What is your Erdős power reducing number?

Luke Walsh
he / him / his
Catawba Valley CC
Hickory, North Carolina
The ultimate tragedy is not the oppression and cruelty by the bad people but the silence over that by the good people.

Martin Luther King Jr.
\[
\begin{align*}
\int \sin^n(x) \, dx &= \frac{-1}{n} \sin^{n-1}(x) \cos(x) + \frac{n-1}{n} \int \sin^{n-2}(x) \, dx \\
\int \cos^n(x) \, dx &= \frac{1}{n} \cos^{n-1}(x) \sin(x) + \frac{n-1}{n} \int \cos^{n-2}(x) \, dx \\
\int \tan^n(x) \, dx &= \frac{1}{n-1} \tan^{n-1}(x) - \int \tan^{n-2}(x) \, dx \\
\int \cot^n(x) \, dx &= \frac{-1}{n-1} \cot^{n-1}(x) - \int \cot^{n-2}(x) \, dx \\
\int \sec^n(x) \, dx &= \frac{1}{n-1} \sec^{n-2}(x) \tan(x) + \frac{n-2}{n-1} \int \sec^{n-2}(x) \, dx \\
\int \csc^n(x) \, dx &= \frac{-1}{n-1} \csc^{n-2}(x) \cot(x) + \frac{n-2}{n-1} \int \csc^{n-2}(x) \, dx
\end{align*}
\]
Integration of \( u \) do \( u \).

\[ \int u^n \, du \]

Be conscious of the power \( U \) have.
I HAVE THE POWER OF

○ WHITENESS
○ MALE
○ WHITE MALE
\[
\left( \frac{1 \text{ of me}}{2 \text{ not like me}} \right) \text{ me}
\]
Square Root
EQUALITY
Half Power
HALVE
I HAVE THE POWER OF

\[ \sqrt{\text{whiteness}} \]

\[ (\text{male})^{1/2} \]

\[ (\text{white male})^{1/2} \]
MATHMATICS
GENIUS

@lukeselfwalker
Like Einstein

Not Like Einstein

Einstein Power Reducing Formula

2019

@lukeselfwalker
OPENS THE SPACE FOR OTHERS

2019 @lukeselfwalker
What do you notice / wonder?

Phyllis Chinn
Fan Chung
Yolanda Debose
Yael Dowker
Maria Klawe
Ewa Maria Kubicka
Carole B. Lacampagne
Jean Larson
Renu Chakravarti Laskar
Wen-Ching Winnie Li
Janice L. Malouf
Anja Gabriele Meyer
Ortrud R. Oellermann
Mary Ellen Estill Rudin
Ruth Silverman
Vera Turan Sos
Hendrika Cornelia Scott Swa
Claudia Alison Spiro-Silverm
Esther Klein Szekeres
Katalin L. Vesztergombi
Foong Frances Yao

https://medium.com/q-e-d/whos-important-a-tale-from-wikipedia-a370dc6ef078
Paul Erdős # of 1

21 women

= 512 people

≈ 96% male
What is your Erdős power reducing number?
"I tell my students, 'When you get these jobs that you have been so brilliantly trained for, just remember that your real job is that if you are free, you need to free somebody else. If you have some power, then your job is to empower somebody else. This is not just a grab-bag candy game.'

TONI MORRISON

@lukeselfwalker
Improving Mathematical PROWESS And College Teaching

PR – Proficiency
OW – Ownership
E – Engagement
SS – Student

2019

@lukeselfwalker
ZERO IS MY WE ARE ALL POWER REDUCING NUMBER

\[(we)^0 = 1\]
Use of Complex Numbers

… in Calculus II (and beyond)

Violeta Kovacev-Nikolic
College of the Canyons
Partial Fractions

Let $f(x) = \frac{P(x)}{Q(x)}$ be a proper rational function, deg $P < \text{deg } Q$.

Goal: integrate $f(x)$.

