Incorporating Team-Based Learning in General Education Mathematics Courses
Laura Buch, Wisconsin Lutheran College
AMATYC Conference Session S029A  November 14, 2019

The following pages are excerpts of Application Activities which I have used with TBL in PreCalculus and Mathematical Problem Solving over the past 10 years.

Application Activity - Matching

Find the functions which are inverses of each other

\[ y = 2x - 3 \quad y = 3(x - 2) \quad y = \frac{1}{3}x + 2 \]

\[ y = 3x - 2 \quad y = \frac{1}{2}x - 3 \quad y = \frac{x + 3}{2} \]

\[ y = x^3 + 2 \quad y = (x - 2)^3 \quad y = (x + 3)^2, \text{ for } x \geq -3 \]

\[ y = \sqrt[3]{x - 2} \quad y = \sqrt[3]{x - 3} \quad y = \sqrt[3]{x} + 2 \]

Application Activity - Matching

Without the use of technology, determine which of the following matrices are inverses of each other.

\[ A = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 0 \\ 3 & 0.5 \end{bmatrix} \quad C = \begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix} \]

\[ D = \begin{bmatrix} -6 & -2 \\ 4 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 0.5 & 1 \\ -2 & -3 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 2 \\ -4 & 6 \end{bmatrix} \]

\[ G = \begin{bmatrix} -1 & 0 \\ 6 & 2 \end{bmatrix} \quad H = \begin{bmatrix} -0.5 & 3 \\ 0 & 1 \end{bmatrix} \quad J = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix} \]
Application Activity - Matching

The nine tiles are cut apart. Students position them on square with equivalent expressions adjacent.

Application Activity - Creating

1. Give an example of a rational function that has the given characteristic.
   a) a vertical asymptote at $x = 3$

   b) a vertical asymptote at $x = 3$ AND horizontal asymptote at $y = 2$

   c) vertical asymptotes at $x = 1$ AND $x = -1$, horizontal asymptote $y = 0$, AND $x$ intercept of 4.
Create a trigonometric function with the following characteristics:

- Amplitude is 6
- Period length is $4\pi$
- Passes through the point $(\pi, 0)$

Graph the function and label the 5 framing points with EXACT values.

Write the function in symbolic form

$$y = a \sin[b(x - h)] + k \quad \text{or} \quad y = a \cos[b(x - h)] + k$$

**BONUS:** Write another function which is equivalent to your original.
Application Activity – ASN (Always, Sometimes, Never)

<table>
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<tr>
<th></th>
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<th>NEVER TRUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\sqrt{a^2} = a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$(\sqrt{a})^2 = a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\sqrt{a} + \sqrt{b} = \sqrt{a + b}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\sqrt{a} - \sqrt{b} = \sqrt{a} - \sqrt{b}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$\sqrt{ab} = \sqrt{a} \sqrt{b}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{a}}{\sqrt{b}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$\frac{a}{\sqrt{b}} = \frac{a \sqrt{b}}{b}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$\frac{1}{a + \sqrt{b}} = \frac{a - \sqrt{b}}{a^2 - b}$</td>
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Application Activity – ASN (Always, Sometimes, Never)

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<td>1</td>
<td>The graph of a polynomial function has a $y$-intercept.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>The graph of a polynomial function has an $x$-intercept.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>The domain of a polynomial function is $(-\infty, \infty)$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>The range of a polynomial function is $(-\infty, \infty)$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>An even degree polynomial has an odd number of turning points.</td>
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Application Activity - Sorting

As a group, sort the eight angles in two different ways and record your answers below.

1. Sort by quadrant in which the terminal side of the angle is located when in standard position.


If time permits, create an additional rule by which you could sort these angles. Certainly sorting into degree vs. radian measure is one possibility, but is there something more interesting?

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<td>( \theta_G = 300^\circ )</td>
<td>( \theta_H = 315^\circ )</td>
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Application Activity - Sorting

Sort the cards into equivalent expressions, i.e. \( \arccos \left( -\frac{\sqrt{3}}{2} \right) = \frac{5\pi}{6} \)

Some cards may not be used. Record the groups you found below.

