Mathematics and its Application for Careers Community

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MAC-AMATYC

• Technical Math Committee
  • Pittsburgh, 1999
• Math for AAS Programs
• Math and its Applications for Careers (MAC)

• Stigmas
  • Vo-Tech; Tech Math
• Offerings
  • AAS Degree
MAC-AMATYC

• Applications
  • Real Apps
• Original Themed Session
  • Allied Health/Health Sciences
• Grants & Other Outside Opportunities
  • CRAFTY (NSF)
  • Needed Math
  • MAA PIC MATH (NSF and NSA)
• Non-Traditional Students
Traditional Careers

• Manufacturing; Drafting; CAD/CAM
• Horticulture
• Culinary; Hospitality
• Fashion
• Welding
• Fire Fighting (Wilderness, Other)
• Many, many more
  • Clackamas CC has 30 degree programs that allow the math requirement to be satisfied by Technical Math
Teaching Math - The First Two Years in a Four Year College

• Gen Z going to college online (with no Gen Z students)
• Many non traditional students
  • Military transition to civilian careers
  • Returning (or new) student who haven’t had a math class in 10 or more years
• Online in 9 week terms with 5 major terms a year and 7 additional terms; all time zones
• Typical degrees
  • Aeronautics
  • Aviation Maintenance
  • Aviation Business Administration
  • Engineering Fundamentals
  • Technical Management
Math Skills Needed in the Future Workplace

• New job descriptions appearing every day
  • What other math related skills should we prepare them for?
    • Problem modeling and solving
    • Analytical thinking
    • Critical thinking
    • Decision analysis
    • Reading and constructing charts and graphs
    • Logic
    • Collaboration
    • Break down large or complicated problems into smaller manageable pieces
    • Attention to detail
    • Learning how to Learn (technical reading texts and industry publications)
    • Mathematical Communication
“New-ish” Careers
Commercial & Technical Diving

- Gobs of Jobs
- Salary
  - Start: $55K
  - Experienced: $95K
- Growth: 37% over the next 10 years
- Education: Associate’s Degree or Degree Certificate (specialty school)
- Math
  - Welding (esp UW); Logs/Exp
Haldane’s Model

John Scott Haldane
UK Royal Navy; 1905

Determined that logarithmic functions could be used to construct a reasonable model for nitrogen absorption by cells under additional pressure.

Fig. 1. Curve showing the progress of saturation of any part of the body with nitrogen after any given sudden rise of air pressure. The percentage saturation can be read off on the curve, provided the duration of exposure to the pressure, and the time required to produce half-saturation of the part in question, are both known. Thus
Traditional Careers, Modernizing
Allied Health & Health Sciences

• Medical Technicians and Licensed Nurse Practitioners (LPNs)
  • X-Ray Tech; Phlebotomist; Radiologist Tech; Dialysis Tech; EMTs
  • Gobs of Jobs; Specialties within each category

• Salary
  • Start: $35K to $65K or more

• Growth: 10-25% over the next 10 years (depending on sub-field)

• Math
  • Proportions; Percents; Algebraic Manipulation; Measurement; Geometry; Reading Graphs; Statistics & Probability
Projects for Students

• Renewable Energy Technology (RET)
  • Wind Turbines
    • Teach Me
Projects for Students

• Horticulture
  • Two Projects
    • Fertilizer Analysis
      • Ratios & Proportions; Geometry & Measurement
      • Technical Communication
    • Tree Counts & Statistical Surveys
      • Geometry & Measurement
      • Statistics & Probability
      • Fun Spring Term Project
Projects for Students

• Geometry & CNC
  • Really Cool Geometry
    • Area of a Polygon
  • CNC
    • Discipline-specific project
Projects for Students

Critical thinking in a business transaction
  • Present Value of money in a car purchase
Projects for Students

Payments and Present Value of Larger Loan
Projects for Students

• OMG: I don’t know squat about (choose your favorite tech field)
  • Teach Me
  • Students Create a Project
  • Instructor Investigation Through Partnership With a Company
Projects in Online Classes

• The standard assign groups
• Using the discussion board for mini projects
Thank You

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• Join Us
  • MAC Committee Meeting
    • Friday, 4:15-5:45 in 203AB
  • Focus on a Function Presentation: Trig Functions (S131)
    • Saturday, 11:55-12:45 in 202C
Group Project: Geometry & CNC

Part I: Computing the Area of a Polygon
Recall the definitions of polygons and composite figures from MTH-050.

