S138

Saturday 11/17/18
1:05 PM–1:55 PM

Teaching and Learning Power Series Expansions of Taylor Series Using Desmos

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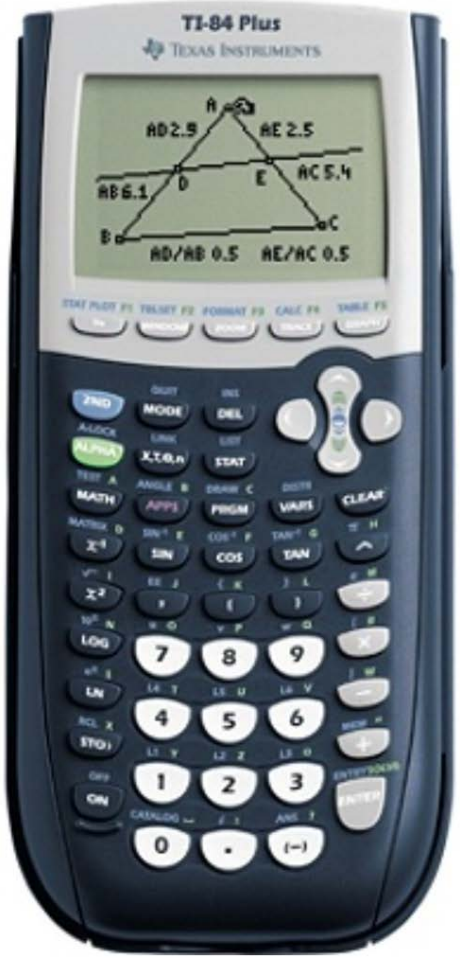
TI-84 Plus vs. Desmos



desmos

Taylor Series

A **Taylor Series** is a polynomial with an infinite number of terms that represents another function. This function can be approximated with a finite number of terms of its Taylor series. The Taylor series of a real function $f(x)$ that is everywhere differentiable at a real number a is the power series

$$f(x) = \sum_{n-0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \cdots$$

If $a = 0$, then the series is called a **Maclaurin series**.

Example 1

The geometric series formula

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$$

converges for $-1 < x < 1$.

Example 1 (cont.)

That is, the graph of

$$f(x) = \frac{1}{1-x}$$

can be approximated by the graph of the Taylor series

$$T(x) = \sum_{n=0}^{p} x^n$$

where p is a positive integer, but only for $-1 < x < 1$. Outside of this interval, however, the graphs of f and T diverge.

Example 1 (cont.)

For the geometric series in this example, you will find an animation of the Taylor series for the approximating polynomial here:

https://tinyurl.com/amatyc1

Please click the play button  in the third line to animate the graph of the Taylor series.

Your Turn: Examples 2–4

Ex.	Function	Taylor Series	Radius of Convergence
2	$f(x) = e^x$	$T(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$	$R = \infty$
3	$f(x) = \cos x$	$T(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$R = \infty$
4	$f(x) = \ln(1+x)$	$T(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$	$R = 1$ for $x \in (-1, 1]$

Solutions: Examples 2–4

2. https://tinyurl.com/amatyc2

3. https://tinyurl.com/amatyc3

4. https://tinyurl.com/amatyc4

Discussion Questions

- What year is it? **1994** or **2018**?
- Why are many instructors still using TI-84 graphing calculators (or similar)?
- How can Desmos be used in your classes?
- How can the test mode in Desmos be used for proctored testing?