<table>
<thead>
<tr>
<th>( \sin^{-1}(1) )</th>
<th>0</th>
<th>( \sin(\pi) )</th>
<th>( \tan^{-1}(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \arctan(0) )</td>
<td>( \cos^{-1} \left( \frac{\sqrt{3}}{2} \right) )</td>
<td>( \frac{\pi}{3} )</td>
<td>( -\frac{\pi}{4} )</td>
</tr>
<tr>
<td>( \frac{\pi}{6} )</td>
<td>( \arcsin(0) )</td>
<td>( \cos^{-1}(0.5) )</td>
<td>( \arcsin(0.5) )</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>( \arccos(0) )</td>
<td>( \frac{\pi}{4} )</td>
<td>1</td>
</tr>
<tr>
<td>( \arcsin \left( -\frac{\sqrt{3}}{2} \right) )</td>
<td>( \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) )</td>
<td>( \frac{3\pi}{4} )</td>
<td>( \frac{7\pi}{4} )</td>
</tr>
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</table>
Application Activity - Sorting

Use the labels for each figure above to state which tetromino(s) would belong in each group. Give mathematical explanation for your choices.

a) Sort the tetrominos into three groups. Group 1 contains tetromino(s) which have no lines of reflective symmetry. Group 2 contains tetromino(s) which have exactly one line of reflective symmetry. Group 3 contains tetromino(s) which have more than one line of reflective symmetry.

b) Sort the tetrominos into two groups. One group contains tetromino(s) which have rotational symmetry. The other group contains tetromino(s) which do not have rotational symmetry.

Application Activity – Stations

Tasks:

A. OVERESTIMATE the cost of the groceries in this bag.

B. UNDERESTIMATE the number of candies in this container (w/o opening the container).

C. Estimate the number of students who have completed Math 110 at BLC while I have been the instructor of the course.

D. Estimate the number of students who could stand in this classroom.

E. Estimate the number of sheets of paper in this stack.

F. Estimate the number of golf balls which would fill this classroom.

Directions:

Your group will have 3 minutes to give an estimate. Record your response on the sheet provided.

You may ask ONE clarifying question which can be answered in one sentence or less by the instructor.

You may ask me nearly anything except to provide you with the answer.

The golf ball task was then assigned for the next homework in which each group measured the dimensions of the room and devised a method by which they would most accurately estimate the number of golf balls needed to fill the classroom.
Application Activity – Gallery Walk

Instructions: For the function that your group has been given, determine the information to complete the table. In some cases the correct response might be DNE (does not exist) or none. You can approximate the numerical values from the graph but also see if you can verify them analytically with the function that is given.

| Color of the graph & Type of line (solid, dotted dashed) | |
| Domain | |
| Range | |
| X-intercept(s) | |
| Y - Intercept | |
| Symmetry (Even, Odd or Neither) | |
| Intervals where function is increasing | |
| Intervals where function is decreasing | |

The functions that I used for this activity are listed below. I graphed them using Desmos Grapher in various colors and line types (solid, dotted, etc.) to distinguish each. After collecting the group work, I checked them and then returned the table minus the first row (identifying the color of the graph) to a different group. Using the clues, the group had to identify the graph which matched the description. I posted each graph on legal paper around the classroom.

\[
f(x) = \frac{3}{x^2} \quad f(x) = \frac{3x}{x^2 + 1} \quad f(x) = \frac{3x^2}{x^2 - 1} \quad f(x) = x \sqrt{5 - x^2}
\]

\[
f(x) = \sqrt{5 - x^2} \quad f(x) = 5 - 3 \sqrt[3]{x^2} \quad f(x) = \frac{3}{\sqrt{(5 - 3x)^2}}
\]

\[
f(x) = x^3 - 5x \quad f(x) = \frac{\sqrt{5-x}}{x} \quad f(x) = \frac{\sqrt{5-x^2}}{x}
\]
Application Activity – Add ‘Em Up

INSTRUCTIONS
⇒ Each group will get 4 different problems to solve. Distribute one problem to each student. At first, work individually to solve the problem.

⇒ When your group is finished solving the 4 problems, then add the 4 solutions together. Report the sum of the four solutions to me.
   - If the sum is correct, I will bring you the next set of problems. Repeat the process.
   - If the sum is not correct, check over each other’s work until you detect any errors.

Inspired by https://www.saravanderwerf.com/add-em-up-one-of-my-favorite-review-activities/

Application Activity – Which One Doesn’t Belong?

For each of the squares below, choose a number that “doesn’t belong”. State a reason why you think it is different from the others.

Be sure to use correct mathematical vocabulary!