Definition: Polygon
A polygon is a simple closed figure with three or more sides in which each side is a line segment.

Definition: Composite Figure
A composite figure is a geometric figure which is formed by adjoining two or more basic geometric figures.

In MTH-050, you learned to find the area of such a figure by decomposing it into simpler figures such as triangles and rectangles.

Note: In MTH-050, some composite figures included parts of circles. The method that will be shown here requires the figure to have sides that are line segments.

A different method that can be used with polygons involves positioning the figure in a rectangular coordinate system and identifying the vertices.

Note: If you have never worked with a coordinate system, like the one shown below, or if you would like a refresher, see the supplement ProjGeomSupp M080.

Consider the triangle shown.

Exercises
1. Identify the coordinates \((x, y)\) of each vertex of the triangle shown.
2. Compute the area of the triangle using any convenient method.

Note: You can find the length of the sides and use Heron’s formula or you can name a side the base and find the height with that side as the base.

In the first exercise, you should have identified the vertices as \((2,1), (1,7),\) and \((3,4)\). We can use these coordinates and an easy algorithm to find the area of this triangle.
**Step by Step: Finding the Area of a Polygon**

**Step 1** List the coordinates in a column. Begin at one vertex and list the remaining vertices by rotating counterclockwise around the figure. Complete the list by writing the first vertex at the end of the list.

**Step 2** Draw an arrow from the $x$-coordinate of the first vertex (on top) through the $y$-coordinate of the second vertex (on the second line).

**Step 3** Next, draw an arrow from the $x$-coordinate of the second vertex through the $y$-coordinate of the third vertex. Continue drawing these arrows until you have drawn an arrow through the $y$-coordinate of the bottom vertex.

**Step 4** Multiply each pair of numbers indicated by the arrows and write each product near the arrowhead. These are called the right-diagonal products.

**Step 5** Find the sum of the right-diagonal products.

Using the triangle from before, and beginning at the vertex $(2,1)$, we begin to apply the algorithm.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Steps 2–3</th>
<th>Step 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(2,1)$</td>
<td>$(2,1)$</td>
<td>$(2,1)$</td>
</tr>
<tr>
<td>$(3,4)$</td>
<td>$(3,4)$</td>
<td>$(3,4)$</td>
</tr>
<tr>
<td>$(1,7)$</td>
<td>$(1,7)$</td>
<td>$(1,7)$</td>
</tr>
<tr>
<td>$(2,1)$</td>
<td>$(2,1)$</td>
<td>$(2,1)$</td>
</tr>
</tbody>
</table>

**Step 5**

$8 + 21 + 1 = 30$

We continue with Step 6.

**Step 6** Rewrite the list of vertices from Step 1.

**Step 7** Draw an arrow from the $y$-coordinate of the first vertex (on top) through the $x$-coordinate of the second vertex (on the second line). Continue sketching arrows down the list of vertices until you reach the bottom vertex.

**Step 8** Find the sum of the left-diagonal products, as indicated by the arrows.

<table>
<thead>
<tr>
<th>Step 6</th>
<th>Step 7</th>
<th>Step 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(2,1)$</td>
<td>$(2,1)$</td>
<td>$(2,1)$</td>
</tr>
<tr>
<td>$(3,4)$</td>
<td>$(3,4)$</td>
<td>$(3,4)$</td>
</tr>
<tr>
<td>$(1,7)$</td>
<td>$(1,7)$</td>
<td>$(1,7)$</td>
</tr>
<tr>
<td>$(2,1)$</td>
<td>$(2,1)$</td>
<td>$(2,1)$</td>
</tr>
</tbody>
</table>

**Step 8**

$3 + 4 + 14 = 21$
We complete the algorithm with two final steps.