Method: write $f(x)$ as a sum of simpler fractions ("partial fractions")
Partial Fractions: cases

1. *Distinct Linear Factors* in the denominator

\[ Q(x) = (a_1x + b_1)(a_2x + b_2) \cdots (a_kx + b_k) \]

\[
\begin{array}{c}
\frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \cdots + \frac{A_k}{a_kx + b_k}
\end{array}
\]
Partial Fractions: cases

2. Repeated Linear Factor \((a_1x + b_1)^r\)

\[
\frac{A_1}{a_1x + b_1} + \frac{A_2}{(a_1x + b_1)^2} + \cdots + \frac{A_r}{(a_1x + b_1)^r}
\]
Partial Fractions: cases

3. Irreducible Quadratic: \((ax^2 + bx + c)\), \(b^2 - 4ac < 0\)

\[
\frac{Ax + B}{ax^2 + bx + c}
\]
Partial Fractions: cases

4. **Repeated Irreducible Quadratic**: $\left( {ax^2 + bx + c} \right)^r$

\[
\frac{A_1 x + B_1}{ax^2 + bx + c} + \frac{A_2 x + B_2}{\left( ax^2 + bx + c \right)^2} + \cdots + \frac{A_r x + B_r}{\left( ax^2 + bx + c \right)^r}
\]
Partial Fractions: strategy

1) Factor the denominator
2) Write $f(x)$ as a sum of partial fractions
3) Multiply both sides by the denominator
4) Solve for the coefficients
Partial Fractions: two methods

1) Undetermined Coefficients
   - “Method of Atoms”
   - Compare like terms on both sides
   - Set coefficients equal
   - Solve system
Partial Fractions: methods

2) Sample Method

- “Plug and Chug Method”
- Choose suitable values for $x$
- Each value of $x$ yields an equation
- Solve system for remaining constant(s)
Partial Fractions: usual method

\[
\frac{5x^2 - 7x + 4}{x(x^2 + 1)} = \frac{4}{x} + \frac{x - 7}{x^2 + 1}
\]

\[
\Rightarrow 5x^2 - 7x + 4 = (A + B)x^2 + Cx + A
\]

\[
\Rightarrow A = 4
\]

\[
\Rightarrow C = -7
\]

\[
\Rightarrow A + B = 5 \quad \Rightarrow B = 1
\]
Partial Fractions: suitable x

\[
\frac{5x^2 - 7x + 4}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}
\]

\[
\Leftrightarrow 5x^2 - 7x + 4 = A(x^2 + 1) + (Bx + C)x
\]

Let \( x = 0 \) \( \Rightarrow \boxed{A = 4} \)

Let \( x = i \) \( \Rightarrow \quad -5 - 7i + 4 = (Bi + C)i \)

\[
-1 - 7i = -B + Ci
\]

\boxed{B = 1} , \quad \boxed{C = -7}
Integrals

Find:

\[ I_1 = \int e^{ax} \cos(bx) \, dx \quad , \quad I_2 = \int e^{ax} \sin(bx) \, dx \]

- **Usual method**: integration by parts
- **Another way**: use complex numbers
Integrals: Euler’s formula

Euler’s formula:

\[ e^{(a+ib)x} = e^{ax} (\cos(bx) + i \sin(bx)) \]

\[
\int e^{(a+ib)x} \, dx = \left[ \int e^{ax} \cos(bx) \, dx \right] + i \left[ \int e^{ax} \sin(bx) \, dt \right] - I_1 + i I_2
\]
Integrals: solving procedure

\[
\int e^{(a+ib)x} \, dx
\]

\[
= \frac{1}{a+ib} e^{(a+ib)x} + C
\]

\[
= \frac{1}{a+ib} \cdot \frac{a - ib}{a - ib} \cdot e^{(a+ib)x} + C
\]

\[
= \frac{e^{ax}}{a^2 + b^2} \cdot (a - ib)(\cos bx + i \sin bx) + C
\]

\[
= e^{ax} \cdot \frac{(a \cos bx + b \sin bx)}{a^2 + b^2} + i \left( \frac{a \sin bx - b \cos bx}{a^2 + b^2} \right) + C
\]
Integrals

\[ I_1 = \int e^{ax} \cos(bx) \, dx \]
\[ = \frac{e^{ax}}{a^2 + b^2} \left[ a \cdot \cos(bx) + b \cdot \sin(bx) \right] + C \]

\[ I_2 = \int e^{ax} \sin(bx) \, dx \]
\[ = \frac{e^{ax}}{a^2 + b^2} \left[ a \cdot \sin(bx) - b \cdot \cos(bx) \right] + C \]
“I tell you, with complex numbers you can do anything.”