Which number doesn’t belong?_______

Reason:

Inspired by “Which One Doesn’t Belong – A Shapes Book” by Christopher Danielson http://wodb.ca/
Application Activity – Mystery Clues

<table>
<thead>
<tr>
<th>Matrix A has more rows than columns</th>
<th>Matrix B is a square matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Every entry in row 2 of Matrix A is the number 5</td>
<td>Element $b_{2,1}$ in Matrix B is zero. (i.e. $b_{2,1} = 0$)</td>
</tr>
<tr>
<td>Element $a_{3,1}$ in Matrix A is eight (i.e. $a_{3,1} = 8$)</td>
<td>Matrix B is an identity matrix</td>
</tr>
<tr>
<td>Matrix A can be added to a 3 x 2 matrix.</td>
<td>Matrix B can be multiplied by Matrix A (i.e. 8 x A is defined)</td>
</tr>
<tr>
<td>The remaining elements of Matrix A are the number 3</td>
<td></td>
</tr>
</tbody>
</table>

I cut out and fastened the clues in each column separately into a packet so that groups revealed one clue at a time.

Application Activity – Recommendation

Section 13.4 Measures of Center
Math 110 Buch

You have to be in class one hour from now but you are hungry for pizza. Here are the several delivery times of some local pizza places (in minutes).

Manny’s: 24, 38, 31, 27, 40, 21, 34, 19

Papa Sam’s: 18, 69, 22, 27, 32, 25, 16, 25

Pizza Shack: 19, 42, 23, 42, 24, 39, 20, 25

Using the data above as a guide, from which place do you think that you should order?
Application Activity – Other

From Prime Climb board game

Based on the patterns for the numbers 1-20 that are shown in the image above, determine the coloring for the following numbers:

37  25  42  60

Application Activity – Other

Which figure on the page has:

Greatest Area_____________________

Smallest Area_____________________

Greatest Perimeter_____________________

Smallest Perimeter_____________________

Additional TBL Information and Resources

https://teambasedlearning.site-ym.com/page/started

https://commons.erau.edu/cgi/viewcontent.cgi?article=1016&context=aviasian

*Team-Based Learning: A Transformative Use of Small Groups in College Teaching* Michaelsen, Knight, Fink 2004

Feel free to use, edit, modify any of these activities to fit your courses. Hopefully they provide inspiration for additional ideas as well!

Contact me with questions [laura.buch@wlc.edu](mailto:laura.buch@wlc.edu)
Incorporating Team-Based Learning in General Education Mathematics Courses

Laura Buch, Wisconsin Lutheran College
AMATYC Conference S029A    November 14, 2019
In 2008...

Mathematical Problem Solving (MATH110)
Liberal Arts General Education course
Required for Elementary Education Majors
Initial challenges with MATH110

MN State Standards for pre-service teachers

Some material too easy/review and some material never seen before

Problem solving is a SKILL to be practiced
In 2010...

Searching to improve the course

http://www.teambasedlearning.org/
In 2017...

PreCalculus MAT120

Remediation for Calculus

Taught in fairly traditional manner
In 2018...

PreCalculus MAT120

Used modified Team-Based Learning
In 2019...

Attended the TBLC Annual Meeting

Certified in Fundamentals of TBL

http://www.teambasedlearning.org/tblc-certifications/
Overview of Team-Based Learning Approach

Before class
1. Learn with pre-work on your own
2. Quiz (forced recall)

In class: Closed book
3. Team Quiz (peer learning; practice with feedback)
4. Clarify Doubts (explain by peers; just-in-time teaching)

In class: Open
5. Team Cases (apply to solve problems)

Source: Brian O'Dwyer, Flipped Classroom Approach Forum, Hong Kong, December 8, 2017
Student’s Pre-work

Assigned reading & exercises from textbook

Web resources - Khan Academy, other

Future idea - record mini-lecture videos
IRAT - Individual Readiness Assurance Task

Multiple choice quiz

First 10 minutes of class

Approximately 2 times per week.
TRAT - Team Readiness Assurance Task

SAME Multiple choice quiz

Next 10 minutes of class

Group shares and discusses their thinking
RAT Format

Group Quiz 4
Math 110 Buch
Sections 8.1 & 8.3

Group:__________________

1. \(16.34\) centiliters is _____ liters
   A) \(16.340\) ℓ  B) \(1634\) ℓ  C) \(0.1634\) ℓ  D) \(0.01634\) ℓ

