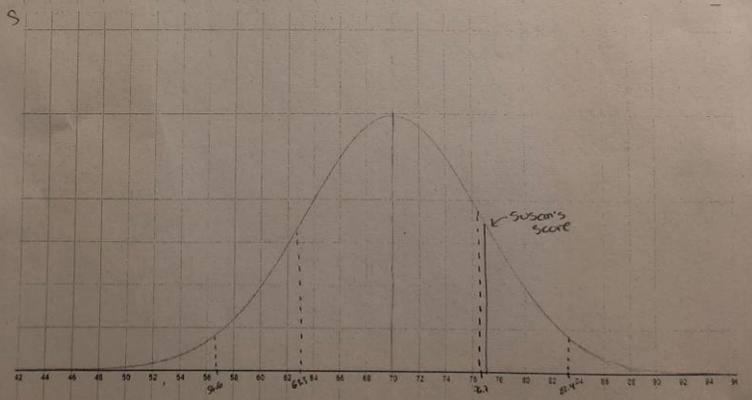
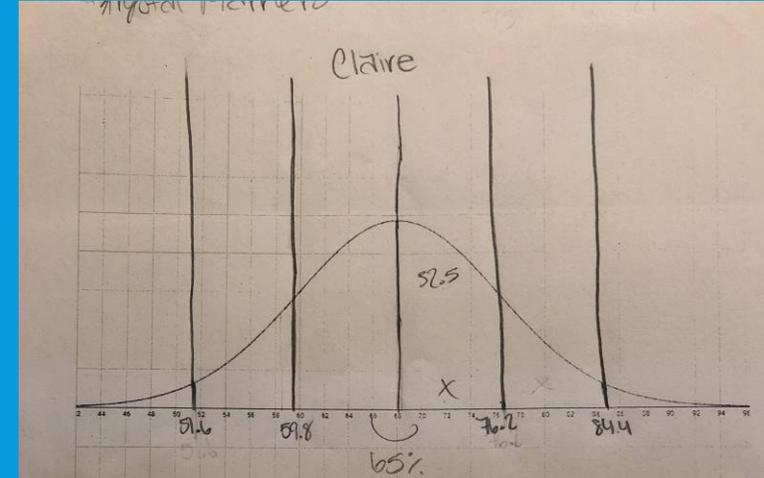
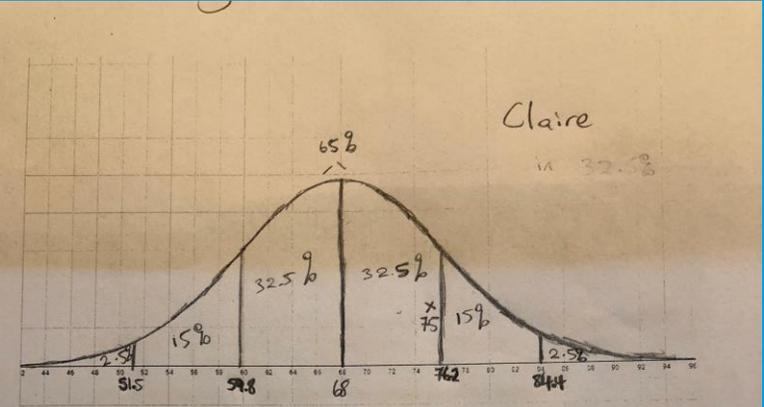
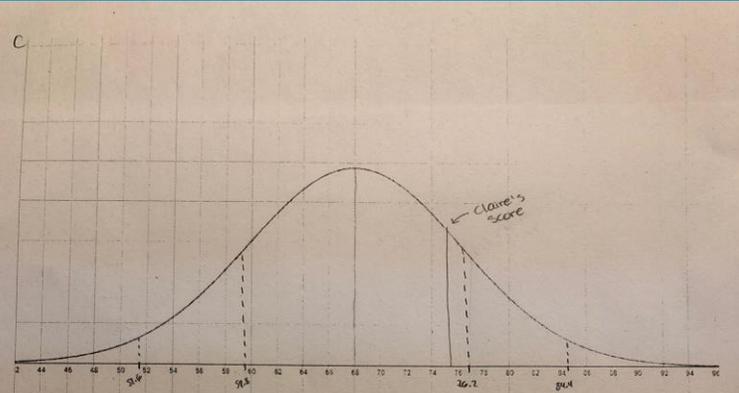
Claire - 68 / 8.2 (7 pts.)

Susan - 70 / 6.7 (7 pts.)

Susan

- Susan got higher grade: her score is greater
- Their score is the same because the mean deviation is equal to each other.
- What about the standard deviation?

# NORMAL DISTRIBUTION



# NORMAL DISTRIBUTION



each one's performance, and evaluate who gets a higher grade (score). One semester, they are taking statistics classes; but different classes with different professors. After the first exam, Claire got 75, and Susan got 77 on their first exam. The average of the Claire's class is 68 with the standard deviation 8.2 and the average of the Susan's class is 70 with the standard deviation 6.7.

Question: Who got the higher grade on the first exam?