- John Derbyshire, writer
References:

AN ODE TO MATH & GOLF

MARI MENARD
LSC-KINGWOOD
AMATYC TRAVELING WORKSHOP COORDINATOR

2019
AN ODE

A lyrical stanza praising or glorifying an event

Usually with music
JUST 18 HOLES OF GOLF

After the 1st hole...
JUST NOT HERE!
SUBTRACTION IS COOL ...

KIND OF LIKE THE WEATHER
STUPID BALL!

GO IN THE HOLE!
GIVE ME A BREAK!
LINEAR APPLICATION
WHEN YOU HURL YOUR CLUB…
THAT PERFECT SHOT.
NOT CLOSE BUT...
“Do you see that?” How visualization can increase understanding of real-world applications in math.

Sherrie Serros
Mount Mary University
How much of a 60% acid solution should be mixed with 20 liters of a 35% solution to obtain a 50% solution.

\[
\begin{align*}
20 \times 15 & = x \times 10 \\
300 & = 10x \\
30 & = x
\end{align*}
\]
There were 35 more students than teachers at a school party. Altogether there were 49 people at the party. How many teachers were at the school party?
15 gallons fill a tub 2/5 full. What is the capacity of the tub?
Sherrie is baking. She has 5 cups of flour which is enough for 1 ½ batches of rolls. How much flour is used in each batch?
Kahlia spends \( \frac{1}{5} \) of her dog-sitting money on a CD and has $44.00 left. How much dog-sitting money did Kahlia have?
When a box of chocolates was full, it weighed 1.1 kg. After half of the chocolates were eaten, the box (with the remaining chocolates) weighed 0.7 kg. How much did the box weigh without the chocolates?
1.1 kg

- box: chocolates
  - box: \( \frac{1}{2} \) choc
    - 0.7 kg
    - 0.4 kg

0.3

box alone weighs 0.3 kg

\( \frac{1}{2} \) chocolates weigh 0.44 kg

all chocolates weigh 0.8 kg

box alone weighs 0.3 kg

0.7 kg

1.1 kg

remove 0.4 kg

remove 0.4 kg
b: weight of box in kg

C: weight of chocolates in kg

\[
\begin{align*}
\begin{cases}
    b + c &= 1.1 \\
    b + \frac{1}{2}c &= 0.7
\end{cases}
\end{align*}
\]

elimination

\[
\frac{1}{2}c = 0.4
\]

\[
c = 0.8
\]

\[
b + 0.8 = 1.1
\]

\[
b = 0.3
\]
At O’Snare airport, flights take off on-time in stormy weather just 20% of the time, but take off on-time in good weather 70% of the time. The forecast for tomorrow indicates a 60% chance of a storm at noon.

What is the probability that a noon flight will take off on-time at O’Snare tomorrow?
\[ P(\text{on-time}) = 0.12 + 0.28 = 0.40 \]
\[ P(\text{on-time @ noon}) = 0.12 + 0.28 = 0.40 \]
GW: good weather
OT: on time

\[ P(\text{on-time}) = 0.40 \]
\[ P(\text{on-time}) = P(GW \cap OT) + P(SW \cap OT) \]
\[ = P(GW) \cdot P(OT|GW) + P(SW) \cdot P(OT|SW) \]
\[ = (0.4)(0.7) + (0.6)(0.2) \]
\[ = 0.28 + 0.12 = 0.40 \]
Working alone, Maria can complete a task in 100 minutes. Shaniqua can complete the same task in two hours. They work together with Liu, the three-person team completes the task in 40 minutes. How long would it take Liu to complete the task alone?
Maria 100 min 3/5 40/100
Shaniqua 120 min 1/3 40/120
Liu ?

\[
\frac{6}{15} + \frac{5}{15} = \frac{11}{15}
\]

Liu completes
\[
\frac{4}{15} \text{ task in 40 minutes}
\]
\[
\frac{1}{15} \text{ task in 10 minutes}
\]

1 = \frac{15}{15} \text{ task in 150 minutes}
Together Evan, Katie, and McKenna had $865 when they left to go shopping. Evan spent 2/5 of his money. Katie spent $40. McKenna spent twice as much as Evan. They each have the same amount of money left. How much money did each take shopping with them?
Together Evan, Katie, and McKenna had $865. Evan spent 2/5 of his money. Katie spent $40. McKenna spent twice as much as Evan. They each have the same amount of money left.