2. Anna ran 100 yards and Sara ran 100 meters in the same length of time. Who ran faster?
   A) Sara  B) Anna  C) they ran at the same speed

3. Sean takes two 250mg chewable calcium tablets each day. How many grams will he take in a week?
   A) \(3.50\) g  B) \(35\) g  C) \(350\) g  D) \(3500\) g

4. The mass of a pair of sunglasses would likely be measured in
   A) kilograms  B) grams  C) milligrams

5. A comfortable room temperature might be
   A) \(5^\circ\)C  B) \(20^\circ\)C  C) \(45^\circ\)C  D) \(70^\circ\)C

6. Which one of the following statements is true about a person’s mass if measured on Earth and then on
   the moon where the gravitational pull is about \(1/6\) as much as on Earth.
   A) the mass on the moon would be greater  B) the mass on the Earth would be greater
   C) the mass would be the same
Clarify RAT Questions

Address any unresolved questions on RAT

Email appeal of poorly worded questions
4S’s of Application

- Significant problem
- Specific choice
- Same problem
- Simultaneous reporting
My Structure for the Application Activity

Open-ended questions applying the concepts
(TBLC recommends multiple choice)

Report group work orally or written submission

Group work often reappears as individual work
(weekly homework or exam)
Find the functions which are inverses of each other

\[ y = 2x - 3 \quad \quad y = 3(x - 2) \quad \quad y = \frac{1}{3}x + 2 \]

\[ y = 3x - 2 \quad \quad y = \frac{1}{2}x - 3 \quad \quad y = \frac{x + 3}{2} \]

\[ y = x^3 + 2 \quad \quad y = (x - 2)^3 \quad \quad y = (x + 3)^2, \text{ for } x \geq -3 \]

\[ y = \sqrt[3]{x - 2} \quad \quad y = \sqrt[3]{x - 3} \quad \quad y = \sqrt[3]{x} + 2 \]
Without the use of technology, determine which of the following matrices are inverses of each other.

\[
A = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 3 & 0.5 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & -2 \\ 1 & -3 \end{bmatrix}
\]

\[
D = \begin{bmatrix} -6 & -2 \\ 4 & 2 \end{bmatrix}, \quad E = \begin{bmatrix} 0.5 & 1 \\ -2 & -3 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 2 \\ -4 & 6 \end{bmatrix}
\]

\[
G = \begin{bmatrix} -1 & 0 \\ 6 & 2 \end{bmatrix}, \quad H = \begin{bmatrix} -0.5 & 3 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}
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The nine tiles are cut apart. Students position them on square with equivalent expressions adjacent.
As a group sort the eight angles in two different ways and record your answers below.

1. Sort by quadrant in which the terminal side of the angle is located when in standard position.


If time permits, create an additional rule by which you could sort these angles. Certainly sorting into degree vs. radian measure is one possibility, but is there something more interesting?

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<td>$\theta_H = 315^\circ$</td>
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### Application Activity - Sorting

<table>
<thead>
<tr>
<th>E1</th>
<th>f(x) = x^2 + 2</th>
<th>E2</th>
<th>f(x) = (x - 1)^2</th>
<th>E3</th>
<th>f(x) = -x^2 + 1</th>
<th>E4</th>
<th>f(x) = \frac{1}{2}x^4 - 2</th>
</tr>
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<tr>
<td>E5</td>
<td>f(x) = \lvert x + 1 \rvert</td>
<td>E6</td>
<td>f(x) = \lvert 2x - 1 \rvert</td>
<td>E7</td>
<td>f(x) = \frac{1}{2}\lvert x + 1 \rvert</td>
<td>E8</td>
<td>f(x) = -2\lvert x + 2 \rvert - 1</td>
</tr>
<tr>
<td>E9</td>
<td>f(x) = -\sqrt{x + 1}</td>
<td>E10</td>
<td>f(x) = \sqrt{-x - 3} + 2</td>
<td>E11</td>
<td>f(x) = 2\sqrt{x + 2}</td>
<td>E12</td>
<td>f(x) = -\sqrt{2x - 1} + 2</td>
</tr>
<tr>
<td>E13</td>
<td>f(x) = -x^3 + 2</td>
<td>E14</td>
<td>f(x) = (x + 2)^3 - 1</td>
<td>E15</td>
<td>f(x) = -(x + 2)^3 - 1</td>
<td>E16</td>
<td>f(x) = (-x - 1)^3</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>D1</th>
<th>Reflection across y-axis</th>
<th>Horizontal Shift: Right 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>Vertical Stretch: 2</td>
<td>Horizontal Shift: Left 2</td>
</tr>
<tr>
<td>D3</td>
<td>Horizontal Shift: Right 1</td>
<td></td>
</tr>
<tr>
<td>D4</td>
<td>Vertical Shift: Up 2</td>
<td></td>
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<tr>
<td>D5</td>
<td>Reflection across x-axis</td>
<td></td>
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<td>D6</td>
<td>Horizontal Shift: Left 2</td>
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<td>D7</td>
<td>Reflection across x-axis</td>
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<tr>
<td>D8</td>
<td>Vertical Shift: Down 1</td>
<td></td>
</tr>
<tr>
<td>D9</td>
<td>Vertical Compression: \frac{1}{2}</td>
<td></td>
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<tr>
<td>D10</td>
<td>Reflection across y-axis</td>
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<td>D11</td>
<td>Horizontal Compression: 2</td>
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</tr>
<tr>
<td>D16</td>
<td>Vertical Shift: Up 2</td>
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### Graphs