	Score	Class Ave	Standard D
Susan	77	70	6.7
Claire	75	68	8.2

  
$$z = \frac{x - \bar{x}}{s}$$
$$z = \frac{77 - 70}{6.7} = \frac{7}{6.7} = 1.0447$$
$$z = \frac{75 - 68}{8.2} = \frac{7}{8.2} = 0.8536$$

- One group began to use the z-score
- Watch the online lecture video

Normal Distribution

Read the page 44, and answer the following question

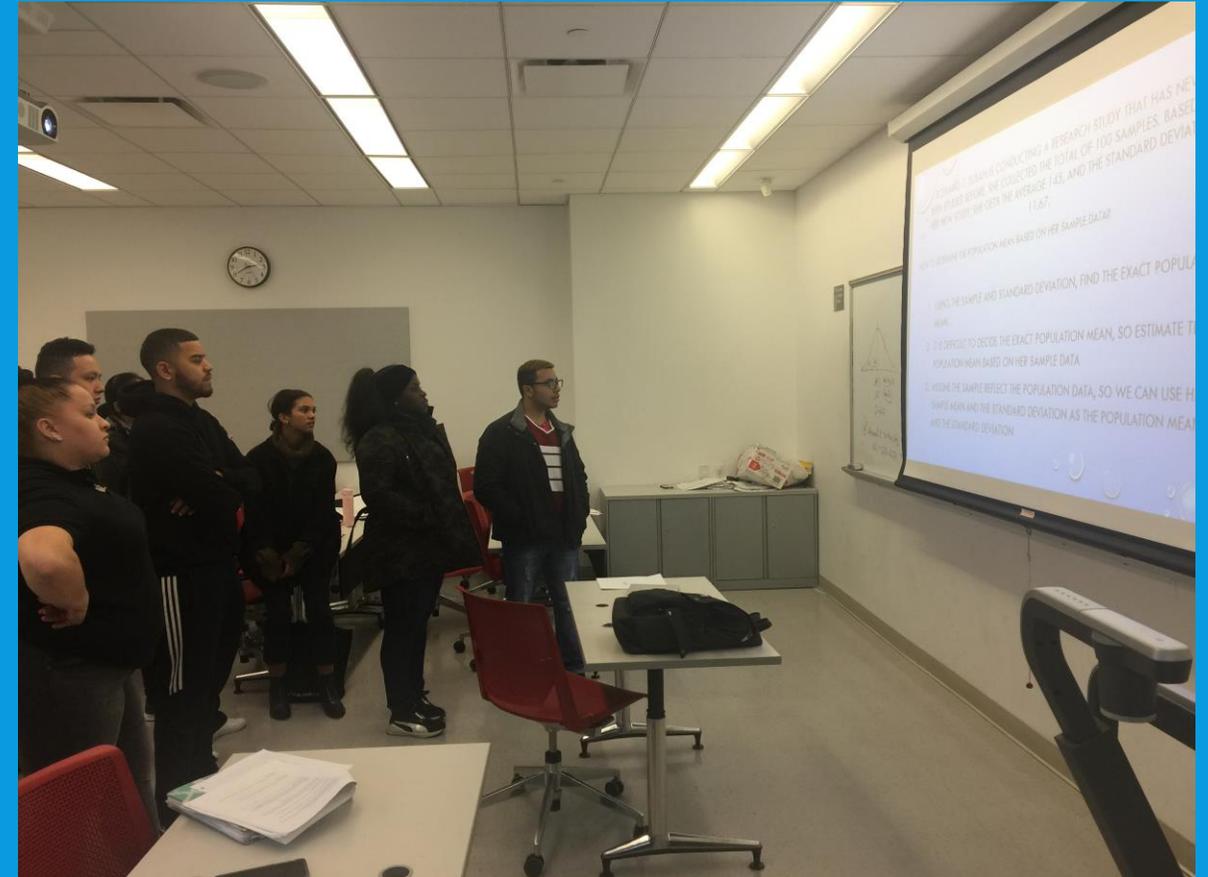
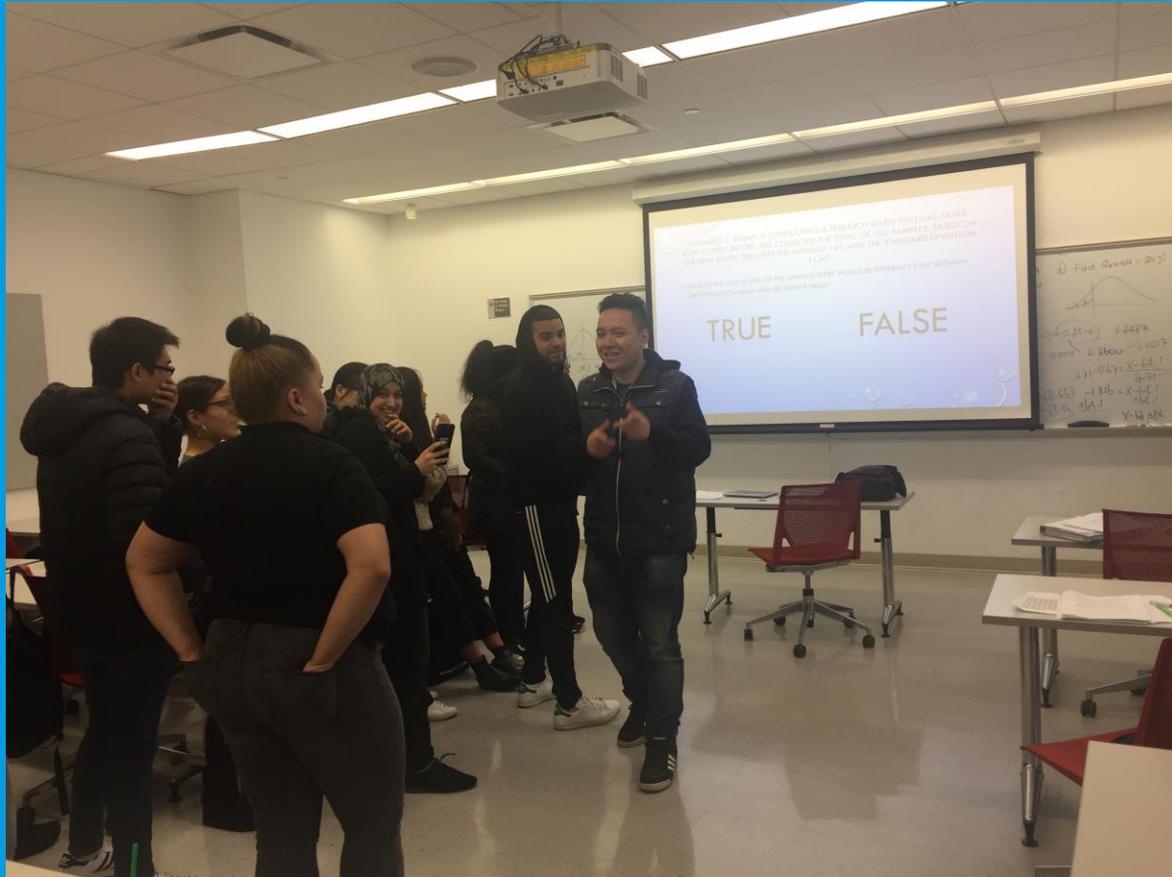
# NORMAL DISTRIBUTION