**Step 9**  Subtract the smaller diagonal product from the larger diagonal product.

**Step 10**  Halve your result from Step 9. This is the area of the polygon.

**Step 9**  
30 – 21 = 9

**Step 10**  
Area = \( \frac{9}{2} \) sq units = 4.5 sq units

This is the area you should have computed in the second exercise.

**Exercises**

3. Use this method to find the area of the trapezoid.

4. Use the formula \( A = \frac{1}{2} h(b_1 + b_2) \) to find the trapezoid’s area. How does your compare to your answer from Exercise 3?

*Use this new method to find the area of each polygon shown.*

5.

6.
Part II: Computer Numerical Control (CNC)

Computer numerical control, or CNC, is a language that allows a machine to cut complex patterns into a product. The key to this language is the rectangular coordinate system. The design is created by entering a series of points into the machine. The machine then cuts from one point to the next to create a pattern.

Exercises

1. On a piece of graph paper, carefully print your name using only straight-line segments. Adjust the lines so that the line segments start and end at intersection points on the graph paper.

2. Label the bottom left corner of the graph paper \((0,0)\). This will be the origin of your design and ensures that all of the coordinates are positive numbers.

3. Label the coordinates at the end of every line segment.

4. Create a list of the line segments that need to be created. Name each line by its endpoints (for instance, \((3,4),(3,12)\) names a particular vertical line segment.

Often, two rates are used to describe the time required to cut a pattern: Cutting speed and feed rate. Cutting Speed describes the speed at which a cutting tool moves through the material. It is given as distance per time – usually surface feet per minute.

Feed Rate describes the speed based on the cutting tool. For instance, if a cutting tool rotates, then the feed rate is given by the distance cut with each revolution of the tool. The units are often inches per revolution.

The time required to cut a pattern is determined by the length of the cut, the material being cut, and the size of the cutting tool.

For a rotating tool, the cutting speed can be calculated using the formula

\[
V = \frac{\pi \cdot d \cdot n}{12}
\]

In this formula, \(d\) is the diameter of the cutting tool (inches), \(n\) is the speed of the cutting tool (in revolutions per minute), and \(V\) is the cutting speed in surface feet per minute.
Exercises
5. When cutting aluminum with a \( \frac{1}{4} \)-in. diameter tool, the cutting speed is given as 1,000 surface feet per minute. Find the speed of the cutting tool (in revolutions per minute).

6. Using your name, mapped out on graph paper (from earlier), calculate the length of each line segment in your name.

7. Find the total length of the pattern to be cut.

8. If the feed rate is \( \frac{1}{128} \) in. per revolution, how long will it take to cut one inch in the aluminum, using the tool in Exercise 5?

9. If your pattern (name) were to be cut into aluminum with this tool, how long would it take to cut?

In fact, you also need to consider the time it takes for the cutter to move when it is not cutting into the material.
One type of movement is engaging and disengaging from the material. For instance, to switch from one letter to the next, the tool needs to disengage from the first letter and then engage with the next letter.

10. Count the number of times the tool will need to engage or disengage in your pattern.

11. If it takes 1.5 sec to engage or disengage, how much time does this add to the cutting time?

The other type of movement is simply movement from one cut to the next. Since it is not cutting during this movement, the tool travels faster. While disengaged, this tool moves at a rate of 20 ft per minute.

12. Calculate the distance the tool needs to move while disengaged from your pattern. How long will it take to move the total non-cutting distance?

13. What is the total time required for cutting your name into the piece of aluminum?
All fertilizers have three numbers on the label, which indicate the fertilizer analysis, or "percentage by weight" of nitrogen, phosphate (P$_2$O$_5$) and potash (K$_2$O), always in that order. The bag of lawn fertilizer, pictured to the right, has 25% nitrogen (N), 5% phosphate, and 5% potash. P is the symbol for the element phosphorous, K is the symbol for the element potassium, and O is the symbol for the element oxygen.