- **G1** to **G16**: Graphs corresponding to the functions and transformations listed above.
Application Activity - Creating

1. Give an example of a rational function that has the given characteristic.
   a) a vertical asymptote at $x = 3$

   b) a vertical asymptote at $x = 3$ AND horizontal asymptote at $y = 2$

   c) vertical asymptotes at $x = 1$ AND $x = -1$, horizontal asymptote $y = 0$, AND $x$ intercept of 4.
Application Activity - Creating Groups

created cipher and then exchanged with another group

1) Choose your own four letter word and represent it in a $2 \times 2$ matrix, $M$. The letters are represented using $A=1$, $B=2$, $C=3$, etc. and inserted into the matrix starting at top left, top right, bottom left, bottom right.

2) Use the cipher matrix $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ to multiply $C \times M$ to create the secret matrix, $S$.

3) Use the inverse of the cipher matrix to "decode" the secret matrix using the fact that a matrix multiplied by its inverse results in the identity matrix.
Application Activity - Creating

Create a trigonometric function with the following characteristics:

- Amplitude is 6
- Period length is $4\pi$
- Passes through the point $(\pi, 0)$

Graph the function and label the 5 framing points with EXACT values.

Write the function in symbolic form

$$y = a \sin[b(x - h)] + k \quad \text{or} \quad y = a \cos[b(x - h)] + k$$

**BONUS:** Write another function which is equivalent to your original.
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<tr>
<td>6</td>
<td>$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$\frac{a}{\sqrt{b}} = a \frac{\sqrt{b}}{b}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$\frac{1}{a + \sqrt{b}} = \frac{a - \sqrt{b}}{a^2 - b}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Application Activity - Gallery Walk

<table>
<thead>
<tr>
<th>Color of the graph &amp; Type of line (solid, dotted dashed)</th>
<th>Domain</th>
<th>Range</th>
<th>X-intercept(s)</th>
<th>Y-intercept</th>
<th>Symmetry (Even, Odd or Neither)</th>
<th>Intervals where function is increasing</th>
<th>Intervals where function is decreasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orange Graph $f(x) = \frac{x^2}{x^3 + 1}$</td>
<td><img src="image1" alt="Orange Graph" /></td>
<td><img src="image2" alt="Orange Graph" /></td>
<td><img src="image3" alt="Orange Graph" /></td>
<td><img src="image4" alt="Orange Graph" /></td>
<td><img src="image5" alt="Orange Graph" /></td>
<td><img src="image6" alt="Orange Graph" /></td>
<td><img src="image7" alt="Orange Graph" /></td>
</tr>
<tr>
<td>Purple Graph $f(x) = x \sqrt{5 - x^2}$</td>
<td><img src="image8" alt="Purple Graph" /></td>
<td><img src="image9" alt="Purple Graph" /></td>
<td><img src="image10" alt="Purple Graph" /></td>
<td><img src="image11" alt="Purple Graph" /></td>
<td><img src="image12" alt="Purple Graph" /></td>
<td><img src="image13" alt="Purple Graph" /></td>
<td><img src="image14" alt="Purple Graph" /></td>
</tr>
<tr>
<td>Green Dotted Graph $f(x) = x^3 - 5x$</td>
<td><img src="image15" alt="Green Dotted Graph" /></td>
<td><img src="image16" alt="Green Dotted Graph" /></td>
<td><img src="image17" alt="Green Dotted Graph" /></td>
<td><img src="image18" alt="Green Dotted Graph" /></td>
<td><img src="image19" alt="Green Dotted Graph" /></td>
<td><img src="image20" alt="Green Dotted Graph" /></td>
<td><img src="image21" alt="Green Dotted Graph" /></td>
</tr>
</tbody>
</table>
**Application Activity - Add ‘Em Up**