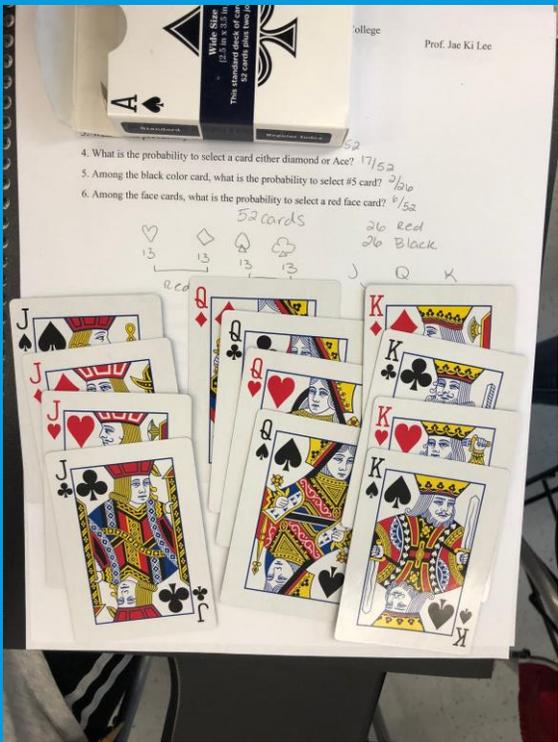
**Students found out that**

- **A smaller standard deviation generates**
  - **narrower and taller normal curve**
  - **Greater z-score**

# CLASSROOM ACTIVITY



# CLASSROOM ACTIVITY



# CLASSROOM ACTIVITY



# HYPOTHESIS PROJECT



Topic: Number of Siblings

Based on the previous Prof. Lee's statistic project, the mean number of siblings is 3, with the standard deviation of 0.85.

Day 1

1. Do you agree with the claim?
2. Set up the Hypothesis set up.
3. Collect 15 data: You can search it using the internet, taking survey, or other resources.

Day 2

4. Decide the Hypothesis Test method (z-test, P-value test, or t-test)
5. Using the Hypothesis Test, Conclude the result

Day 3

Create the Power Point Slide and Hand Out paper

Day 4

Review the Project Document

## Topic: Number of Siblings

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Create the Power Point Slide and Hand Out paper