**Nitrogen (N)** is the main nutrient for new, green growth. Plants that are almost all leaf (such as lawn grasses) need plenty of nitrogen, so the first number is especially high in fertilizers for lawns because grass must continuously renew itself after mowing. The higher the number, the more nitrogen the fertilizer provides.

**Phosphate (P$_2$O$_5$)** Contains phosphorous, which promotes root development to help strengthen plants. It also increases blooms on flowers. Lots of phosphorous is great for bulbs, fruit development, perennials, and newly planted trees and shrubs. They depend on strong roots, so fertilizers meant for these plants often have high middle numbers.

**Potash (K$_2$O)** Contains potassium, which improves the overall health of plants. It helps them withstand very hot or cold weather and defend against diseases. Most soils already have some potassium, so the third number in the fertilizer analysis is usually smaller than the other two. Fertilizers for some tropical plants, especially palms, contain extra potassium because these plants have a special need for it. Fertilizers meant for fall, such as *Winterizer*, also contain extra potassium to help prepare plants for cold weather.

Other nutrients are often found in fertilizer, but usually not in as large amounts as nitrogen, phosphate and potash (sometimes referred to as N, P, and K). These nutrients can include calcium (Ca), magnesium (Mg), sulfur (S), iron (Fe), and other trace elements.
The three numbers for fertilizers form ratios which indicate the relative amounts of the nutrients to each other. In our picture above, the 25-5-5 lawn fertilizer would have a 5-1-1 ratio (divide each number by 5 to simplify this ratio). That is, there is a 5:1 ratio of nitrogen to phosphate (there is 5 times as much nitrogen as phosphate) and a 5:1 ratio of nitrogen to potash and a 1:1 ratio of phosphate to potash (they have the same amounts in the bag).

Ratios can be helpful when looking for the "right mix" for a certain type of plant or situation. For example, vegetable gardens often call for a 1-2-1 ratio. But, you probably won’t find a bag of fertilizer with those three numbers on it. You can probably find a bag of fertilizer which has a ratio of 5-10-5 or 10-20-10. These bags have the nutrients in the same ratio. Bags with smaller numbers indicate that the fertilizer is less concentrated than in bags with bigger numbers. That means those with larger numbers on the label could be applied at lower amounts to yield the same results. In other words, 5 lbs of a 20-20-20 fertilizer would yield the same amount of actual nutrients as 10 lbs of a 10-10-10 fertilizer.

The advantage of a bag of fertilizer with larger numbers comes largely in transportation and storage costs savings: A smaller bag of more concentrated fertilizer takes up less space and is easier to carry around than a bigger bag with the same ratio (but smaller numbers).

Most trees like a 2-1-1 ratio, which would be a fertilizer product such as 10-5-5 or 20-10-10. Many lawns prefer a 3-1-2 ratio fertilizer, so a fertilizer product with 30-10-20 on the label would be a good ratio match. To really determine the amount of fertilizer you might need for a particular job, you should test the soil.

1. For the fertilizer shown in the photo below, how many pounds of nitrogen, phosphate and potash are there? How many pounds of filler are there?

![22-3-14 fertilizer photo]

2. A bag of fertilizer at Home Depot is labeled: 4-lb bag Scotts 20-27-5 Starter Fertilizer; 5,000 sq ft; $11.98 each. You just resodded your lawn and the salesman at Home Depot says this is the fertilizer your new lawn needs. Your lawn is 7,400 sq ft and you plan on fertilizing it twice this year. How many bags should you buy for the year?

3. In the past you fertilized your lawn every spring with 5 pounds of 36-6-6. You could only find 18-3-3 at the store this spring. How many pounds of 18-3-3 should you use on your lawn to get the same results?

4. How much of a 2-2-4 organic fertilizer do you need to get the same amount of potash as if you were using 15 pounds of a 15-10-12 chemical fertilizer?
5. A soil test on a large flowerbed comes back with the recommendation that it receive fertilizer in a 3-2-4 ratio. Your local home garden store only has the following fertilizers: 12-6-18, 12-8-14, 21-14-28, 21-9-13, 27-18-32, and 24-16-32. Which of these fertilizers could you buy?