\[
\begin{align*}
\frac{15}{5} E + 40 &= 865 \\
3E &= 825 \Rightarrow E = 275
\end{align*}
\]
E: amount Evan starts with
\( \frac{2}{5} E \): amount Evan spends
\( \frac{3}{5} E \): amount Evan has left

K: amount Katie starts with
40: amount Katie spends
K-40: amount Katie has left

M: amount McKenna starts with
\( \frac{4}{5} E \): amount McKenna spends
M-\( \frac{4}{5} E \): amount McKenna has left

\[ E + \frac{2}{5} E + 40 + \frac{3}{5} E = 865 \]
\[ \frac{15}{5} E = 825 \]
\[ E = \frac{825}{3} = 275 \]

Evan: 275
Katie: \( \frac{3}{5} E + 40 \rightarrow 1205 \)
McKenna: \( \frac{7}{5} E \rightarrow 385 \)
Why would I wear pants?! 
and other math analogies

Luke Audette, a rookie
Analogy??

- Studies have shown...
- Any comparison for any topic
- 7-12 students worked well but...
Priming the engine

- My informal studies have shown
  - Creating Relationships
  - Laughs
  - Having Fun
  - Linking topics
Let’s Get Rolling

MARRIED 2008
Following the rules
Why would I wear Pants!
Upscale Math!

\[
\frac{5}{\sqrt{2}} \quad \frac{5\sqrt{2}}{2}
\]
Upscale Math!

\[
\frac{5}{\sqrt{2}} \quad \text{VS} \quad \frac{5\sqrt{2}}{2}
\]
I’ll take the number 1 permo
Fractions and Word Problems
<table>
<thead>
<tr>
<th>QTY</th>
<th>ITEM</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MCDouble</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>HOT&amp;SPICY MCCHICKEN</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>SML FRENCH FRIES</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>LRG COKE</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td><strong>Subtotal</strong></td>
<td><strong>4.00</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Tax</strong></td>
<td><strong>0.24</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Eat-In Total</strong></td>
<td><strong>4.24</strong></td>
</tr>
</tbody>
</table>
Don’t end up in a roadside ditch
Let's talk about Functions

Why is it a function? Besides clearly passing the VLT….

Domain of a function… CAREFUL!
CRAP!
Washers and Dryers
This one is approved
Where is the love for point slope!
Crying Kids
Wooden Puzzles

Prove the following identity:

\[ \csc(x) - \cos(x) \cot(x) = \sin(x) \]
The mother of all analogies

\[(a + b)^2 \neq a^2 + b^2\]
What is in it for Me?

ignite.amatyc

What's in it for ME?

right now!

itune.apple.com

mychidesigns.com
Mind-Set Matters, Exercise and Placebo Effect
Mind-Set Matters, Exercise and Placebo Effect

- Mean Weight (pounds)
- BMI (weight/height²)
- Percentage Body Fat
- Waist/Hip Ratio
- Systolic Blood Pressure
- Diastolic Blood Pressure

Comparison between Informed and Control groups:
- Time 1 vs Time 2
# Smart Phones Versus Our Brain

<table>
<thead>
<tr>
<th>Hardware</th>
<th>Software</th>
<th>Reception</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smart Phones</td>
<td>Smart Phones</td>
<td>Smart Phones</td>
</tr>
<tr>
<td>digital trends.com</td>
<td>lifewire.com</td>
<td>androidauthority.com</td>
</tr>
<tr>
<td>Processors, Electronic components</td>
<td>Programs for processing digital data</td>
<td>onlineclassnotes.com</td>
</tr>
<tr>
<td>Brain Cells and their connections, Neurons and Axons, Gray and white matter part of the brain</td>
<td>Mind, Thoughts, Feelings, Behaviors</td>
<td>blogs.discovermagazine.com</td>
</tr>
<tr>
<td>futurism.com</td>
<td>futurism.com</td>
<td>theconversation.com</td>
</tr>
</tbody>
</table>

Neurotransmitter receptors communicate with one another through chemical signals.
Pruning Phase and Use it or Lose it Principle!

Source: Sarah-Jayne Blakemore
Information is NOT Knowledge!
Brain Plasticity

Brain’s ability to reorganize neural pathways throughout the lifespan as a result of experience.
Brain Plasticity, Timing is Everything!