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Review the Project Document



SHELENE

COOPER

JOEL

# HYPOTHESIS PROJECT

# TOPIC :NUMBER OF SIBLINGS

- The mean number of siblings is 3, with the standard deviation of 0.85

YES WE AGREE  
WITH THE CLAIM

$H_0: \mu = 3$   
Claim

$H_a: \mu \neq 3$

3	6	2	0	2	1	8	
2	4	1	2	5	5	1	2

# TEST METHOD

## T-TEST

- We decided to use the t test method because our data set is smaller than 30

$N < 15$

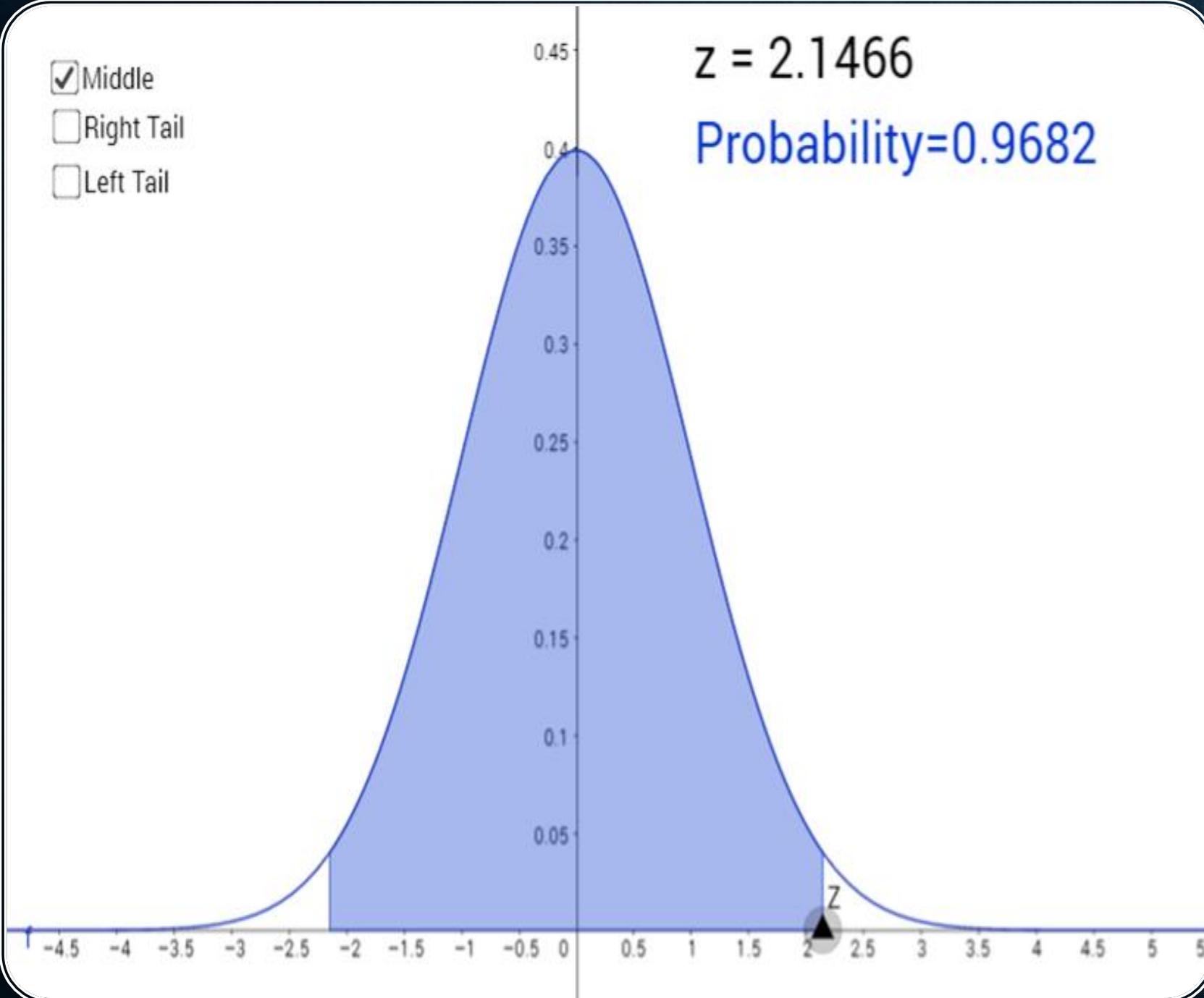
$\mu = 3$

$df = 14 - 2 = 14$

$\bar{x} = 2.9333$

$S = 0.85$

- Middle
- Right Tail
- Left Tail



$$z = 2.1466$$

$$\text{Probability} = 0.9682$$

- Fail to reject  $H_0$  due to the lack of evidence to reject the claim
- There is not enough evidence to support the claim

