Challenges of Teaching Calculus

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Question

What are the major concerns regarding teaching calculus today?
Concerns

Nationally only about 25% of Calculus I students successfully complete the course. (MAA)

Students do not seem to be fully engaged with learning the material.

Large numbers of students seem unprepared for calculus.

Many students need to be motivated to study and learn.

Too many students and teachers believe that the key to success in mathematics is memorization and mimicry.
What are some of the distractions that students face today?
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Cell Phones
Social Media
Video Games
Socializing
What are some of the distractions that students face today?

- Cell Phones
- Social Media
- Video Games
- Socializing

Can mathematics faculty use any of these distractions to support learning?
Myth of Multitasking

I am a great multitasker

https://www.psychologytoday.com/blog/creativity-without-borders/201405/the-myth-multitasking
The use of cell phones in the classroom causes distraction to the class (Burns & Lohenry, 2010; Campbell, 2006; Gilroy, 2004; Shelton et al., 2009; Tindell & Bohlander, 2012).

Cell Phones

• More than 90% of students sent text messages during classes. Tindell and Bohlander (2012)

• Cell phones distract 89% of students during study hours and nearly 77% of students during class. Potharst (2010)

• Student performance improved once cell phones were banned. Bugeja (2007)
Poor Student Preparation

• National Assessment of Educational Progress - NAEP
• Program for International Student Assessment - PISA
• ACT
What are the five most important prerequisite concepts that calculus students must understand?
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My List
Algebraic Fractions
Functions in All Their Glory
Properties of Exponents
Inequalities
Solving Trigonometric Equations
What can be done to assist students who are lacking in these areas?
What can be done to assist students who are lacking in these areas?

• Spend some time in class reviewing concepts like adding fractions.
• Encourage students to watch videos from sources like Khan academy, PatrickJMT and mathispower4u.
• Strongly encourage students to seek assistance.
• Develop co-requisite models for weaker students.
• Use diagnostic computer-based packages with weaker students.
National Assessment of Educational Progress

Proficient or Higher
25% of the overall population
  32% of Caucasian students
  7% of African American students
  12% of Hispanic students

No significant change in the overall number over the last ten years

https://www.nationsreportcard.gov/
Program for International Student Assessment

The PISA is a worldwide exam administered every three years that measures 15-year-olds in 72 countries. About 540,000 students took the exam in 2015.

US saw an 11 point drop in mathematics

Math

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41% of students scored a 22 or above on the mathematics section, indicating a 75% likelihood of passing college algebra.

Poor Student Background Knowledge

The rush to calculus in high school may have shortchanged students on topics from algebra, geometry, trigonometry, and analytic geometry.

43% increase in calculus enrollment at two-year colleges from 2000 – 2015. (CBMS 2015)

42% increase in calculus enrollments at the university level.
Sadler and Sonnert (2018)

Mastery of pre-calculus concepts was found to have more than double the impact of taking a high school calculus course on students’ later performance in college calculus.
Poor Study Habits

Cerrita and Levi (1999) found that students have sufficient time to study mathematics but they are choosing not to do so. In addition, the regular collection and grading of homework was highly correlated with increased study time in mathematics.

Question

How can calculus teachers motivate students?
Motivation

• The concern shown by teachers in taking the time to find the techniques that best fit the class and allow the teachers to feel comfortable and professional is essential to motivating students.

  http://math.coe.uga.edu/TME/Issues/v03n2/Eggleton.pdf

• Nonjudgmental verbal reinforcement and positive feedback can stimulate interest or at least keep it from evaporating. Stress opportunities to improve and find ways to stimulate advancement.

• Students rebel if they believe the reward structures are designed to control them.

• Encourage cooperation and collaboration

  Kenneth Bain (2004): What the Best College Teachers Do
How to Motivate Adult Learners

• Focus on short-term goals
• Emphasize that all progress matters
• Let them know it’s okay to get it wrong
• Make it relevant to their lives
• Tap into their intrinsic motivation

https://www.seedsofliteracy.org/5-ways-to-motivate-adult-learners/
Research-Based Motivation Strategies

http://www.vanderbilt.edu/cft/resources/teaching_resources/interactions/motivating.htm


Become a role model for student interest.