Source: Sarah-Jayne Blakemore
Brain Basics

Brain Stem

Used with permission from BrainLine.org
Amygdala, Reactive Part of the Brain

Used with permission from BrainLine.org
Prefrontal Cortex, Reflective Part of the Brain

- Planning & organizing
- Problem solving & decision making
- Memory & attention
- Controlling behavior, emotions & impulses

\[
\begin{align*}
  y &= x + 1 \\
  y &= x + 1 \\
  + y &= -x \\
  2y &= 1 \\
  y &= \frac{1}{2} \\
  \downarrow \\
  y &= -x \\
  -\frac{1}{2} &= x
\end{align*}
\]

\[\text{Solution} \left(-\frac{1}{2}, \frac{1}{2}\right)\]
Reactive versus Reflective Part of the Brain

$y = f(x) = 2^x + 4 = 36$

$y = f(x) = 2x + 4 = 14$
What is in it for Me Right Now?

Brain Cell Connections

acceleratedlearning.com
Power Supply
Brain’s Blood Flow

Research/scan compliments of Dr. Chuck Hillman, University of Illinois

Baseline
Meditation
Orientation Area
Orientation Area

Depression Changes Brain
Metabolism and Blood Flow:
Frontal Cortex

Healthy
Chronic Depression

Normal
Mild cognitive impairment
Alzheimer’s disease

researchgate.net
Look at the Similarities of...

Increased Blood flow, Increases Attention
Price to Pay???

Brain with Adderall  Brain without Adderall
Red= Increased Activity

youtube.com  researchgate.net

vimeo.com

hlswakeview.com  michigandaily.com
Younger Generation’s IQ and Their Attention Span

The U.S. population has shown an increase of about 3 points in the average IQ score every decade.
Helping Our Student to Care More About Their Brain Than Their Grade.

How your brain activity spikes when we see something new.

Thank You!
Math Oxymorons & Names that are Verbs

Eddie Tchertchian, LA Pierce College
Math oxymorons & verb names

- It would be an **accurate estimate** to say you use math oxymorons in your math classroom!
- Nick – to jot down
Math oxymorons & verb names

- One might say in algebra class that two expressions are **approximately equal**.
- Bob – to strike with a quick light blow
Let’s split some quantity into halves. Now, let me talk about the **larger half**… is it a **complete half**… what about the **smaller half**? Is it a **whole half**?

Chip – to cut or hew with an edged tool
Math oxymorons & verb names

- Did you get that previous slide about the halves? The **whole hemisphere** was paying attention…
- Pat – to tap or stroke gently with the hand
- Hello fellow presenter, Pat Riley!
I really am starting to think this conversation on halves is straight out of line. And this line does extend to infinity and beyond.

Chuck – to pat

Hi again, Pat Riley!
Math oxymoron & verb names

- I was told this conference is in close **proximity** to Chicago. I figured it’s AMATYC, I only have **one choice** – I’m going!
- Grant – to consent to carry out for a person
Math oxymorons & verb names

- Ignite is the one breakout at AMATYC where you can be sure the room will not be fully empty.
- Flip – to toss so as to cause to turn over in the air
Math oxymorons & verb names

- I went to a Bucks game on Thursday night – I’m confident betting on them to win the east at even odds was the right move!
- Skip – to move or proceed with leaps and bounds or with a skip
Math oxymorons & verb names

- An **accurate prediction** – this conference rocks! **Fuzzy logic** at best, I know, but you’ve all been to AMATYC before – it’s **consistently random** yet always great!
- Chase – to follow rapidly
Math oxymorons & verb names

- Did you think this slide was not going to have a name at the bottom? You must have been expecting a different pattern.
- Mark – to designate as if by a mark
Math oxymorons & verb names

- Rob – to take something away from by force
- Did you think this slide was not going to have a math oxymoron because I put the name at the top? **False negative** – again, just a **different pattern**.
- Hi fellow presenter, Rob Eby!
After some random order on the last slide, we are now back to normal. I promise, no more sharp curves.

Rip – to tear or split apart or open
Math oxymorons & verb names

- You might say I’m predictably irrational due to these procedural variations. It’s that fuzzy logic again…
- Carry – to move while supporting
Math oxymorons & verb names

- I wonder how many “gifts” for us all Jon Oaks brought to this presentation? I hope the number is not a fixed variable. Nor that it is extremely average.