# CONCLUSION

- The mean number of siblings is 3

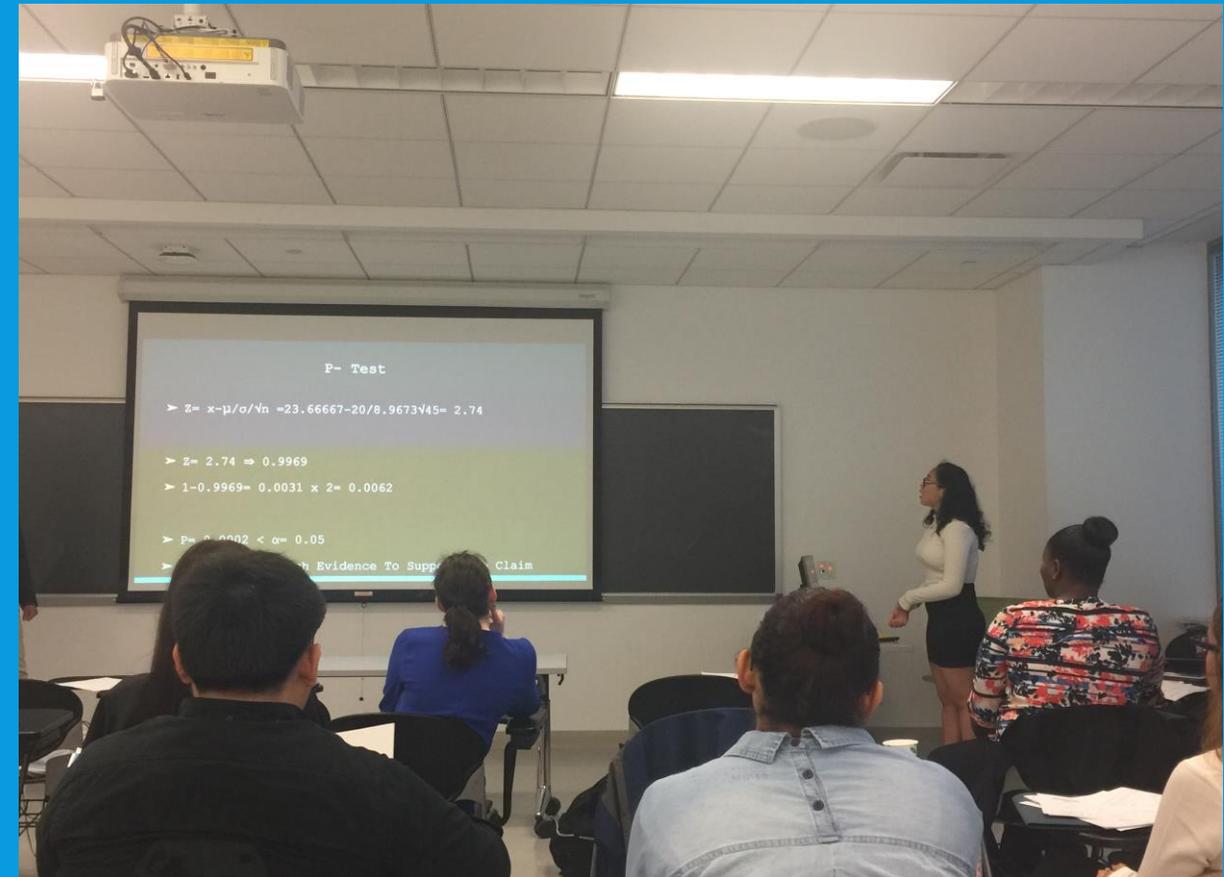
# STUDENTS' PRESENTATION



- Students must use
  - Power Point Slide show
  - Wear Formal dress code
  - Divide role to present, so that all group members present.



# STUDENTS' PRESENTATION



# STUDENT'S PRESENTATION



## P-VALUE TEST

- ❖ We used excel to calculate mean, variance and standard deviation.
- ❖  $N=40$   $\bar{X}=7.7$   $s=\delta=7.8503$   $\alpha=0.05$
- ❖ P-value test:  $z = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{7.7-8.5}{7.8503/\sqrt{40}} = -0.64$
- ❖  $Z(-0.64) - P = 0.2611 > \alpha = 0.05$   $H_0: \mu \geq 8.5$
- ❖ Fail to Reject  $H_0$  Claim is incorrect  $H_a: \mu < 8.5$

## Z - TEST

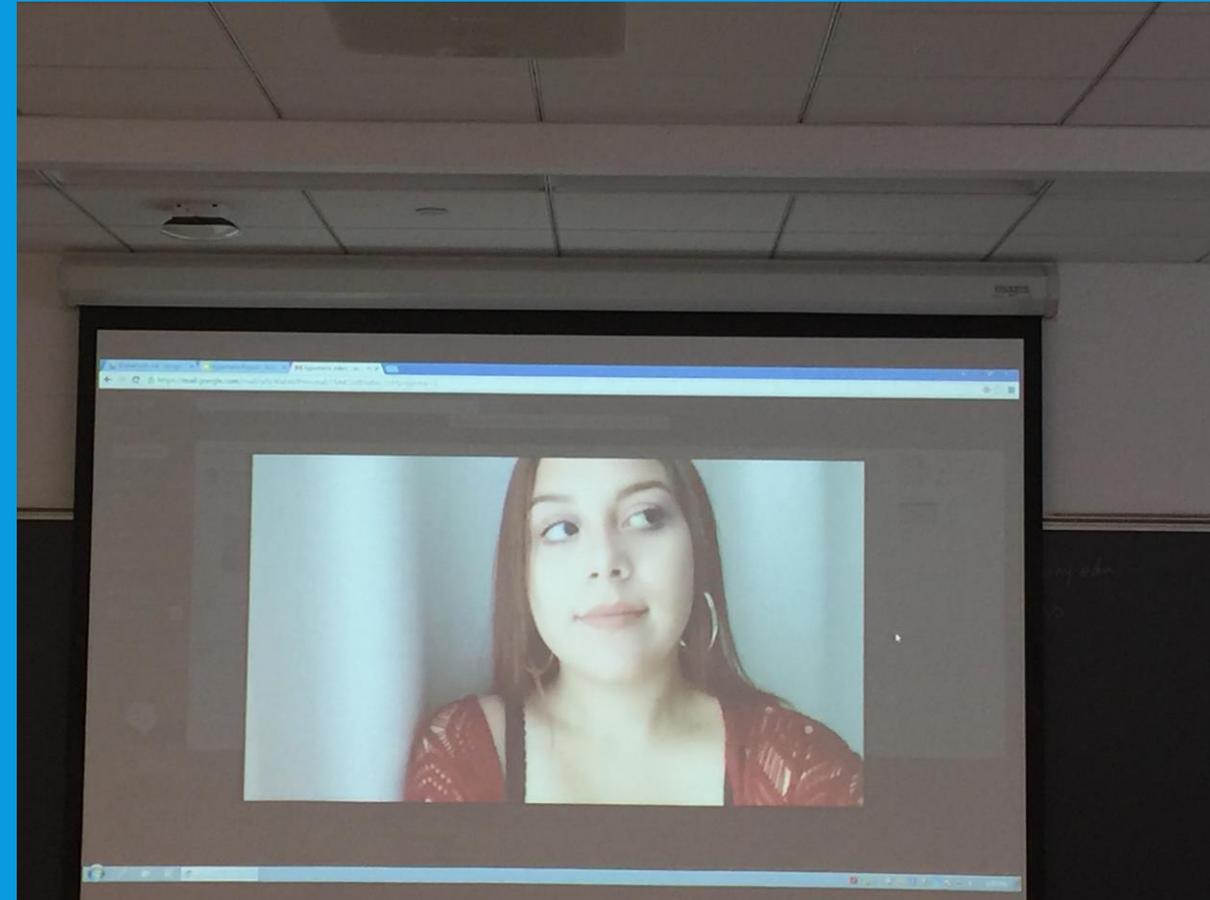
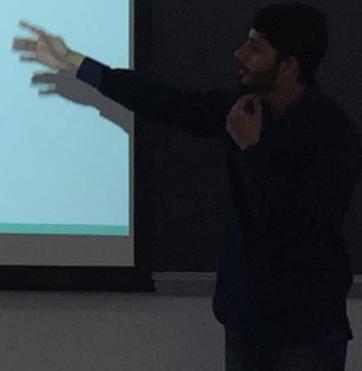
- ❖  $Z = -0.64$   $\alpha = 0.05 = -1.645$   $n = \infty$   $H_0 \downarrow$
- ❖ Because  $-0.64$  belongs to  $H_0$  are:
- ❖ Fail to Reject  $H_0$
- ❖ Claim is Incorrect
- ❖  $H_0: \mu \geq 8.5$
- ❖  $H_a: \mu < 8.5$  Claim