**INSTRUCTIONS**

⇒ Each group will get 4 different problems to solve. Distribute one problem to each student. At first, work individually to solve the problem.

⇒ When your group is finished solving the 4 problems, then add the 4 solutions together. Report the sum of the four solutions to me.
   - If the sum is correct, I will bring you the next set of problems. Repeat the process.
   - If the sum is not correct, check over each other’s work until you detect any errors.

<table>
<thead>
<tr>
<th>Left Column</th>
<th>Right Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given that ( \cos(\theta) = \frac{35}{37} ), determine the exact value of ( \csc(\theta) )</td>
<td>Given that ( \cot(\theta) = \frac{12}{35} ), determine the exact value of ( \sec(\theta) )</td>
</tr>
<tr>
<td>Given that ( \csc(\theta) = \frac{50}{14} ), determine the exact value of ( \tan(\theta) )</td>
<td>Given that ( \sec(\theta) = \frac{50}{48} ), determine the exact value of ( \tan(\theta) )</td>
</tr>
</tbody>
</table>

Application Activity - Which One Doesn’t Belong?

For each of the squares below, choose a number that “doesn’t belong”.
State a reason why you think it is different from the others.
Be sure to use correct mathematical vocabulary!

Which number doesn’t belong?_______

Reason:

Inspired by
“Which One Doesn’t Belong – A Shapes Book”
by Christopher Danielson

http://wodb.ca/
Application Activity - Stations

Tasks:

A. OVERESTIMATE the cost of the groceries in this bag.
B. UNDERESTIMATE the number of candies in this container (w/o opening the container).
C. Estimate the number of students who have completed Math 110 at BLC while I have been the instructor of the course.
D. Estimate the number of students who could stand in this classroom.
E. Estimate the number of sheets of paper in this stack.
F. Estimate the number of golf balls which would fill this classroom.

Directions:

Your group will have 3 minutes to give an estimate. Record your response on the sheet provided.

You may ask ONE clarifying question which can be answered in one sentence or less by the instructor.
You may ask me nearly anything except to provide you with the answer.

Each group receives single task with pertinent items. Rotate every 3 minutes until complete all six tasks.
### Application Activity - Mystery Clues

<table>
<thead>
<tr>
<th>Matrix A has more rows than columns</th>
<th>Matrix B is a square matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Every entry in row 2 of Matrix A is the number 5</td>
<td>Element $b_{2,1}$ in Matrix B is zero. (i.e. $b_{2,1} = 0$)</td>
</tr>
<tr>
<td>Element $a_{3,1}$ in Matrix A is eight (i.e. $a_{3,1} = 8$)</td>
<td>Matrix B is an identity matrix</td>
</tr>
<tr>
<td>Matrix A can be added to a $3 \times 2$ matrix.</td>
<td>Matrix B can be multiplied by Matrix A (i.e. $B \times A$ is defined)</td>
</tr>
<tr>
<td>The remaining elements of Matrix A are the number 3</td>
<td></td>
</tr>
</tbody>
</table>

Cut out and fastened the clues in each column into a packet so that groups reveal one clue at a time.
Application Activity - Recommendation

Section 13.4 Measures of Center
Math 110 Buch

You have to be in class one hour from now but you are hungry for pizza. Here are the several delivery times of some local pizza places (in minutes).