6. Knowing how much nitrogen is in a bag of fertilizer is fairly easy. The percentage is listed right on the bag. To determine the actual amount of the element phosphorus (P) that is in phosphate (P₂O₅) and how much the element potassium (K) is in potash (K₂O) you have to do a little math. Use these formulas: 44% of P₂O₅ is phosphorous, and 83% of K₂O is potassium. If you have a 50 lb bag of 20-5-10 fertilizer, how many pounds of nitrogen, phosphorus and potassium are there in the bag?

7. The putting greens on a 9-hole golf course total 14,000 sq ft. The bag of fertilizer shown below is 40 pounds. If you need to apply nitrogen to the greens at a rate of 1 pound of nitrogen per 1,000 sq ft, how many bags of this fertilizer should you buy?

8. A plot of land in South Florida is 100 ft wide and 200 ft long. The 200-ft length lies along a canal that eventually drains into the Everglades. Knowing that runoff of phosphate-rich fertilizer is a major source of groundwater pollution, you decide not to spread the fertilizer within 10 ft of the water (your spreader has a deflector shield that keeps the spread pattern semicircular). Using a 20-2-6 slow release fertilizer at a rate of 1 lb nitrogen per 1,000 sq ft, how many pounds of fertilizer do you need to do the job?

9. A homeowner buys a bag of fertilizer with a label that says to apply 2 lb of the packaged product per 1,000 ft² of surface area. If the area of the homeowner’s lawn to be treated totals 1,575 ft², how much product will the homeowner need to apply?

10. A gardener applies a dry 20-10-20 fertilizer to her flowerbeds, using 2 pounds of potash per 1,000 square feet, 2 times a year. If a 50 lb bag of this fertilizer costs $9.50, how much is this gardener paying a year for this application if her flowerbeds total 1,450 ft²?
Project II: Group

Topic: Tree Counts and Statistical Surveys

Recall that 640 acres = 1 mi².

1. If an acre were square-shaped, what would the length of a side be (in feet)?

*Note*: Your answer is a definition not a measurement – that is, it is an exact values (or decimal approximation, based on definitions not measurements).

2. Convert your answer in (1) to meters.

Go to a park or other wilderness area containing a large stand of trees (at least two acres).

3. What is the overall size of the stand of trees in the area found?

4. Measure off a square area, 20 feet per side, and count the number of trees in the square. Record your counts for each different type of tree in the square.

5. Repeat Exercise (4) for a second and third square area (20 feet per side).

6. How do the overall tree-count totals in each of the three areas compare?

7. How does the tree-type distribution in each of the areas compare?

8. Find the average tree-count (total, and by type) for the three areas surveyed.

9. How many squares, 20 feet per side, are in an acre?

10. Extrapolate from your answer to exercises 8-9 to give an estimate of the total number of trees in an acre and the distribution, by type.

11. Estimate the total number, and number of each type, of trees in the whole stand.

12. Write two or three paragraphs discussing which answers in exercises 1-11 are exact values and which are approximations. Discuss your approximations in terms of precision and accuracy.

13. Choose one tree and measure the circumference of its trunk. Record your results to the proper precision, based on your measuring device. Record the height at which you measured the circumference of the tree.

14. Estimate the height of the tree. Report your estimate, and write a paragraph discussing the method used to determine the height of the tree.

15. Use the results of Exercises 13-14 to estimate the total volume of wood in the trunk of the tree.
Buying a car – Present value of money
Present Value of money

• Present Value can be described as:
  • The amount invested today to reach a specific goal in the future
• PV = present value of all payments
• PMT = periodic payment
• i = rate per period
• n = number of periods

\[ PV = PMT \frac{1 - (1 + i)^{-n}}{i} \]
Present Value of a Loan

• A car company is offering five year loans at 6% compounded monthly. If you can afford monthly payments of $200, how much can you borrow? What is the total interest you will pay for this loan?