# STUDENTS' PRESENTATION



## Results

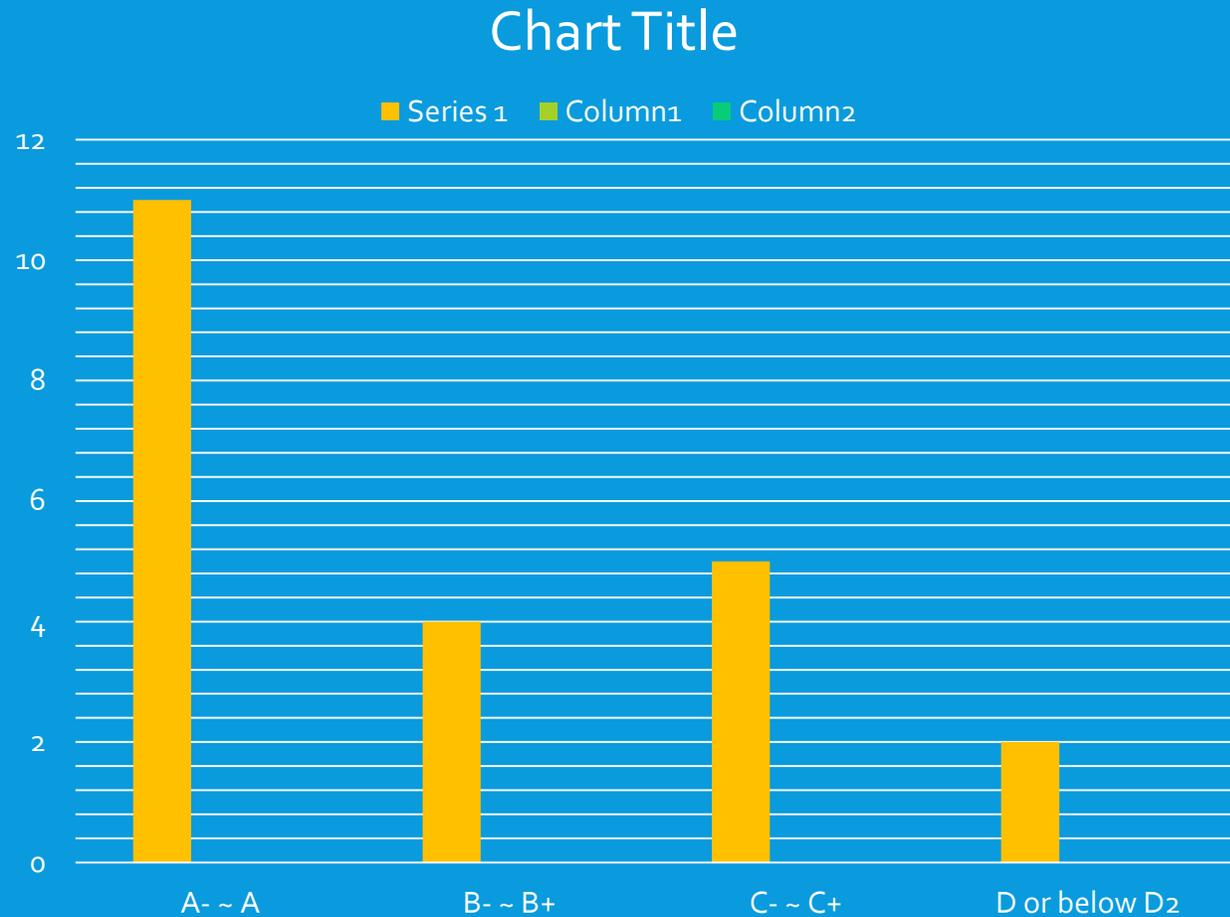
- $\mu = 20$   $\alpha = 0.10$   $\sigma = 1.46507 = 1.4651$   $n = 40$   
$$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{24 - 20}{1.4651/\sqrt{40}} = 17.2672$$
- Z-test  $\rightarrow z = -17.2672 < \alpha = -2.326$
- P-test  $\rightarrow p = 0.0002 < \alpha = 0.10$