Manny’s: 24, 38, 31, 27, 40, 21, 34, 19

Papa Sam’s: 18, 69, 22, 27, 32, 25, 16, 25

Pizza Shack: 19, 42, 23, 42, 24, 39, 20, 25

Using the data above as a guide, from which place do you think that you should order?
Application Activity - Other

Which figure on the page has:

- Greatest Area
- Smallest Area
- Greatest Perimeter
- Smallest Perimeter
Application Activity - Other

Based on the patterns for the numbers 1-20 that are shown in the image above, determine the coloring for the following numbers:

37  25  42  60
Practical Tips for using TBL

Group Folders

Facilitation skills

IFAT forms for TRAT
Benefits from using TBL

Active learning

“Soft” skills
(conflict resolution, inclusion, time management)

Multiple solution methods

Emphasis on process
Forming Groups for TBL

Heterogeneous
(majors, FR SO JR SR, ability, etc.)

4-5 members per group (suggested 5-7)

Seating chart

Same groups for entire semester
Forming Groups for TBL

Student Information Card:

Major

Class (FR, SO, JR, SR)

Previous Course

Extra- and Co-curriculars

Math 110 Buch
Mathematical Problem Solving
Affective Inventory

Name: _______________________________________
Date: __________________

Rate yourself using the following statements using this scale
1 – Never  2 – Rarely  3 – Sometimes  4 – Often  5 – Always

_____ I enjoy solving mathematical problems.

[Red Box] _____ I enjoy working cooperatively with others.

_____ I give up easily when I get frustrated with a problem.

_____ I check my solution to a mathematical problem.

_____ I try different strategies until I find one that works.

_____ I organize my work in a systematic way.

_____ I am willing to try a variety of problems.

[Red Box] _____ I know many different techniques/strategies for solving mathematical problems.

[Red Box] _____ I am confident in my ability to solve problems.

_____ I feel that mathematical problem solving is a useful life skill.
Feedback for Group Members

At the completion of Unit 1, all members of this class will assess the contributions that each member of their group made to the work of the group. This contribution should reflect your judgment of such things as:

- **Preparation** – Were group members prepared when they came to class?
- **Contribution** – Did all members contribute productively to the group discussion and work?
- **Respect** for other’s ideas – Did your group encourage each other to contribute ideas?
- **Flexibility** – Was your group flexible when disagreements occurred?

It is important that you identify the people who truly worked hard for the good of the group and constructively comment on ways in which the members of your group can be more successful at learning together. Your comments will be compiled and communicated *anonymously* with the group.

<table>
<thead>
<tr>
<th>Please format your comments in the following manner:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make at least 2 comments about each member of your group</td>
</tr>
<tr>
<td>Name: __________________________</td>
</tr>
<tr>
<td>:</td>
</tr>
<tr>
<td>State one problem that hinders learning within your group.</td>
</tr>
<tr>
<td>State one characteristic that helps your group the most.</td>
</tr>
</tbody>
</table>
2. Make an argument for/against using team-based learning in an upcoming semester of Pre-Calculus. In what way(s) did working together as a team enhance or detract from learning mathematics? In what way(s) did working together as a team enhance or detract from learning the skill of solving problems? Is it worthwhile to structure the course this way in order to promote learning? Give a rationale for alternative formats to the current structure to change or improve the course.

***I am interested in your reflective and constructive feedback. You will not be graded on how well I agree with your response. Rather, I am looking to improve the course and my instruction – so any/all well-thought out comments (both positive and negative) are appreciated.***
“if you don’t understand a concept, one of your group members can help you understand and teach you. There are a lot of times that I used this to teach other people how to do a concept and many times that group members taught me.”

“You can see that you can solve a math problem in many different ways and you learn which one works better for you.”

“It was easier to ask them the question than it is to ask in front of the whole class.”
“We were focused on math but still had a fun time and got to know each other.”

“It works because everyone remembers something different from high school and is able to lead the group.”

“It allows for cooperative communication on problems that are difficult to solve.”

“It helps me remember the ways to solve a problem when I have to explain it to another person.”
Student Comments: PreCalculus

“I am not in support of team-based learning simply because students learn at different pace than others... Moreover, if the group as a whole is struggling on a specific topic, we cannot help each other.”

“There were many times when my group mates did not put in any effort to learn the material prior to quizzes.”

“Difficult to learn if the group cannot stay on task and becomes distracted.”

“I did not take as many notes as I would have liked to.”
Future Plans for TBL in My Courses

MAT118 Modelling Approach to College Algebra
MAT119 Trigonometry

Pre-work teaching videos/enhanced textbook

Directly address detrimental group behavior
Thank you for this opportunity to share!

Slides & some handouts in AMATYC app

Contact me with questions
laura.buch@wlc.edu

Enjoy your visit to MKE!