• What do we know?
  • PMT = $200
  • r = rate = 6% or 0.06
  • m = compounding periods = 12
  • i = 0.06/12 = 0.005
  • n = 5 years * 12 months per year = 60 payments
  • PV = present value which is missing since problem asks how much can we borrow now
Present Value of a Loan

• What do we know?
  • PMT = $200
  • \( i = \frac{0.06}{12} = 0.005 \)
  • \( n = 5 \text{ years} \times 12 \text{ months per year} = 60 \text{ payments} \)

• How much can we borrow?

\[
PV = (PMT) \frac{1-(1+i)^{-n}}{i}
\]

\[
PV = (200) \frac{1-(1+0.005)^{-60}}{0.005}
\]
Present Value of a Loan

\[ PV = (200) \frac{1 - (1.005)^{-60}}{0.005} \]

First, simplify the numerator. Using excel with the function power \((1.005, -60)\) we get 0.741372196

Completing the numerator we compute
\[ 1 - .741372196 = 0.258627804 \]

Our new expression is

\[ PV = 200 \left( \frac{0.258628}{0.005} \right) \]
Present Value of a Loan

\[ PV = 200 \left( \frac{0.258628}{0.005} \right) \]

Now divide the fraction

\[ PV = 200(51.72556) \]

Therefore, we can borrow $10,345.11 today.

The last step is to multiply by 200 to get the PV

\[ PV = $10,345.11 \]
Present Value of a Loan

• What is the total interest you will pay for this loan?
• Total Interest = (# of payments * payment) – initial loan

Total Interest = (60 * 200) – 10,345.11
Total Interest = 12,000 – 10,345.11
Total Interest = 1654.89

• Therefore, you will pay a total of 1,654.89 in interest after 60 months.
Present Value of a Loan- Follow up questions

• Compare answers to class mate. Are they the same? Why or why not?
• Did you use your technology correctly to get the most accurate answer?
• What experience do you have purchasing a vehicle?
• What other factors must you take into consideration before committing to the purchase?
• What is the advantage to the buyer in this setting?
• How may the sales person try to change your mind?
• Does knowing the total interest paid prior to purchase have an effect on your decision?
• ?
• ?
Payments on Compound Interest Loan

You find that a small business loan in the amount is the amount you need to purchase a restaurant location. After researching banks to find the best interest rate, you find the best rate of 9% interest that compounds monthly for 7 years.

* Could have students do the research online as part of the project

1. What is the monthly payment for the loan? Show the formula and values used for each variable to calculate the monthly payment.

2. What is the unpaid balance of the loan at the end of the first year? Show the formula and values used for each variable to calculate the unpaid balance at the end of the first year.

3. What is the unpaid balance of the loan at the end of the 6th year? Show the formula and values used for each variable to calculate the unpaid balance at the end of the 6th year.
Present Value of money

• Present Value can be described as:
  • The amount invested today to reach a specific goal in the future
• PV = present value of all payments
• PMT = periodic payment
• i = rate per period
• n = number of periods

\[ PV = PMT \frac{1 - (1 + i)^{-n}}{i} \]
Present Value of Money

The formula is the same as PV but solved for PMT

*Include showing the formula manipulation to solve for PMT as part of the mathematical communication skills.

\[
PMT = PV \frac{i}{1 - (1 + i)^{-n}}
\]
To find “i” and “n” for the above equation, use the following equations:

\[ i = \frac{r \text{ (rate)}}{m \text{ (compounding periods)}} = \frac{0.09}{12} = 0.0075 \]

\[ n = m \text{ (compounding periods)} \times t \text{ (time in years)} = 12 \times 7 = 84 \]

payments
Input the values into the equation:

\[ PMT = PV \left[ \frac{i}{1 - (1 + i)^{-n}} \right] \]

\[ PMT = 50,000 \left[ \frac{0.0075}{1 - (1 + 0.0075)^{-84}} \right] \]

\[ PMT = 50,000 \left[ \frac{0.0075}{1 - (1.0075)^{-84}} \right] \]

\[ PMT = 50,000 \left[ \frac{0.0075}{1 - 0.533845266} \right] \]
Solution part 1 continued

\[ \text{PMT} = 50,000 \left[ \frac{0.0075}{0.4661547} \right] \]

\[ \text{PMT} = 50,000[0.01608908] \]

\[ \text{PMT} = 804.45 \]
Solution part 2

The unpaid balance, or present value “PV”, of the loan at the end of the first year will be $44,628.35