Deliver your presentations with energy and enthusiasm. As a display of your motivation, your passion motivates your students. Make the course personal, showing why you are interested in the material.
Get to know your students

You will be able to better tailor your instruction to the students' concerns and backgrounds, and your personal interest in them will inspire their personal loyalty to you. Display a strong interest in students' learning and a faith in their abilities.
Use examples freely

Many students want to be shown why a concept or technique is useful before they want to study it further. Inform students about how your course prepares them for future opportunities.
Use a variety of student-active teaching activities

Teach by discovery. Students enjoy reasoning through a problem and discovering the underlying principles on their own.

Cooperative learning activities are particularly effective as they also provide positive social pressure.
Set realistic performance goals

Help students achieve them by encouraging them to set their own reasonable goals. Design assignments that are appropriately challenging in view of the experience and aptitude of the class.
Place appropriate emphasis on testing and grading.

Tests should be a means of showing what students understand, rather than a process to identify misconceptions.
Be free with praise and constructive in criticism.

Negative comments should pertain to particular performances, not the performer. Offer nonjudgmental feedback on students' work, stress opportunities to improve, look for ways to stimulate advancement. Avoid categorizing students based on performance.
Give students as much control over their own education as possible.

Let students choose paper and project topics that interest them. Assess them in a variety of ways (tests, papers, projects, presentations, etc.) to give students more control over how they show their understanding to you.
Question

Why do so many students struggle with the basics of mathematics?
The methods used to teach mathematics in K-12 schools are not succeeding.

Many community college students are not able to perform basic tasks in arithmetic, pre-algebra and algebra.
K-12 Mathematics –
Stigler, Givvin, Thompson

• The Trends In International Mathematics and Science Study (TIMSS) showed that the most common teaching methods used in the U.S. focus almost entirely on practicing routine procedures with virtually no emphasis on understanding of core mathematics concepts.

• The high-achieving countries use instructional methods that focus on actively engaging students with understanding mathematical concepts.
Ganem

Steps can be memorized but it will be a long time, if ever, before the concept and motivation for the process is understood. That raises the question of what exactly is being accomplished with such a curricula?

A Math Paradox:
The Widening Gap Between High School and College Math
Learning techniques without understanding them does no good in preparing students for college. At the college level emphasis is on understanding, not memorization and computational prowess.

A Math Paradox:
The Widening Gap Between High School and College Math
We have also become aware that students increasingly seek someone to provide structure, direction, and praise in a way previous generations of students did not. . .

This generation of college students has been raised on interactive technology and entertainment-style communication. We have been told by our students that straight lectures or PowerPoint presentations rarely hold their attention. Experiences that involve students and require them to interact as a part of their own learning are more likely to maintain their interest.
Active learning increases student performance in science engineering and mathematics.
Active Learning

Any instructional method that engages students in the learning process. In short, active learning requires students to do meaningful learning activities and think about what they are doing. Active learning is often contrasted to the traditional lecture where students passively receive information from the instructor.

Question

What are some of the strategies that a calculus teacher can use to foster an active student learning environment?
Active Learning Strategies

Ask students to complete problems in class; discuss the solutions with classmates; present solutions to the class.

Ask questions to students to see if they understand the concepts
Pause during the lecture to give students an opportunity to reflect.
Get student feedback on muddiest point or clearest concept from a lesson.
Have students present solution strategies in class.
Ask students to construct multiple strategies to solve a given problem.
Assign roles like questioner, writer, reporter, etc.
Ask students to question each other in order to develop understanding.
Ask the students to take turns teaching each other.
Conclusions

• Instructors must help students resist the many tempting distractions.

• Instructors must assist poorly-prepared students in seeking the necessary assistance to overcome that challenge.

• Good teachers are good motivators. It is not enough to deliver well prepared professional lessons.

• Active student learning approaches can be effective in improving student performance.
Thank you!

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See you next year in Milwaukee