- Hope – to cherish a desire with anticipation
Math oxymoron & verb names

- The **one hundred percent plus** at this conference believe in **united divisions** when it comes to all branches of math. Do you?
- Sue – to seek justice or right from (a person) by a legal process
Math oxymorons & verb names

- Have you ever told your students to **multiply by a denomination** when they do currency conversion problems?
- Grace – to confer dignity or honor on
Math oxymorons & verb names

- An infinitely finite number of math educators are gathering at AMATYC next year at Spokane. (Yes, I was far yet near from using an actual math example here!)
- Carol – to sing especially in a joyful manner
Math oxymorons & verb names

- I hope this presentation was relatively perfect.
- Eddy – to cause to move in an eddy
An alternative perspective on linear regression

especially for those of us who do not teach linear regression
Standard elementary view of linear regression

The least-squares regression line minimizes the sum of squares of distances between the data points \((x_i, y_i)\) and the line.

The correlation coefficient \(r\) measures how tightly the points cluster about the straight line: the points lie exactly on the line if \(\left|r\right| = 1\).
Another interpretation: $r$ is the cosine of an angle

Pearson's correlation coefficient, $r$, is the cosine of the angle between two vectors in $n$-dimensional space.

This implies that $r = 1$ when the two vectors point in the same direction, $r = -1$ when the two vectors point in opposite directions, and $r = 0$ when the vectors are orthogonal.
From the physicist's definition of the dot product, $\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \|\mathbf{w}\| \cos \theta$, where $\theta$ is the angle between $\mathbf{v}$ and $\mathbf{w}$.

Even in dimensions greater than 3, we can define the angle $\theta$ between $\mathbf{v}$ and $\mathbf{w}$ by

$$\cos \theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\| \|\mathbf{w}\|}$$
$r$ is the cosine of an angle

Suppose we have the $n$ data points $(x_1, y_1)$, $(x_2, y_2)$, ..., $(x_n, y_n)$. We denote the associated vectors

$$
\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}
$$

and

$$
\vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}
$$

and we define $\overline{x}$ to be the mean of the $x_i$'s and $\overline{y}$ to be the mean of the $y_i$'s.
If we now define
\[
\vec{v} = \begin{pmatrix}
  x_1 - \bar{x} \\
  x_2 - \bar{x} \\
  \vdots \\
  x_n - \bar{x}
\end{pmatrix}
\quad \text{and} \quad
\vec{w} = \begin{pmatrix}
  y_1 - \bar{y} \\
  y_2 - \bar{y} \\
  \vdots \\
  y_n - \bar{y}
\end{pmatrix}
\]
then
\[
r = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} = \cos \theta
\]
If we want $r$ to measure how well $n$ ordered pairs line up in the plane, then $r$ should be unchanged if we simply translate all the points horizontally, vertically, or a combination of both.

A horizontal shift corresponds to adding a fixed value $h$ to all the $x$'s.
Why use $\vec{v}$ and $\vec{w}$ rather than $\vec{x}$ and $\vec{y}$?

That is, we should get the same $r$ if we replaced $\vec{x}$ with $\vec{x} + h \vec{1}$, where

$$\vec{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

To ignore the effect of adding scalar multiples of $\vec{1}$, we only need to pay attention to the orthogonal complement of $\vec{x}$ with respect to $\vec{1}$.
Why use $\vec{v}$ and $\vec{w}$ rather than $\vec{x}$ and $\vec{y}$?

Again from physics (or linear algebra), we can write any vector $\vec{x}$ as a sum

$$\vec{x} = \vec{x}_\parallel + \vec{x}_\perp$$

where $\vec{x}_\parallel$ is parallel to $\vec{1}$ and $\vec{x}_\perp$ is perpendicular to $\vec{1}$. 
Why use $\vec{v}$ and $\vec{w}$ rather than $\vec{x}$ and $\vec{y}$?