# GRADING DISTRIBUTION



	Frequency
A- ~ A	11
B- ~ B+	4
C- ~ C+	5
D or Below D	2
Total	22



# STUDENTS' FEEDBACKS



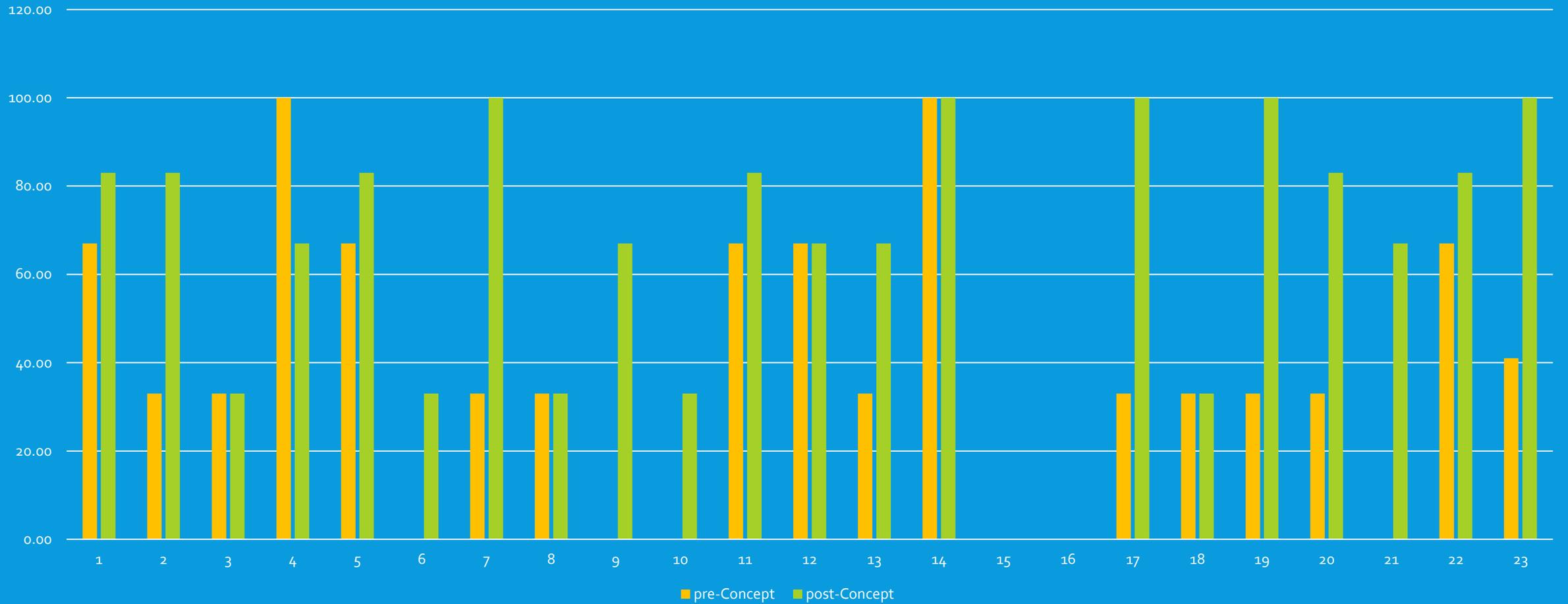
- Enjoyed your class!!
- I did not even realized the semester is over.
- Keep doing this professor.
- I truly learned statistics. I can analyze other hypothesis test.