This is determined using the following formula where

“PMT” is the monthly payment calculated above, “i” is the rate per period as calculated above, and “n” is the number of periods remaining on the loan after one year or 72:

\[ PV = PMT \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \]
Solution part 2 continued

Input the values into the equation:

\[ PV = PMT \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \]

\[ PV = 804.45 \left[ \frac{1 - (1 + 0.0075)^{-72}}{0.0075} \right] \]

\[ PV = 804.45 \left[ \frac{1 - (1.0075)^{-72}}{0.0075} \right] \]

\[ PV = 804.45 \left[ \frac{1 - 0.583923634}{0.0075} \right] \]
Solution part 2 continued

\[ PV = 804.45 \left[ \frac{0.4160764}{0.0075} \right] \]

\[ PV = 804.45 [55.476853] \]

\[ PV = 44,628.35 \]
Solution part 3

The unpaid balance, or present value “PV”, of the loan at the end of the sixth year will be $9,198.82

This is determined using the following formula where

“PMT” is the monthly payment calculated above, “i” is the rate per period as calculated above, and “n” is the number of periods remaining on the loan after 6 years or 12.

\[ PV = \text{PMT} \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \]
Solution part 3 continued

Input the values into the equation:

\[ PV = PMT \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \]

\[ PV = 804.45 \left[ \frac{1 - (1 + 0.0075)^{-12}}{0.0075} \right] \]

\[ PV = 804.45 \left[ \frac{1 - (1.0075)^{-12}}{0.0075} \right] \]

\[ PV = 804.45 \left[ \frac{1 - 0.914238155}{0.0075} \right] \]
Solution part 3 continued

\[ PV = 804.45 \left( \frac{0.08576184}{0.0075} \right) \]

\[ PV = 804.45[11.434912] \]

\[ PV = 9,198.82 \]
Additional ideas

1. Have students research the loan information for a given purchase.
2. Emphasize the technology use so rounding doesn’t get out of hand.
3. Consider using Excel or similar to do the computations as another technology form.
4. Discuss the cost of borrowing money— the total paid and interest amounts
**Project II: Individual**

**Topic: Wind Turbines**

East of Clackamas CC, both sides of the Columbia River Gorge are dotted with wind turbines. In this project, you will have the opportunity to explore aspects of these wind turbines.

Write a brief “pamphlet,” at the level of an educated layperson, describing how wind turbines generate energy. Be sure to summarize the difference between a windmill and a wind turbine. Add information about the specifications of the wind turbines used in the Columbia River Gorge, including their size, capacity, rotation angle (that is, direction that each turbine faces), and other useful information. If there is more than one type of turbine, describe each type.

How many wind turbines are there on the Oregon side of the Columbia River? The Washington side? Compare the energy generation levels of the Oregon and Washington turbines.

If possible, take a drive out to the wind turbines along the Gorge. Describe anything relevant from your experience.

Discuss the rotation of each set of turbines in sequence. That is, in a series of turbines, each successive turbine faces a slightly different direction than the turbine that preceded it. Discuss the reasons for this; include cons, as well as pros, for setting them up this way. How much additional energy is generated because of this, versus having all of the turbines facing in the same direction?

How does the rate of air flow affect the power generation capacity of the wind turbines along the Columbia River Gorge? What is the average air flow through the Gorge? What does this translate to in terms of expected power generation levels?

Finally, discuss the initial costs of the turbines and any ongoing costs of operation and compare this to their energy generation levels. How this compare to fossil-fuel-based energy generation?

Please feel comfortable making suggestions about how to improve this project topic.