We can compute

$$\vec{x}_\parallel = \frac{\vec{x} \cdot \vec{1}}{\vec{1} \cdot \vec{1}} \vec{1} = \frac{\sum x_i \vec{1}}{\sum 1} = \bar{x} \vec{1}$$

$$= \begin{pmatrix} \bar{x} \\ \bar{x} \\ \vdots \\ \bar{x} \end{pmatrix}$$

and

$$\vec{x}_\perp = \vec{x} - \bar{x} \vec{1} = \vec{v}$$
Note: $\sigma_x = \frac{\|\vec{x}_\perp\|}{\sqrt{n}}$

That is, the standard deviation is $1/\sqrt{n}$ times the length of the orthogonal complement to $\vec{1}$:

$$\frac{\|\vec{x}_\perp\|}{\sqrt{n}} = \frac{\|\vec{x} - \bar{x}\vec{1}\|}{\sqrt{n}} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sigma_x$$
Finding the least squares regression line is solving an over-determined system of equations.

We try to find two parameters $m$ and $b$ so that the equations $mx_i + b = y_i$ are satisfied for all $n$ of the $(x_i, y_i)$.
Finding the least squares regression line is solving a vector equation.

We try to find two parameters $m$ and $b$ so that $m\vec{x} + b\vec{1} = \vec{y}$, but typically, $\vec{y}$ is not in the span of the vectors $\vec{x}$ and $\vec{1}$. The best we can do is to solve $m\vec{x} + b\vec{1} = \vec{y}_{\text{proj}}$, where $\vec{y}_{\text{proj}}$ is the projection of $\vec{y}$ into the plane spanned by $\{\vec{x}, \vec{1}\}$.
Finding the least squares regression line is solving a vector equation.

When we solve \( m\bar{x} + b \bar{1} = \bar{y}_{proj} \), we find that

\[
m = r \frac{\sigma_y}{\sigma_x}
\]

and

\[
\bar{y} = m\bar{x} + b
\]
$r^2$ gives the fraction of the variance in $y$ that is predicted by a linear dependence on $x$.

Because

$$m = r \frac{\sigma_y}{\sigma_x}$$

we get

$$m^2 = r^2 \frac{\sigma_y^2}{\sigma_x^2}, \text{ or } m^2 \sigma_x^2 = r^2 \sigma_y^2$$
$r^2$ gives the fraction of the variance in $y$ that is predicted by a linear dependence on $x$.

But if $y = mx+b$, then the variance in $y$ should be the variance in $x$ scaled by $m^2$, that is, the variance in $y$ should be $m^2 \sigma_x^2$.

And because we know that $m^2 \sigma_x^2 = r^2 \sigma_y^2$ we see that the variance predicted by a linear dependence on $x$ is $r^2$ times $\sigma_y^2$, the actual variance in $y$. 
"Lucky Larry" joke?


\[ Y_i - \bar{Y} = (Y_i - Y') + (Y' - \bar{Y}) \]

If we squared each \( Y_i - \bar{Y} \) and summed over all the scores, we would obtain

\[ \sum (Y_i - \bar{Y})^2 = \sum (Y_i - Y')^2 + \sum (Y' - \bar{Y})^2 \]
"Lucky Larry" joke?

Using my notation,

\[ y_i - \bar{y} = (y_i - y_{proj,i}) + (y_{proj,i} - \bar{y}) \]

Then the questionable claim becomes

\[ \|\hat{y} - \bar{y}\|^2 = \|\hat{y} - \hat{y}_{proj}\|^2 + \|\hat{y}_{proj} - \bar{y}\|^2 \]
"Lucky Larry" joke?

and

\[ \| \mathbf{y} - \mathbf{y}^1 \|^2 = \| \mathbf{y} - \mathbf{y}_{proj} \|^2 + \| \mathbf{y}_{proj} - \mathbf{y}^1 \|^2 \]

is true because (as in the Pythagorean theorem) the sum of the squares of the norms of two orthogonal vectors is the square of the norm of their sum:
"Lucky Larry" joke?

If \( \vec{c} = \vec{a} + \vec{b} \), \( \|\vec{c}\|^2 = \vec{c} \cdot \vec{c} \)

\[
\begin{align*}
&= (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) \\
&= \vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} \\
&= \|\vec{a}\|^2 + 2\|\vec{a}\|\|\vec{b}\|\cos \theta + \|\vec{b}\|^2
\end{align*}
\]

which is \( \|\vec{a}\|^2 + \|\vec{b}\|^2 \) exactly when \( \cos \theta = 0 \), that is, when \( \vec{a} \) and \( \vec{b} \) are orthogonal.
“Inspiring” Quotes

Jennifer Ackerman
Jefferson Community & Technical College
You can’t go back and change the beginning, but you can start where you are and change the ending.