# STATISTIC ANALYSIS

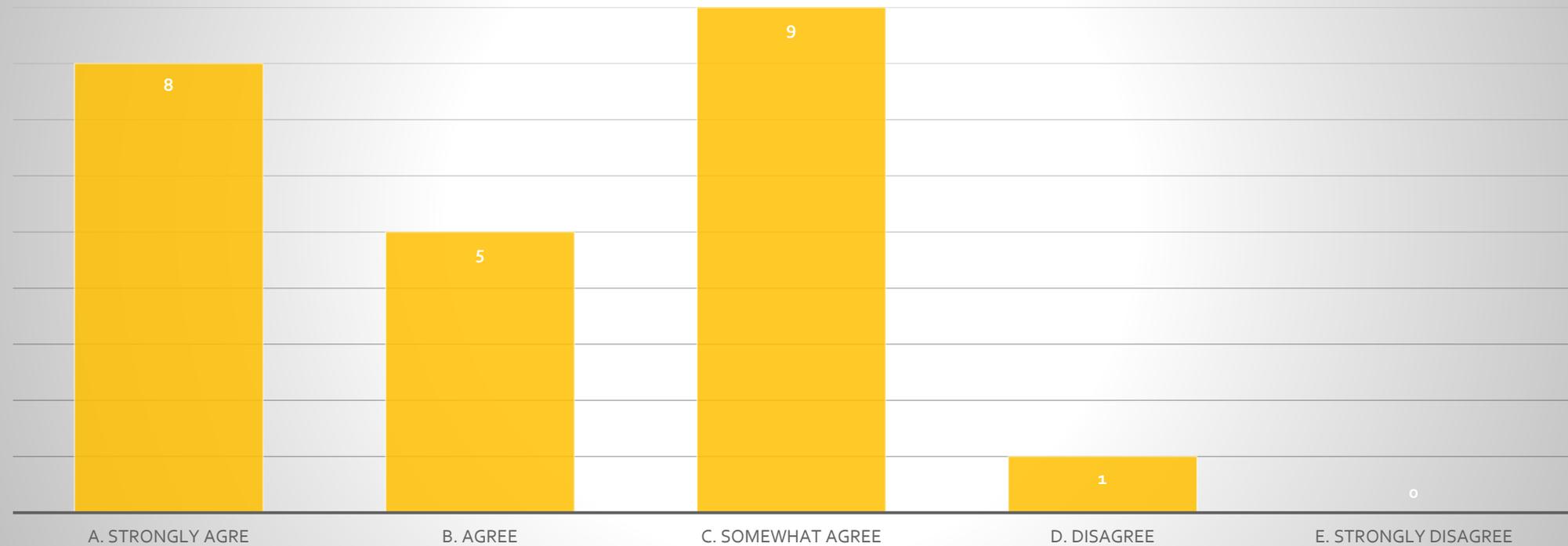


Group Statistics					
	group	N	Mean	Std. Deviation	Std. Error Mean
Pre and post	posttest	24	62.0033	28.39851	5.79682
	pretest	24	45.7429	18.86852	3.85152

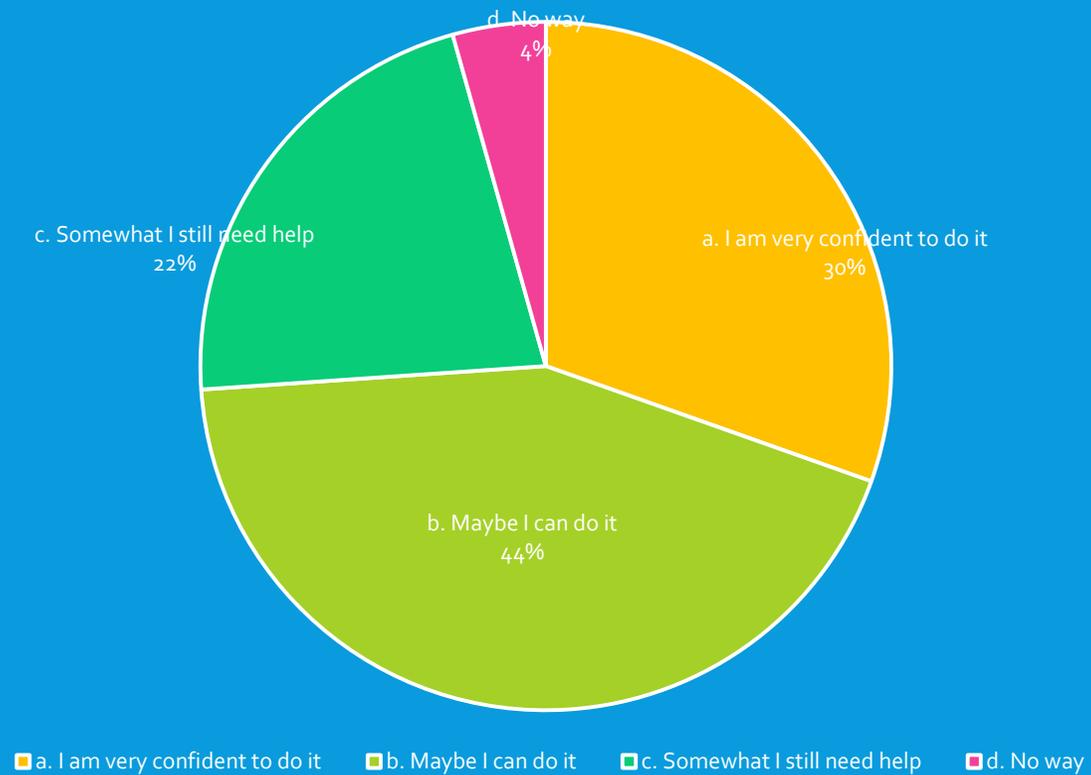
Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Pre and post	Equal variances assumed	4.767	.034	2.336	46	.024	16.26042	6.95969	2.25128	30.26956
	Equal variances not assumed			2.336	39.995	.025	16.26042	6.95969	2.19429	30.32654



Do you think the Hypothesis group project improve your understanding of the Hypothesis Test?



After the Hypotehsis Project, do you feel to conduct a hypothesis porejct if any situation is given to you?



# CONCLUSION



- Combining between Inquiry Based Learning and Project Based Learning is a great tool to teach Intro Statistics Class
  - IBL provided students to explore questions deeply and conceptually.
  - They developed own approaches: A student thought the combination  $nCr$  for Binomial Probability before the class discussed the topic.
- Students are engaged, and improved grade significantly

# CONCLUSION



- Group Project provides them to explore statistical contents, and enhance to understand them
  - Students had an experience to apply all the class materials into their own project
  - The Hypothesis Test Project Provided all the students to be ready to answer for their presentation.
  - Collaborative work
  - Role play: All students must participate in group presentation.
  - All group members must understand their presentation material thoroughly.

QUESTIONS?



# Thank You

Contact Information

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