C. S. Lewis

Mickie Hennig, JCTC (math)
"The person who says she can and the person who says she can't are both right."

Adaptation of Confucius quote

Dr. Michael Ackerman, Bellarmine University (math)
Attitude is the mind’s paintbrush; it can color any situation

Barbara Johnson

Laura Weinstein, Bellarmine University (advising)
“Education does not transform the world. Education changes people. People change the world.”

Jill Adams, JCTC (English)

Paulo Freire
“Nobody cares how much you know, until they know how much you care.”
—Theodore Roosevelt

adapted to:
“Students don’t care how much you know until they know how much you care.”
—John C. Maxwell

Dr. Steve Pruitt, JCTC (chemistry)
DONT GIVE UP.
DONT EVER GIVE UP.

-JIM VALVANO

Pat Riley, HCC (math)
Shoot for the Moon

Even if you miss, you’ll land among the Stars

Les Brown

Arthur Schultz, HCC (math)
EDUCATION COSTS MONEY

BUT THEN SO DOES IGNORANCE

~SIR CLAUS MOSER

Sherry McCormack, HCC (math)
If we teach today’s students as we taught yesterday’s, we rob them of tomorrow.
The struggle is real

Attributed to:
Jon Oaks

Ben Aschenbrenner, Ivy Tech (math)
"Long range planning does not deal with future decisions, but with the future of present decisions."

PETER DRUCKER

Maria Galyon, JCTC (First Year Experience)
Your education is a dress rehearsal for the life you want to lead.
-Nora Ephron

Venetia Lacy, JCTC (Deaf and Hard of Hearing Coordinator)
“Do what you can, with what you have, where you are.”
—Theodore Roosevelt

adapted to:
“Start where you are. Use what you have. Do what you can.”
—Arthur Ashe

Terry Lutz, JCTC (commercial arts technology)
“You cannot get through a single day without having an impact on the world around you. **What you do makes a difference**, and you have to decide what kind of difference you want to make.” —Jane Goodall

Virginia Hosono, University of Louisville (Japanese, Art, Study Abroad)
“Without community, there is no liberation.”
Audre Lorde
I hear and I forget
I see and I remember
I do and I understand

Confucius

Caryn Walker, Jefferson County Public Schools
(science specialist, retired)
Our Education System

“Everybody is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid.”

- Albert Einstein
“Never have so many owed so much to so few”

John J. Sinai, University of Louisville (retired physics)

Winston Churchill
“No good deed goes unpunished”

John J. Sinai
Why I Hate Email

Jon Oaks
Macomb Community College
Warren, MI
www.jonoaks.com
G-MAIL ACCOUNT?

NO, I WANT AN E-MAIL ACCOUNT
BRACE YOURSELVES

A POINTLESS EMAIL IS COMING
I got an email

asking for my email address
THERE SHE GOES

WITH ANOTHER REMINDER EMAIL
OH GOOD
ANOTHER LONG EMAIL
SURVIVED ANOTHER MEETING

THAT COULD HAVE BEEN AN EMAIL
I Don't Always Check my Email

But When I Do, I Miss Something Really Important.
I WILL FIND YOU
AND SEND YOU AN EMAIL
OH COME ON!

DID YOU NOT READ MY EMAIL!?
DID YOU GET MY EMAIL

ASKING WHETHER YOU GOT MY EMAIL?
Yeah, if you could not reply all for things that are only relevant to you

That'd be great
ONE DOES NOT RESPOND TO MESSAGES IN ALL CAPS

IT IS CALLED BAD ETIQUETTE
SENT EMAIL.....

REMEMBERED THE ATTACHMENT
I changed all my passwords to "incorrect".

So whenever I forget, it will tell me "Your password is incorrect."
WHAT'S GOING ON?

THE EMAIL BOUNCED AGAIN!
WHY ARE ALL THE EMAILS FROM PEOPLE WHO HAVE 3,500,000 FOR ME GOING INTO MY SPAM FOLDER
I LOVE EMAILS

SENDING EMAILS IS MY FAVORITE THING
How did I get this email?

Do they deliver on Sundays now?
PLEASE

NO MORE EMAIL
Thank You!

Ignite AMATYC

enlighten us, but make it quick